[1.] Consider the ODE system of simple electric circuits

$$\frac{d}{dt}\begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} I \\ V \end{pmatrix}, \quad t \geq 0$$

with positive constants $L, R, C > 0$. State and simplify the forward Euler and backward Euler methods for these equations (show that both schemes satisfy an explicit linear iteration scheme of the form $y_{n+1} = A_n y_n$ with appropriate matrices A_n, resp. and $y_n = (I_n \ \ V_n)^T$.)

[2.] Consider the forward Euler method for $dx/dt = 1 - t + Lx$ with $x(t_0) = x_0 \in \mathbb{R}^1$, $t \geq t_0$.
(a) Show that the Euler scheme (FEM) is of the form $y_{n+1} = (1 + Lh_n)y_n + h_n - h_n t_n$, $n \in \mathbb{N}$.
(b) Assuming uniform step sizes $h = (t - t_0)/n$ and $L = 1$, show by induction on n that $y_n = (1+h)^n(y_0 - t_0) + t_n$ for FEM.
(c) Let $y_0 = x_0$. Prove that $\lim_{n\to+\infty} y_n = x(t)$ for FEM. Hints: You may use the fact $\lim_{n\to+\infty}(1 + a/n)^n = e^a$. Moreover, for the error analysis, show first that

$$x(t) = (x_0 - t_0)e^{t-t_0} + t, \quad t \geq t_0.$$