

Do as many problems or their parts as you can. You may not be able to complete all the problems within the given time. Partial credit will be given only in Problem 5 and in no other problem.

1. Let P , Q and R be the following propositions.

P : "You get an A in the examination."

Q : "You do every problem in your text book."

R : "You get an A in this course."

Express each of the following statements using P , Q , R and logical operators. (5 points/part)

(a) You get an A in the course, but you do not do every problem in your text book.

(b) You get an A in the examination, you do every problem in your text book, and you get an A in this course.

(c) To get an A in the course, it is necessary for you to get an A in the examination.

(d) You get an A in the examination, but you do not do every problem in your text book; nevertheless, you get an A in the course.

(e) Getting an A in the examination and doing every problem in your text book is sufficient for getting an A in this course.

(f) You get an A in this course if and only if you either do every problem in your text book or you get an A in the examination.

2. Using truth tables or otherwise classify each of the following propositions as a tautology, contradiction or neither. (5 pts/part)

(a) $P \rightarrow \bar{P}$

(b) $\bar{P} \vee (P \rightarrow Q)$

(c) $(P \rightarrow Q) \wedge (\bar{P} \rightarrow Q) \leftrightarrow Q$

(d) $(\bar{Q} \wedge (P \rightarrow Q)) \rightarrow \bar{P}$

(e) $\overline{(P \oplus Q)} \leftrightarrow (P \leftrightarrow Q)$

(f) $P \wedge \overline{(P \vee Q)}$

3. Let $P(x,y)$ be the statement that "x is a contestant on y". Express each of the following statements using $P(x,y)$, logical operators and quantifiers. The universe for x is the set of all students and for y is the set of all quiz shows on television. (6 pts/part).

- (a) There is a student who is a contestant on a television quiz show.
- (b) There is a student who is a contestant on Wheel of Fortune and on Jeopardy.
- (c) No student is a contestant on a television quiz show.
- (d) Every television quiz show has a student as a contestant.
- (e) At least two students are contestants on Wheel of Fortune.

4. Let $P(x)$ denote "x is prime," $Q(x)$ denote "x is even," and $R(x)$ denote "x is divisible by 6". The universe for both x and y is the set of natural numbers. Determine the truth values of the following. (5 pts/part)

- (a) $\exists x (P(x) \wedge R(x))$
- (b) $(\forall x P(x)) \rightarrow (\forall x \overline{Q(x)})$
- (c) $\forall x (P(x) \rightarrow \overline{Q(x)})$
- (d) $\exists x \exists y ((x=y) \rightarrow (x>y))$
- (e) $\forall x \exists y ((x=y) \rightarrow (x>y))$
- (f) $\exists x \forall y ((x=y) \rightarrow (x>y))$

5. Prove or disprove the following propositions.

- (a)(i) (3 pts) If p and q are prime numbers, then $pq+1$ is a prime number.
- (ii) (3 points) Every even number can be written as the sum of an odd number and a perfect square.
- (b) (6 pts) If x and y are positive integers with $x>y+1$, then x^2-y^2 is not prime.
- (c) (6 pts) The product of two odd numbers is odd.
- (d) (6 pts) The product of a nonzero rational number and an irrational number is irrational.
- (e) (6 pts) If x is the product of four distinct prime numbers, then $x > 200$.