Algebra Checkpoint ✓

Check your knowledge of expanding quadratic expressions and completing the square by doing the following problems. You can review these topics in **Algebra Toolkits** ~~**8.1**~~ and ~~**8.2**~~ on pages T25 and T33.

1. Expand each expression.

(a) $(x + 2)(x - 3)$ **(b)** $(2t - 5)(t + 1)$

(c) $(3r - 2)(2r - 3)$ **(d)** $(6 + 3v)(6 - 3v)$

2. Determine whether the given expression is a perfect square.

(a) $x^2 + 6x + 9$ **(b)** $t^2 - 10t + 100$ **(c)** $t^2 - 20t + 100$ **(d)** $w^2 + 2w + \frac{4}{9}$

3. Factor the given perfect square.

(a) $x^2 + 14x + 49$ **(b)** $t^2 - 16t + 64$ **(c)** $w^2 - w + \frac{1}{4}$ **(d)** $s^2 + 5s + \frac{25}{4}$

4. Add the appropriate constant to make a perfect square.

(a) $x^2 + 10x + \_\_\_\_ = (x + \_\_\_\_)^2$

(b) $t^2 - 20t + \_\_\_\_ = (t - \_\_\_\_)^2$

(c) $3u^2 + 18u + \_\_\_\_ = 3(u + \_\_\_\_)^2$

(d) $4v^2 - 20v + \_\_\_\_ = 4(v - \_\_\_\_)^2$

5. Add and subtract the appropriate constants to complete the square.

(a) $x^2 + 6x = (x^2 + 6x + \_\_\_\_) - \_\_\_\_$

(b) $y^2 - y = (y^2 - y + \_\_\_\_) - \_\_\_\_$

(c) $2w^2 + 8w = 2(w^2 + 4w + \_\_\_\_) - \_\_\_\_$

(d) $3s^2 - s + 1 = 3(s^2 - \frac{1}{3}s + \_\_\_\_) - \_\_\_\_ + 1$

5.2 Exercises

Fundamentals

1. To put the quadratic function $f(x) = ax^2 + bx + c$ in standard form, we complete the _____.

2. The quadratic function $f(x) = a(x - h)^2 + k$ is in standard form.

(a) The graph of f is a parabola with vertex (_____, _____).

(b) If $a > 0$, the graph of f opens _____ (upward/downward).

(c) If $a < 0$, the graph of f opens _____ (upward/downward).

3. The graph of $f(x) = 2(x - 3)^2 + 5$ is a parabola that opens _____, with its vertex at (_____, _____).

4. The graph of $f(x) = -2(x + 3)^2 + 5$ is a parabola that opens _____, with its vertex at (_____, _____).

Think About It

5. Consider the quadratic function $y = (x - 2)(x - 4)$.

(a) In general form, $y =$ _____.

(b) From the general form we see that the y-intercept is _____.

6. Consider the quadratic function $y = (x - m)(x - n)$.

(a) In general form, $y =$ _____.

(b) From the general form we see that the y-intercept is _____.

(c) Using the formula from part (b), we see that the y-intercept of $y = (x - 3)(x - 5)$ is _____.

7–8 ■ Determine whether the given function is a quadratic function.

7. **(a)** $f(x) = \frac{1}{2}x^2 + 2x + 1$ **(b)** $g(x) = 3x^3 + 2x^2 + 1$ **(c)** $f(x) = 5^2x + 1$

8. **(a)** $f(x) = \sqrt{2}x^2 - 3x + \sqrt{2}$ **(b)** $g(x) = 5x^4 + 2x^2 + x$ **(c)** $f(x) = 0.7x^2 + \sqrt{5}$

9–16 ■ Express the function in the general form of a quadratic function.

9. $f(x) = (2x + 3)(4x - 1)$

10. $f(x) = (5x - 7)(3x - 2)$

11. $f(x) = (2x + 5)(2x - 5)$

12. $f(x) = (3x - 2)(3x + 2)$

13. $f(x) = (x - 1)^2 + 5$

14. $f(x) = (x + 2)^2 - 3$

15. $f(x) = 4(5x + 3)^2 + 7$

16. $f(x) = -(4x - 1)^2 + 10$

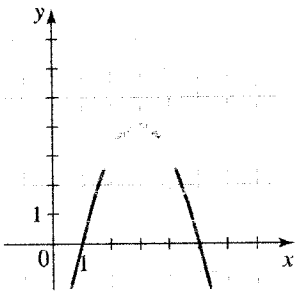
17–20 ■ A quadratic function f and its graph are given.

(a) Express the function in standard form.

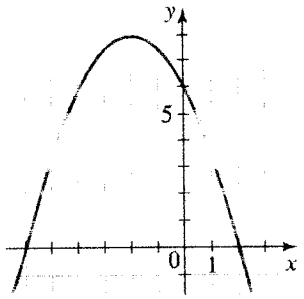
(b) Find the coordinates of the vertex from the standard form.

(c) Confirm your answer to part (b) from the graph.

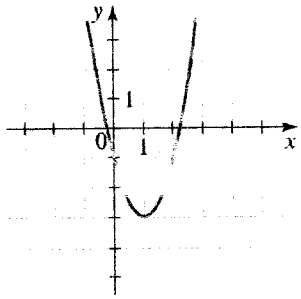
17. $f(x) = -x^2 + 6x - 5$



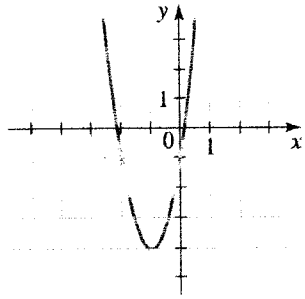
18. $f(x) = -\frac{1}{2}x^2 - 2x + 6$



19. $f(x) = 2x^2 - 4x - 1$



20. $f(x) = 3x^2 + 6x - 1$



21–24 ■ Express the function in the standard form of a quadratic function.

21. $f(x) = x^2 + 2x - 5$

22. $f(x) = x^2 - 4x + 2$

23. $f(x) = 2x^2 + 20x + 1$

24. $f(x) = 3x^2 - 18x - 5$

25–34 ■ A quadratic function is given.

(a) Express the quadratic function in standard form.

(b) Find its vertex.

(c) Sketch its graph.

25. $f(x) = x^2 - 6x$

26. $f(x) = x^2 + 8x$

27. $f(x) = -x^2 + 10x$

28. $f(x) = 2x^2 + 6x$

29. $f(x) = 2x^2 + 4x + 3$

30. $f(x) = x^2 - 2x + 2$

31. $f(x) = -x^2 + 6x + 4$

32. $f(x) = -x^2 - 4x + 4$

33. $f(x) = -x^2 - 3x + 3$

34. $f(x) = 2x^2 + 10x - 1$

35–38 ■ Find a function f whose graph is a parabola with the given vertex and that passes through the given point.

35. Vertex $(-2, -3)$; passes through the point $(-1, -8)$

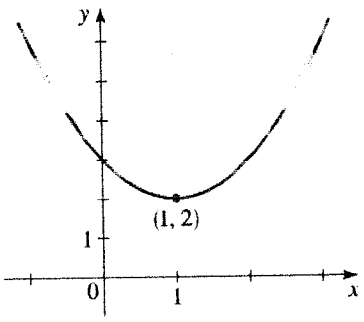
36. Vertex $(1, -2)$; passes through the point $(4, 16)$

37. Vertex $(3, 4)$; passes through the point $(1, -8)$

38. Vertex $(-5, 6)$; passes through the point $(-3, 30)$

39–42 ■ Find a quadratic function f whose graph is shown.

39.



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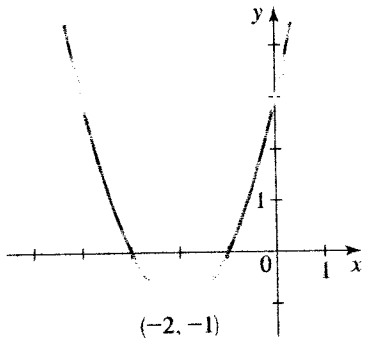

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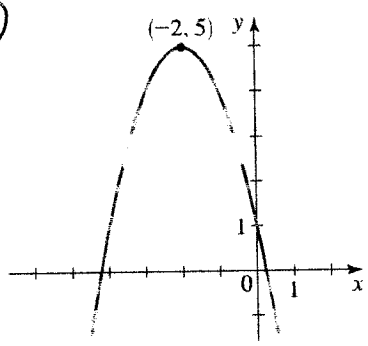

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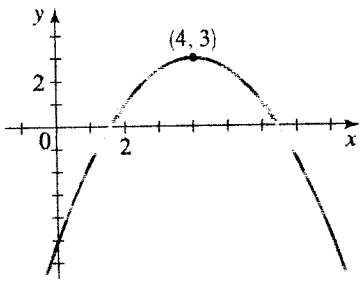


43–44 ■ A linear function f and a quadratic function g are given.

(a) Complete the table, and find the average rate of change of each function on intervals of length 1 for x between 0 and 5.

(b) Sketch a graph of each function.

43. $f(x) = 3x$; $g(x) = 3x^2$

x	$f(x)$	Rate of change
0	0	—
1	3	3
2		
3		
4		
5		

x	$g(x)$	Rate of change
0	0	—
1	3	3
2		
3		
4		
5		

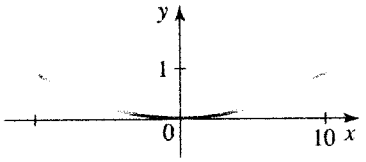
44. $f(x) = 20x$; $g(x) = 15x^2$

x	$f(x)$	Rate of change
0	0	—
1	20	20
2		
3		
4		
5		

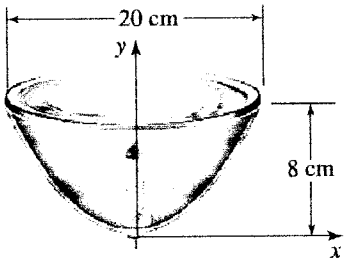
x	$g(x)$	Rate of change
0	0	—
1	15	15
2		
3		
4		
5		

CONTEXTS

45. Satellite Dish A reflector for a satellite dish is parabolic in cross section. The reflector is 1 ft deep and 20 ft wide from rim to rim (see the figure). Find a quadratic function that models the parabolic part of the dish.



46. Parabolic Reflector A headlamp of a car headlight has a reflector that is parabolic in cross section. The reflector is 8 cm deep and 20 cm wide from rim to rim, as shown in the figure. Find a quadratic function that models the parabolic part of the reflector, placing the origin of the coordinate axes at the vertex.



In Words	In Algebra
Ticket price	x
Amount ticket price is lowered	$14 - x$
Increase in attendance	$1000(14 - x)$
Attendance	$9500 + 1000(14 - x)$

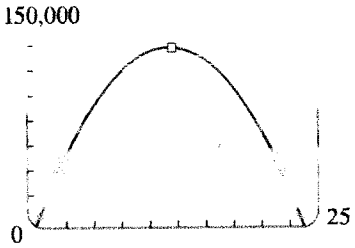
The model that we want is the function R that gives the revenue for a given ticket price x.

$$\text{revenue} = \text{ticket price} \times \text{attendance}$$

$$R(x) = x \times [9500 + 1000(14 - x)]$$

$$R(x) = x(23{,}500 - 1000x)$$

$$R(x) = 23{,}500x - 1000x^2$$



Maximum revenue occurs when the ticket price is \$11.75.

(b) Since R is a quadratic function where a is -1000 and b is 23,500, the maximum occurs at

$$x = -\frac{b}{2a} = -\frac{23{,}500}{2(-1000)} = 11.75$$

So a ticket price of \$11.75 gives the maximum revenue.

■ **NOW TRY EXERCISE 33** ■

5.3 Exercises

CONCEPTS

Fundamentals

1. The quadratic function $f(x) = ax^2 + bx + c$ is in general form.

   (a) The maximum or minimum value of f occurs at $x =$ ____/____

   (b) If $a > 0$, then f has a _____ (maximum/minimum) value.

   (c) If $a < 0$, then f has a _____ (maximum/minimum) value.

2. **(a)** The quadratic function $f(x) = 2x^2 - 12x + 5$ has a _____ (maximum/minimum) value of $f\left(\frac{\square}{\square}\right) =$ _____.

   (b) The quadratic function $f(x) = -2x^2 - 12x + 5$ has a _____ (maximum/minimum) value of $f\left(\frac{\square}{\square}\right) =$ _____.

Think About It

3. Consider the quadratic function $y = (x - 2)(x - 4)$.

   (a) In general form, $y =$ _____.

   (b) The graph is a parabola that opens _____ (up/down).

   (c) From the general form we see that the minimum value occurs at $x =$ ____/____ = _____.

4. Consider the quadratic function $y = (x - m)(x - n)$.

(a) In general form, $y =$ _____.

(b) The graph is a parabola that opens _____ (up/down).

(c) From the general form we see that the minimum value occurs at $x =$ _____.

(d) Using the formula in part (c), the minimum value of $y = (x + 3)(x - 5)$ occurs when $x =$ _____.

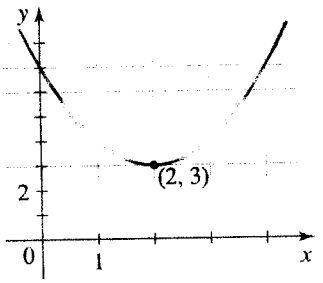
SKILLS

5–8 ■ A graph of a quadratic function is shown.

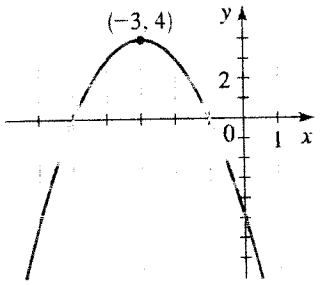
(a) Does the function have a minimum or a maximum value? What is that value?

(b) Find the x-value at which the minimum or maximum value occurs.

5.



6.



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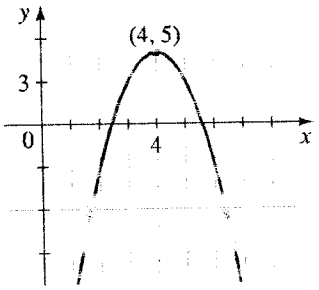

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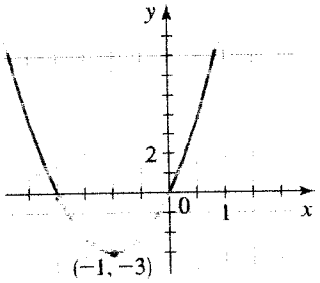


9–14 ■ Find the minimum or maximum value of the quadratic function, and find the x-value at which the minimum or maximum value occurs.

9. $f(x) = -\frac{x^2}{3} + 2x + 7$

10. $g(x) = 2x(x - 4) + 7$

11. $f(x) = x^2 + x + 1$

12. $f(x) = 1 + 3x - x^2$

13. $h(x) = \frac{1}{2}x^2 + 2x - 6$

14. $f(x) = 3 - x - \frac{1}{2}x^2$

15–18 ■ A quadratic function is given.

(a) Sketch its graph.

(b) Find the minimum or maximum value of f, and find the x-value at which the minimum or maximum value occurs.

15. $f(x) = x^2 + 2x - 1$

16. $f(x) = x^2 - 8x + 8$

17. $f(x) = -x^2 - 3x + 3$

18. $f(x) = 1 - 6x - x^2$

19–22 ■ A quadratic function is given.

(a) Use a graphing device to find the maximum or minimum value of the quadratic function f, correct to two decimal places.

(b) Find the maximum or minimum value of f, and compare it with your answer to part (a).

19. $f(x) = x^2 + 1.79x - 3.21$

20. $f(x) = x^2 - 3.2x + 4.1$

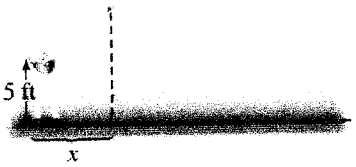
21. $f(x) = 1 + x - \sqrt{2}x^2$

22. $f(x) = 3 - 2x - \sqrt{3}x^2$

CONTEXTS

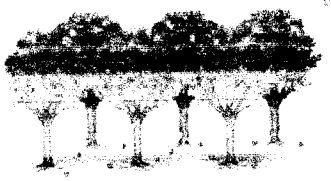
23. Height of a Ball If a ball is thrown directly upward with a velocity of 12 m/s, its height (in meters) after t seconds is given by $y = 12t - 4.9t^2$. What is the maximum height attained by the ball, and after how many seconds is that height attained?

24. Path of a Ball A ball is thrown across a playing field from a height of 5 ft above the ground at an angle of 45° to the horizontal and at a speed of 20 ft/s. It can be deduced from physical principles that the path of the ball is modeled by the function



$$y = -\frac{32}{(20)^2}x^2 + x + 5$$

where x is the distance in feet that the ball has traveled horizontally and y is the height in feet. What is the maximum height attained by the ball, and at what horizontal distance does this occur?



25. Agriculture The number of apples produced by each tree in an apple orchard depends on how densely the trees are planted. If n trees are planted on an acre of land, then each tree produces $900 - 9n$ apples. So the number of apples produced per acre is

$$A(n) = n(900 - 9n)$$

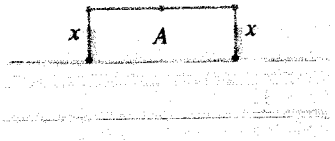
What is the maximum yield of the trees, and how many trees should be planted per acre to obtain the maximum yield of apples?

26. Agriculture At a certain vineyard it is found that each grape vine produces about 10 pounds of grapes in a season when about 700 vines are planted per acre. For each additional vine that is planted, the production of each vine decreases by about 1%. So the number of pounds of grapes produced per acre is modeled by

$$A(n) = (700 + n)(10 - 0.01n)$$

where n is the number of additional vines planted. What is the maximum yield of the grape vines, and how many vines should be planted to maximize grape production?

27. Fencing a Field A farmer has 2400 feet of fencing with which he wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river (see the figure).



(a) Find a function A that models the area of the field in terms of one of its sides x.

(b) What is the largest area that he can fence, and what are the dimensions of that area? [Compare to your graphical solution in Exercise 29, Section 1.8.]

28. Dividing a Pen A rancher with 750 feet of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle (see the figure).

(a) Find a function A that models the total area of the four pens.

(b) Find the largest possible total area of the four pens and the dimensions of that area. [Compare to your graphical solution in Exercise 30, Section 1.8.]

29. Fencing a Horse Corral Carol has 1200 feet of fencing to fence in a rectangular horse corral.

(a) Find a function A that models the area of the corral in terms of the width of the corral.

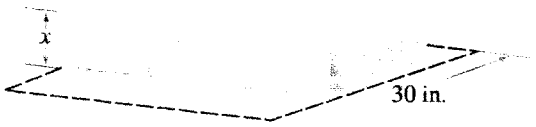
(b) Find the dimensions of the rectangle that maximize the area of the corral.

30. Making a Rain Gutter A rain gutter is formed by bending up the sides of a 30-inch-wide rectangular metal sheet as shown in the figure.

(a) Find a function A that models the cross-sectional area of the gutter in terms of x.

(b) Find the value of x that maximizes the cross-sectional area of the gutter.

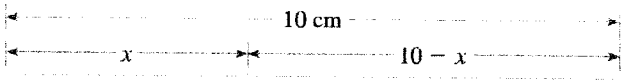
(c) What is the maximum cross-sectional area for the gutter?

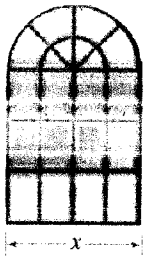


31. Minimizing Area A wire 10 cm long is cut into two pieces, one of length x and the other of length $10 - x$, as shown in the figure. Each piece is bent into the shape of a square.

(a) Find a function A that models the total area enclosed by the two squares.

(b) Find the value of x that minimizes the total area of the two squares.





32. Light from a Window A Norman window has the shape of a rectangle surmounted by a semicircle, as shown in the figure. A Norman window with perimeter 30 feet is to be constructed.

(a) Find a function that models the area of the window.

(b) Find the dimensions of the window that admits the greatest amount of light.

33. Stadium Revenue A baseball team plays in a stadium that holds 55,000 spectators. With the ticket price at \$10, the average attendance at recent games has been 27,000. A market survey indicates that for every dollar the ticket price is lowered, the attendance increases by 3000.

(a) Find a function R that models the revenue in terms of ticket price.

(b) Find the price that maximizes revenue from ticket sales.

34. Maximizing Profit A community bird-watching society makes and sells simple bird feeders to raise money for its conservation activities. The materials for each feeder cost \$6, and the society sells an average of 20 feeders per week at a price of \$10 each. The society has been considering raising the price, so it conducts a survey and finds that for every dollar increase, it loses 2 sales per week.

(a) Find a function P that models weekly profit in terms of price per feeder.

(b) What price should the society charge for each feeder to maximize profits? What is the maximum weekly profit?

According to this model, the rocket hits the ground after 50 seconds. (Of course the rocket is also at ground level at time 0.)

■ **NOW TRY EXERCISE 63** ■

Notice that the model in Example 8 is valid only for $0 \le t \le 50$. For values of t outside this interval the height h would be negative (that is, the rocket would be traveling below ground level).

Algebra Checkpoint

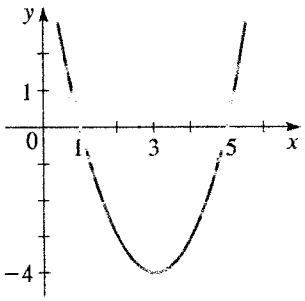
Check your knowledge of factoring quadratic expressions by doing the following problems. You can review this topic in **Algebra Toolkit B.2** on page T33.

1. Factor each expression by factoring out common factors.
   (a) $x^2 + x$ (b) $4t^2 - 6t$
   (c) $2(x + 1)^2 + (x + 1)$ (d) $(2r + 3)(3r + 4) - 5(3r + 4)$
2. Factor the given expression using the Difference of Squares Formula.
   (a) $x^2 - 36$ (b) $4t^2 - 16$
   (c) $(u + 1)^2 - 36$ (d) $4(w - 2)^2 - 49$
3. Factor the perfect square.
   (a) $x^2 + 8x + 16$ (b) $t^2 - 12t + 36$
   (c) $9r^2 - 24r + 16$ (d) $(u + 2)^2 + 10(u + 2) + 25$
4. Factor each expression by trial and error.
   (a) $x^2 + 5x + 6$ (b) $t^2 - t - 12$
   (c) $2s^2 - s - 3$ (d) $6u^2 - 7u - 3$

5.4 Exercises

Fundamentals

1. The Quadratic Formula gives us the solutions of the equation $ax^2 + bx + c = 0$.
   (a) State the Quadratic Formula: $x =$ _____.
   (b) In the equation $\frac{1}{2}x^2 - x - 4 = 0$, $a =$ _____, $b =$ _____, and $c =$ _____. So the solution of the equation is $x =$ _____.
2. To solve the quadratic equation $x^2 - 4x - 5 = 0$, we can do the following.
   (a) Factor the equation as $(x -$ ____$)\cdot(x +$ ____$) = 0$, so the solutions are _____ and _____.
   (b) Use the Quadratic Formula to get $x =$ _____, so the solutions are _____ and _____.
3. Let $f(x) = ax^2 + bx + c$, and let $D = b^2 - 4ac$.
   (a) If $D > 0$, then the number of x-intercepts for the graph of f is _____.
   (b) If $D = 0$, then the number of x-intercepts for the graph of f is _____.
   (c) If $D < 0$, then the number of x-intercepts for the graph of f is _____.
4. The graph of a quadratic function $f(x) = ax^2 + bx + c$ is shown at the top of the next page.
   (a) The x-intercepts are _____.



Graph for Exercise 4

(b) The discriminant of the equation $ax^2 + bx + c = 0$ is _____ (positive, 0, or negative).

(c) The solution(s) to the equation $ax^2 + bx + c = 0$ is (are) _____.

Think About It

5. Consider the quadratic function $y = (x - 2)(x - 4)$.

(a) The graph is a parabola that opens _____.

(b) The x-intercepts are _____ and _____.

(c) From the location of the vertex between the x-intercepts, we see that the x-coordinate of the vertex is _____.

6. Consider the quadratic function $y = (x - m)(x - n)$.

(a) The graph is a parabola that opens _____.

(b) The x-intercepts are _____ and _____.

(c) From the location of the vertex between the x-intercepts, we see that the x-coordinate of the vertex is _____.

(d) Using the formula we found in part (c), we find that the x-coordinate of the vertex of $y = (x - 3)(x - 5)$ is _____.

SKILLS

7–16 ■ Solve the equation by factoring.

7. $x^2 + x = 12$

8. $x^2 + 3x = 4$

9. $t^2 - 7t + 12 = 0$

10. $t^2 + 8t + 12 = 0$

11. $3s^2 - 5s - 2 = 0$

12. $4x^2 - 4x - 15 = 0$

13. $2y^2 + 7y + 3 = 0$

14. $4w^2 = 4w + 3$

15. $6x^2 + 5x = 4$

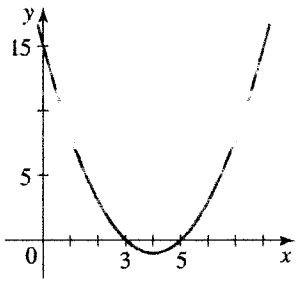
16. $3x^2 + 1 = 4x$

17–20 A graph of a quadratic function f is given.

(a) Find the x-intercepts.

(b) Find an equation that represents the function f (as in Example 3).

17.



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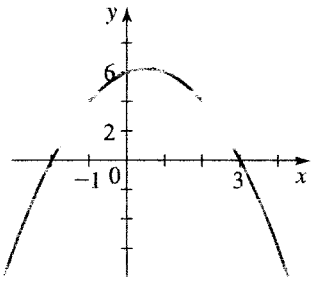

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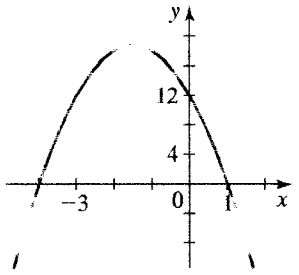

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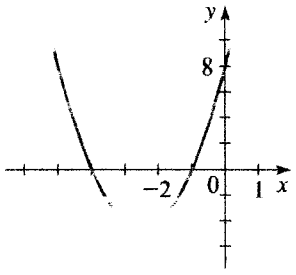


21–26 ■ Solve the equation by both factoring and using the Quadratic Formula.

21. $x^2 - 2x - 15 = 0$

22. $x^2 + 5x - 6 = 0$

23. $x^2 - 7x + 10 = 0$

24. $x^2 + 30x + 200 = 0$

25. $2x^2 + x - 3 = 0$

26. $3x^2 + 7x + 4 = 0$

27–38 ■ Find all real solutions of the equation.

27. $t^2 + 3t + 1 = 0$

28. $2t^2 - 8t + 4 = 0$

29. $y^2 + 12y - 27 = 0$

30. $8y^2 - 6y - 9 = 0$

31. $s^2 - \frac{3}{2}s + \frac{9}{16} = 0$

32. $s^2 - 6s + 1 = 0$

33. $w^2 = 3(w - 1)$

34. $2 + 2z + 3z^2 = 0$

35. $x^2 - \sqrt{5}x + 1 = 0$

36. $5x^2 - 7x + 5 = 0$

37. $10y^2 - 16y + 5 = 0$

38. $25y^2 + 70y + 49 = 0$

39–42 Solve the quadratic equation algebraically and graphically, correct to three decimal places.

39. $x^2 - 0.011x - 0.064 = 0$

40. $x^2 - 2.450x + 1.500 = 0$

41. $x^2 - 2.450x + 1.501 = 0$

42. $x^2 - 1.800x + 0.810 = 0$

43–48 ■ A quadratic function $f(x) = ax^2 + bx + c$ is given.

(a) Find the discriminant of the equation $ax^2 + bx + c = 0$. How many real solutions does this equation have?

(b) Use the answer to part (a) to determine the number of x-intercepts for the graph of the function $f(x) = ax^2 + bx + c$, and then graph the function to confirm your answer.

43. $f(x) = x^2 - 6x + 1$

44. $f(x) = x^2 - 6x + 9$

45. $f(x) = x^2 + 2.20x + 1.21$

46. $f(x) = 0.1x^2 - 0.38x + 0.361$

47. $f(x) = 32x^2 + 40x + 13$

48. $f(x) = 9x^2 - 4x + \frac{4}{9}$

49–52 ■ A graph of a quadratic function $f(x) = ax^2 + bx + c$ is shown.

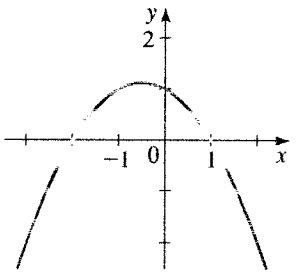
(a) Find the x-intercept(s), if there are any.

(b) Is the discriminant $D = b^2 - 4ac$ positive, negative, or 0?

(c) Find the solution(s) to the equation $ax^2 + bx + c = 0$.

(d) Find an equation that represents the function f.

49.



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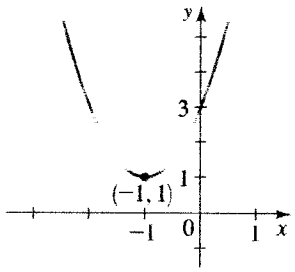

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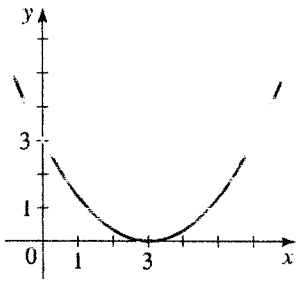

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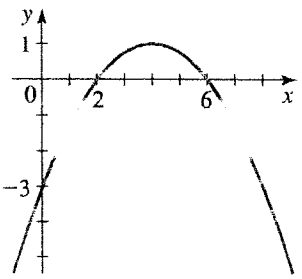


53–60 A quadratic function $f(x) = ax^2 + bx + c$ is given.

(a) Find the x-intercepts of the graph of f.

(b) Sketch the graph of f and label the x- and y-intercepts and the vertex.

53. $f(x) = x^2 + 2x - 1$

54. $f(x) = x^2 - 8x + 8$

55. $f(x) = 3x^2 - 6x + 1$

56. $f(x) = 5x^2 + 30x + 4$

57. $f(x) = -x^2 - 3x + 3$

58. $f(x) = 1 - x - x^2$

59. $f(x) = 3x^2 - 12x + 13$

60. $f(x) = 2x^2 + 8x + 11$

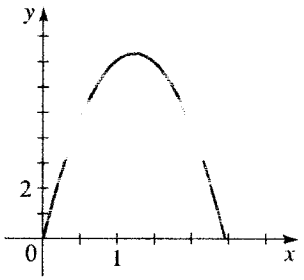
61. Height of a Ball If a ball is thrown directly upward with a velocity of 12 m/s, its height (in meters) after t seconds is modeled by $f(t) = 12t - 4.9t^2$.

(a) When does the ball reach a height of 5 meters?

(b) Does the ball reach a height of 8 meters?

(c) When does the ball hit the ground?

(d) Identify the points on the graph that correspond to your solutions to parts (a)–(c).



62. Path of a Ball A ball is thrown across a playing field from a height of 5 ft above the ground at an angle of 45° to the horizontal at a speed of 20 ft/s. It can be deduced from physical principles that the path of the ball is modeled by the function

$$f(x) = -\frac{32}{(20)^2}x^2 + x + 5$$

where x is the distance in feet that the ball has traveled horizontally.

(a) At what horizontal distance x is the ball 7 ft high?

(b) Does the ball reach a height of 10 ft?

(c) At what horizontal distance x does the ball hit the ground?

(d) Identify the points on the graph in the margin that correspond to your solutions to parts (a)–(c).