5.5 Exercises

CONCEPTS

Fundamentals

1. When modeling data we make a ______________ plot to help us visually determine whether a line or some other curve is appropriate for modeling the data.

2. If the y-values of a set of data increase and then decrease, then a ______________ function may be appropriate to model the data.

Think About It

3–4 ■ The following data are obtained from the function $f(x) = (x - 3)^2$.

x	0	1	2	3	4	5	6	7	8
$f(x)$	9	4	1	0	1	4	9	16	25

3. If you use your calculator to find the quadratic function that best fits the data, what function would you expect to get? Try it.

4. Find the *line* of best fit for this data. Graph the line and a scatter plot of the data on the same screen. How well does "the line of best fit" fit the data?

SKILLS

5–8 ■ A set of data is given.

(a) Make a scatter plot of the data. Is it appropriate to model the data by a quadratic function?

(b) Use a graphing calculator to find the quadratic model that best fits the data. Draw a graph of the model.

(c) Use the model to predict the value of y when x is 7.

5.

x	0.1	0.8	0.8	1.2	1.7	2.3	2.6	3.1	3.3	3.4	3.9	4.1	5.2	5.9
y	101.2	106.7	105.2	110.1	112.7	114.6	113.3	113.1	109.1	110.4	109.2	107.1	97.6	95.5

6.

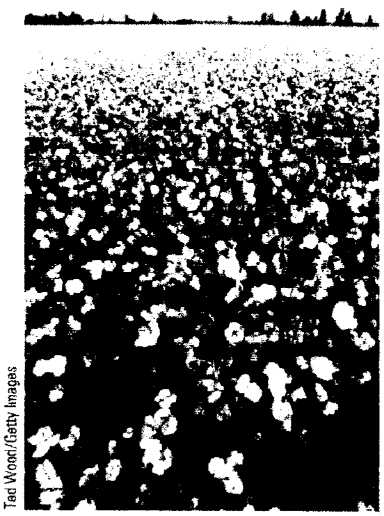
x	0.2	0.3	0.9	1.4	1.5	1.8	2.4	2.6	3.9	4.1	4.7	5.2	5.5	6.3
y	37.8	41.7	45.8	46.2	48.1	49.9	47.3	45.1	42.4	36.2	32.5	29.3	27.9	21.8

7.

x	0.5	0.7	1.3	1.9	2.3	2.8	2.9	3.3	3.7	4.1	4.1	4.4	4.9	5.5
y	52.1	48.2	45.3	40.8	39.7	35.5	34.1	32.5	31.2	32.7	33.4	36.1	38.3	41.2

8.

x	0.0	0.2	0.7	1.2	1.2	2.1	2.9	3.1	3.5	3.7	4.2	4.3	5.5	6.1
y	13.1	12.2	10.1	9.2	9.6	8.7	8.6	9.1	10.7	10.9	11.8	13.4	16.9	18.2



Tad Wood/Getty Images

9. **Rainfall and Crop Yield** Rain is essential for crops to grow, but too much rain can diminish crop yields. The data give rainfall and cotton yield per acre for several seasons in a certain county.
   - **(a)** Make a scatter plot of the data. Does a quadratic function seem appropriate for modeling the data?
   - **(b)** Use a graphing calculator to find the quadratic model that best fits the data. Draw a graph of the model.
   - **(c)** Use the model that you found to estimate the yield if there are 25 inches of rainfall.

Season	Rainfall (in.)	Yield (kg/acre)
1	23.3	5311
2	20.1	4382
3	18.1	3950
4	12.5	3137
5	30.9	5113
6	33.6	4814
7	35.8	3540
8	15.5	3850
9	27.6	5071
10	34.5	3881

10. **Too Many Corn Plants per Acre?** The more corn a farmer plants per acre, the greater is the yield the farmer can expect—but only up to a point. Too many plants per acre can cause overcrowding and decrease yields. The data give crop yields per acre for various densities of corn plantings, as found by researchers at a university test farm.
    - **(a)** Use a graphing calculator to find the quadratic model that best fits the data.
    - **(b)** Draw a graph of the model you found together with a scatter plot of the data.
    - **(c)** Use the model that you found to estimate the yield for 37,000 plants per acre.

Density (plants/acre)	Crop yield (bushels/acre)
15,000	43
20,000	98
25,000	118
30,000	140
35,000	142
40,000	122
45,000	93
50,000	67

Time (s)	Height (m)
0.0	1.28
0.5	7.96
1.0	12.22
1.5	14.02
2.0	13.38
2.5	10.27
3.0	4.82

11. **Height of a Baseball** A baseball is thrown upward, and its height is measured at 0.5-second intervals using a strobe light. The resulting data are given in the table.
    - **(a)** Make a scatter plot of the data. Does a quadratic function seem appropriate for modeling the data?
    - **(b)** Use a graphing calculator to find the quadratic model that best fits the data. Draw a graph of the model on the scatter plot of the data.
    - **(c)** Find the times when the ball is 6 meters above the ground.
    - **(d)** What is the maximum height attained by the ball?

example 6 Products of Functions

An employee invests part of his monthly paycheck in his company's stock. The table gives the price $P(t)$ (in dollars) of the stock on the purchase day and the number $N(t)$ of shares the employee buys.

(a) What is the meaning of the product function $R(t) = N(t)P(t)$?

(b) Calculate the values of $R(t)$ for the indicated months. What do you conclude?

Month t	1	2	3	4	5	6	7	8	9	10	11	12
Price $P(t)$	18	18	20	21	23	23	23	22	20	18	15	15
Number $N(t)$	10	10	12	12	12	12	12	15	20	25	30	30

Solution

(a) The function $R(t)$ gives the amount the employee invests in his company's stock each month.

(b) The value of $R(t)$ at any t is the product of $P(t)$ and $N(t)$. Therefore $R(1) = N(1)P(1) = 18 \cdot 10 = 180$. The other entries in the table below are calculated similarly.

Month t	1	2	3	4	5	6	7	8	9	10	11	12
$R(t)$	180	180	240	252	276	276	276	330	400	450	450	450

It appears that this employee is investing more money every month in his company's stock.

■ NOW TRY EXERCISE 39 ■

6.1 Exercises

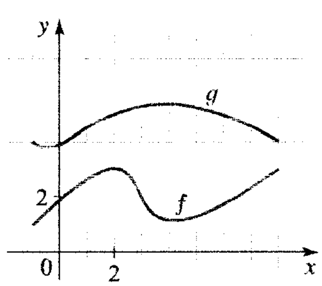
CONCEPTS

Fundamentals

1. Let f and g be functions defined for all real numbers.

(a) By definition, $(f + g)(x) =$ ________ and $(f - g)(x) =$ ________. So if $f(3) = 5$ and $g(3) = 7$, then $(f + g)(3) =$ ________ and $(f - g)(3) =$ ________.

(b) By definition, $(fg)(x) =$ ________ and $(f/g)(x) =$ ________. So if $f(1) = 3$ and $g(1) = 6$, then $(fg)(1) =$ ________ and $(f/g)(1) =$ ________.



2. Use the graphs of f and g in the figure in the margin to find the following.

$(f + g)(2) =$ ________ $(f - g)(2) =$ ________

$(fg)(2) =$ ________ $(f/g)(2) =$ ________

Think About It

3–4 ■ True or false?

3. If $f(x) > 0$ for all x, then the graph of $f + g$ is above the graph of g for all x.

4. If $f(x) < 0$ for all x, then the graph of fg is below the graph of g for all x.

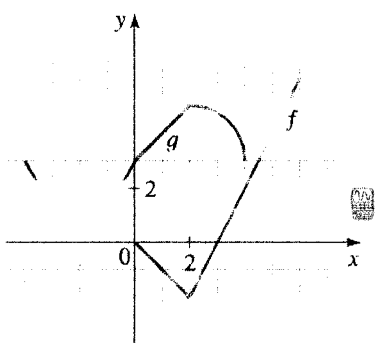
5–6 ■ Functions f and g are given in the table. Complete the table to find $f + g$ and $f - g$.

5.

t	0	1	2	3	4
$f(t)$	30	24	22	17	13
$g(t)$	294	312	331	352	370
$(f+g)(t)$					
$(f-g)(t)$					

6.

t	0	1	2	3	4
$f(t)$	131	282	291	332	504
$g(t)$	459	376	367	274	209
$(f+g)(t)$					
$(f-g)(t)$					



7–12 ■ Use the graphs of f and g shown in the margin to evaluate the expression.

7. $(f+g)(2)$

8. $(f-g)(0)$

9. $(fg)(1)$

10. $(g/f)(0)$

11. $(f/g)(0)$

12. $(fg)(4)$

13–14 ■ The functions f and g are given.

(a) Find the function $f + g$. Draw graphs of f, g, and $f + g$ in the same viewing rectangle.

(b) Find the function $f - g$. Draw graphs of f, g, and $f - g$ in the same viewing rectangle.

13. $f(x) = 2x^2 + 1, \quad g(x) = 3x + 2$

14. $f(x) = 3x^2 + 5, \quad g(x) = 4x + 7$

15–20 ■ The functions f and g are given.

(a) Find the functions $f + g$ and $f - g$ and their domains.

(b) Evaluate the functions $f + g$ and $f - g$ at the indicated value of x, if defined.

15. $f(x) = x^2, \quad g(x) = x - 3; \quad 1$

16. $f(x) = 0.5x^2, \quad g(x) = x + 2; \quad -1$

17. $f(x) = x^2 - 5x, \quad g(x) = 2 - 7x^2; \quad 2$

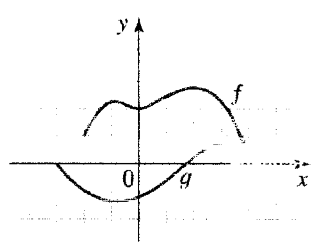
18. $f(x) = x^2 + 2x, \quad g(x) = 3x^2 - 1; \quad 1$

19. $f(x) = \sqrt{7 + x}, \quad g(x) = \sqrt{3 + x}; \quad 3$

20. $f(x) = \sqrt{4 - x}, \quad g(x) = \sqrt{1 + x}; \quad 5$

21–22 Use graphical addition to sketch the graph of $f + g$.

21.



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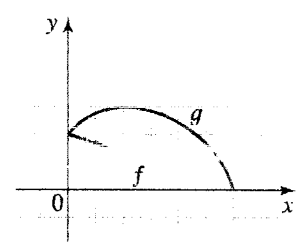


23–26 ■ The functions f and g are given.

(a) Draw the graphs of f, g, and $f + g$ on a common screen to illustrate graphical addition.

(b) Draw the graphs of f, g, and $f - g$ on a common screen to illustrate graphical subtraction.

23. $f(x) = x^2, \quad g(x) = x$

24. $f(x) = x^4, \quad g(x) = x^2$

25. $f(x) = \sqrt{1 + 0.5x}, \quad g(x) = \sqrt{1 - 0.5x}$

26. $f(x) = x^2, \quad g(x) = 0.5x$

27–28 ■ Functions f and g are given in the table. Use the table to find fg and f/g, if defined.

27.

t	0	1	2	3	4
$f(t)$	−3	−0.5	3	5	6
$g(t)$	2	6	7	−5	−2
$(fg)(t)$					
$(f/g)(t)$					

28.

t	0	1	2	3	4
$f(t)$	−1	9	0	5	−2
$g(t)$	−21	−25	−11	0	1
$(fg)(t)$					
$(f/g)(t)$					

29–34 ■ The functions f and g are given.

(a) Find the functions fg and f/g and their domains.

(b) Evaluate the functions fg and f/g at the indicated value, if defined.

29. $f(x) = 3x + 4, \quad g(x) = x + 5; \quad 2$

30. $f(x) = 2x - 1, \quad g(x) = 5x + 10; \quad -1$

31. $f(x) = x - 3, \quad g(x) = x^2; \quad 0$

32. $f(x) = 3x^2, \quad g(x) = 4x + 8; \quad -2$

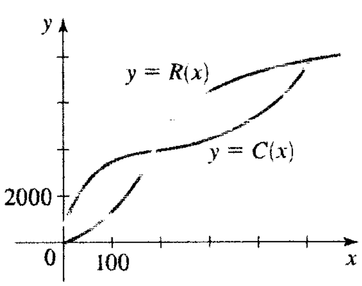
33. $f(x) = \dfrac{2}{x}, \quad g(x) = \dfrac{4}{x + 4}; \quad 4$

34. $f(x) = \dfrac{2}{x + 1}, \quad g(x) = \dfrac{x}{x + 1}; \quad 0$

35. Revenue, Cost, and Profit A store sells a certain type of digital camera. The revenue from selling x cameras is modeled by $R(x) = 450x - 0.5x^2$. The cost of producing x cameras is modeled by $C(x) = 9300 + 150x - 0.1x^2$. These models are valid for x in the interval $[0, 450]$.

(a) Find a function that models the profit from producing and selling x cameras.

(b) Sketch a graph of the revenue, cost and profit functions. How many cameras must be sold in one week before revenue exceeds cost.



36. Revenue, Cost, and Profit The figure in the margin shows graphs of the cost and revenue functions reported by a manufacturer of tires.

(a) Identify on the graph the value of x for which the profit is 0.

(b) Use graphical addition to sketch a graph of the profit function.

37–38 ▪ Revenue, Cost, and Profit A print shop makes bumper stickers for election campaigns. If x stickers are ordered (where $x < 10{,}000$), then the price per sticker is $0.15 - 0.000002x$ dollars, and the total cost of producing the order is $0.095x - 0.0000005x^2$ dollars.

37. Use the fact that revenue = price per item × number of items sold to express $R(x)$, the revenue from an order of x stickers, as a product of two functions of x.

38. Use the fact that profit = revenue − cost to express $P(x)$, the profit on an order of x stickers, as a difference of two functions of x.

39. Home Sales Let $f(t)$ be the number of existing homes sold in the United States in month t of 2007, and let g be the same function as f but for 2008. The table gives the values of f and g.

(a) What is the meaning of the function $h(t) = (f - g)(t)$?

(b) What is the meaning of the function $R(t) = \dfrac{h(t)}{f(t)}$?

(c) Calculate the values of $R(t)$ for each month. What do you conclude?

Month t	1	2	3	4	5	6	7	8	9	10	11	12
2007 sales: $f(t)$ (thousands)	532	550	509	494	494	479	480	458	426	422	418	409
2008 sales: $g(t)$ (thousands)	408	419	412	408	416	404	418	409	428	409	370	396

40. Labor Force The following table shows the number of workers $L(t)$ in the U.S. labor force and the number $E(t)$ of employed workers for each year from 2001 to 2008, as reported by the Bureau of Labor Statistics from the Current Population Survey.

(a) What is the meaning of the function $U(t) = L(t) - E(t)$?

(b) What is the meaning of the quotient functions $R_1(t) = \dfrac{E(t)}{L(t)}$ and $R_2(t) = \dfrac{U(t)}{L(t)}$?

(c) Calculate the values of $R_1(t)$ and $R_2(t)$ for the indicated years. What do you conclude?

t	2001	2002	2003	2004	2005	2006	2007	2008
Labor force $L(t)$ (thousands)	143,734	144,863	146,510	147,401	149,320	151,428	153,124	154,287
Employed $E(t)$ (thousands)	136,933	136,485	137,736	139,252	141,730	144,427	146,047	145,362

41. Smoking in the United States The table below shows estimated numbers of smokers as well as the total population of the United States from 1970 to 2007.

(a) What is the meaning of the quotient function $R(t) = \dfrac{S(t)}{P(t)}$?

(b) Calculate the values of $R(t)$ for the indicated years. What do you conclude?

t	1970	1980	1990	2000	2007
Smokers $S(t)$ (thousands)	76,035	75,212	63,421	65,571	62,737
Population $P(t)$ (thousands)	203,302	226,542	248,710	281,422	301,621

Algebra Checkpoint ✓

Check your knowledge of solving equations involving a power of the variable by doing the following problems. You can review these topics in **Algebra Toolkit C.1** on page T47.

1. Simplify each expression.

   (a) $5^3 \cdot 5^2$ (b) $\left(\frac{3}{5}\right)^3$ (c) $x^{1/2}x^{3/2}$ (d) $\dfrac{x^2}{x^{1/2}}$

2. The given equation involves a power of the variable. Find all real solutions of the equation.

   (a) $x^2 = 25$ (b) $x^3 = -27$ (c) $(3t)^3 = 6$ (d) $10.1(1.1z)^5 = 203$

3. The given equation involves a power of the variable. Find all real solutions of the equation.

   (a) $x^{1/2} = 5$ (b) $A^{4/3} = 81$ (c) $(16y)^{3/4} = 16$ (d) $2.1S^{1.03} = 17.2$

4. An equation in two variables is given. Solve the equation for the indicated variable.

   (a) $x = 2y^2$; y (b) $\dfrac{1}{3A} = \left(\dfrac{2}{3}S\right)^2$; S

   (c) $9w = z^{1/5}$; z (d) $2a_1 = 3(a_2)^{2.12}$; a_2

6.2 Exercises

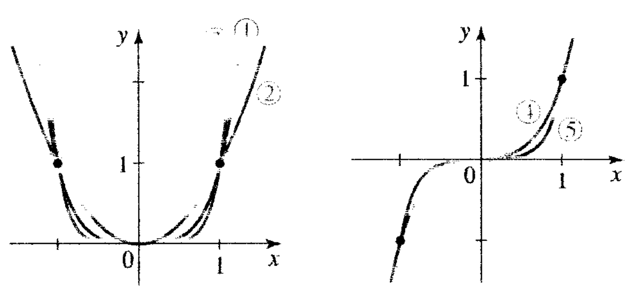
CONCEPTS

Fundamentals

1. (a) When p is an even positive integer, then the graph of the power function $f(x) = x^p$ is similar to the graph of $y =$ ________.

   (b) When p is an odd positive integer, then the graph of the power function $f(x) = x^p$ is similar to the graph of $y =$ ________.

2. Portions of the graphs of $y = x^2$, $y = x^3$, $y = x^4$, $y = x^5$, and $y = x^6$ are plotted in the figures. Match each function with its graph.



3. If the quantities x and y are related by the equation $y = 7x^5$, then we say that y is ________ proportional to the ________ power of x and the constant of proportionality is ________.

4. If y is directly proportional to the third power of x and if the constant of proportionality is 4, then x and y are related by the equation $y = \square\, x^{\square}$.

Think About It

5. True or false?

(a) For $x > 1$ the power function $f(x) = x^4$ is greater than the power function $g(x) = x^3$.

(b) For $0 < x < 1$ the power function $f(x) = x^4$ is greater than the power function $g(x) = x^3$.

6. Graph the functions $y = x^2$, $y = x^3$, $y = x^4$, and $y = x^5$ for $-1 \le x \le 1$ on the same coordinate axes. What do you think the graph of $y = x^{100}$ would look like on this same interval? What about $y = x^{101}$?

SKILLS

7–10 ■ Sketch the graph of each function by transforming the graph of an appropriate function of the form $y = x^n$ from Figure 1, page 494. Indicate all x- and y-intercepts.

7. (a) $P(x) = x^3 - 8$ (b) $Q(x) = -x^3 + 27$

8. (a) $R(x) = -(x + 2)^3$ (b) $S(x) = \frac{1}{2}(x - 1)^3 + 4$

9. (a) $P(x) = x^{1/4} + 2$ (b) $Q(x) = x^{1/4} - 8$

10. (a) $R(x) = (x + 2)^{1/5}$ (b) $S(x) = (x - 1)^{1/5}$

11–16 ■ Graph the family of functions in the same viewing rectangle, using the given values of c. Explain how changing the value of c affects the graph.

11. $R(x) = x^c$; $c = 1, 3, 5, 7$

12. $S(x) = x^c$; $c = 2, 4, 6, 8$

13. $P(x) = cx^3$; $c = 1, 2, 5, \frac{1}{2}$

14. $Q(x) = x^4 + c$; $c = -1, 0, 1, 2$

15. $F(x) = x^c$; $c = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$

16. $G(x) = x^c$; $c = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$

17. Compare the rates of growth of the functions $f(x) = 2^x$ and $g(x) = x^5$ by drawing the graphs of both functions in the following viewing rectangles.

(a) $[0, 5]$ by $[0, 20]$ (b) $[0, 25]$ by $[0, 10^7]$ (c) $[0, 50]$ by $[0, 10^8]$

18. Compare the rates of growth of the functions $f(x) = 3^x$ and $g(x) = x^4$ by drawing the graphs of both functions in the following viewing rectangles.

(a) $[-4, 4]$ by $[0, 20]$ (b) $[0, 10]$ by $[0, 5000]$ (c) $[0, 20]$ by $[0, 10^5]$

19–22 ■ Write an equation that expresses the statement.

19. V is directly proportional to the third power of x.

20. E is directly proportional to the square of c.

21. z is directly proportional to the square root of y.

22. R is directly proportional to the cube root of t.

23–26 ■ Express the statement as an equation. Use the given information to find the constant of proportionality.

23. A is directly proportional to the third power of x. If $x = 2$, then $A = 40$.

24. P is directly proportional to the fourth power of T. If $T = 3$, then $P = 27$.

25. z is directly proportional to square root of t. If $t = 64$, then $z = 2$.

26. S is directly proportional to the fifth root of r. If $r = \frac{1}{32}$, then $S = 12$.

27–28 ■ Use the given information to solve the problem.

27. y is directly proportional to the square of x. If $k = 0.5$ is the constant of proportionality, then find y when $x = 4$.

28. P is directly proportional to the square root of t. If $k = 1.5$ is the constant of proportionality, then find P when $t = 64$.

29. Determine whether the variable w is directly proportional to the third power of x. If so, write the equation of proportionality.

(a)

x	w
0	0
1	2
2	16
3	54
4	128

(b)

x	w
0	0
1	2.1
2	16.8
3	56.7
4	134.4

(c)

x	w
0	0
1	1.4
2	2.8
3	5.6
4	11.2

30. Determine whether the variable w is related to the variable x by a linear, exponential, or power function. In each case, find the function.

(a)

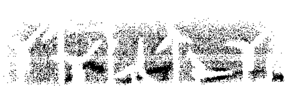
x	r
0	0
1	5
2	80
3	405
4	1280

(b)

x	w
0	51
1	153
2	459
3	1377
4	4131

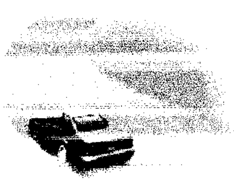
(c)

x	t
0	0
1	2.3
2	4.6
3	6.9
4	9.2



31. **Power from a Windmill** The power P that can be obtained from a windmill is directly proportional to the cube of the wind speed s.
   (a) Write an equation that expresses this proportionality.
   (b) Find the constant of proportionality for a windmill that produces 96 watts of power when the wind is blowing at 20 mi/h.
   (c) How much power will this windmill produce if the wind speed increases to 30 mi/h?

32. **Walking Speed** The natural walking speed s of a person or animal is proportional to the square root of their leg length l.
   (a) Write an equation that expresses this proportionality.
   (b) A person whose legs are 3 ft long has a natural walking speed of 3.7 ft/s. Find the constant of proportionality.
   (c) What is the natural walking speed of a person whose legs are 2.75 ft long?



33. **Skidding in a Curve** A 1000-kilogram car is traveling on a curve that forms a circular arc. The centripetal force F that pushes the car out is directly proportional to the square of the speed s.
   (a) If the car is traveling around the curve at a speed of 15 meters per second, the centripetal force is 2500 newtons. Find the constant of proportionality, and write an equation that expresses this proportionality.
   (b) The car will skid if the centripetal force is greater than the frictional force holding the tires to the road. For this road the maximum frictional force is 5880 newtons. If the car travels around the curve at a speed of 20 meters per second, will it skid?

34. **A Jet of Water** The power P of a jet of water from a fire hose 3 inches in diameter is directly proportional to the cube of the water velocity v. If the velocity is cut in half, by what factor will the power decrease?

35. Radiation Energy The radiation energy E emitted by a heated surface (per unit surface area) is directly proportional to the fourth power of the absolute temperature T of the surface.

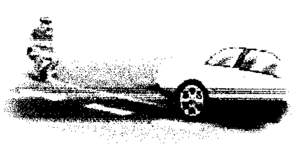
(a) Express this relationship by writing an equation.

(b) The temperature is 6000 K at the surface of the sun and 300 K at the surface of the earth. How many times more radiation energy per unit area is produced by the sun than by the earth?

36. Law of the Pendulum The period of a pendulum (the time elapsed during one complete swing of the pendulum) is directly proportional to the square root of the length of the pendulum.

(a) Express this relationship by writing an equation.

(b) To double the period, how would we have to change the length l?



37. Stopping Distance The stopping distance D of a car after the brakes have been applied is directly proportional to the square of the speed s. The stopping distance of a car on dry asphalt is given by the function $f(s) = 0.033s^2$, where s is measured in mi/h.

(a) Find the inverse function of f.

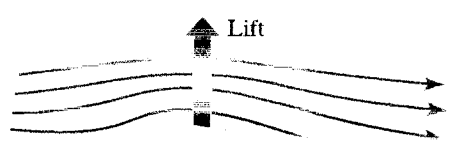
(b) Interpret the inverse function.

(c) If a car skids 50 ft on dry asphalt, how fast was it moving when the brakes were applied?

38. Aerodynamic Lift If a certain airplane wing has an area of 500 square feet, then the lift L of the airplane wing at takeoff is proportional to the square of the speed s of the plane in miles per hour. The lift of the airplane is given by the function $f(s) = 0.00136s^2$.

(a) Find the inverse function of f.

(b) Interpret the inverse function.



39. Ostrich Flight? The weight W (in pounds) of a bird (that can fly) has been related to the wingspan L (in inches) of the bird by the equation $L = 30.6 \cdot W^{0.3952}$. (In Exercise 19 of Section 6.4 this model will be derived from data.)

(a) The bald eagle has a wingspan of about 90 inches. Use the model to estimate the weight of the bald eagle.

(b) An ostrich weighs about 300 pounds. Use the model to estimate what the wingspan of an ostrich should be in order for it to be able to fly.

(c) The wingspan of an ostrich is about 72 inches. Use your answer to part (b) to explain why ostriches can't fly.

40. Biodiversity The number N of species of reptiles and amphibians inhabiting Caribbean islands has been related to the area A (in square meters) of an island by the equation $N = 3.1046A^{0.3080}$.

(a) Use the model to estimate the number of species found in an area that is 65 square meters.

(b) If you discover that 20 species of reptiles and amphibians live on an island, estimate the area of the island.