4.5 Exercises

CONCEPTS

Fundamentals

1. Let's solve the exponential equation $2e^x = 50$.

(a) First, we isolate e^x to get the equivalent equation ________.

(b) Next, we take ln of each side to get the equivalent equation ________.

(c) Now we use a calculator to find $x =$ ________.

2. Let's solve the logarithmic equation $\log 3 + \log(x - 2) = \log x$.

(a) First, we combine the logarithms to get the equivalent equation ________.

(b) Next, we write each side in exponential form to get the equivalent equation

________.

(c) Now we find $x =$ ________.

Think About It

3. To solve the exponential equation $2^{3x} = 2^{x+1}$, we first take logarithms of both sides. What base logarithm leads to the easiest solution? Why does this base work so nicely?

4. Logarithms with any base work equally well to solve the equation $3^x = 4 \cdot 5^x$. Why? First solve the equation using common logarithms, and then solve the equation using natural logarithms. Now use a logarithm with a different base to solve the equation. (You will need to use the Change of Base Formula in the solution.) Do you always get the same answer?

SKILLS

5–18 ■ Solve the exponential equation for the unknown x.

5. $10^x = 25$

6. $e^x = 7$

7. $25 \cdot 3^x = 7$

8. $6 \cdot 10^x = 4$

9. $2e^{12x} = 17$

10. $9 \cdot 4^{3x} = 12$

11. $3^{1-4x} = 2$

12. $e^{2x+1} = 200$

13. $2^{3x+1} = 34$

14. $8^{4x-1} = 5$

15. $6^x = 21 \cdot 2^x$

16. $5^x = 50 \cdot 3^x$

17. $5^x = 4^{x+1}$

18. $(3.2)^x = 35 \cdot (2.6)^x$

19–26 ■ Solve the exponential equation (a) algebraically and (b) graphically.

19. $e^{2x} = 5$

20. $3 \cdot e^x = 10$

21. $2^{1-x} = 3$

22. $2^{3x} = 34$

23. $10^{1-x} = 6^x$

24. $7^{x/2} = 5^{1-x}$

25. $5^x = 212 \cdot 9^x$

26. $5^x = 3 \cdot 2^x$

27–40 ■ Solve the logarithmic equation for the unknown x.

27. $\log_2 x = 10$

28. $\log x = 7$

29. $3 \log x = 12$

30. $2 \log_9 x = 1$

31. $\log_2 8x = -2$

32. $\ln 3x = -4$

33. $\log(x - 4) = 3$

34. $4 \log_5(3x + 5) = 2$

35. $\log(2x + 1) + \log 2 = 2$

36. $\log_2(3x - 1) + \log_2 8 = 5$

37. $\log_2 x - \log_2(x - 3) = 2$

38. $\log_5(x + 1) - \log_5(x - 1) = 2$

39. $\ln 5 + \ln(x - 2) = \ln(3x)$

40. $\log 2x = \log 2 + \log(3x - 4)$

41–48 ▪ Use a graphing calculator to find all solutions of the equation.

41. $\ln x = 3 - x$

42. $\log x = x^2 - 2$

43. $e^x = -x$

44. $2^{-x} = x - 1$

45. $x^3 - x = \log(x + 1)$

46. $x = \ln(4 - x^2)$

47. $4^{-x} = \sqrt{x}$

48. $e^{x^2} - 2 = x^3 - x$

CONTEXTS

49. **Compound Interest** Aviel invests \$6000 in a high-yield uninsured certificate of deposit that pays 4.5% interest per year, compounded quarterly.
   (a) Find a formula for the amount A of the certificate after t years.
   (b) What is the amount after 2 years?
   (c) How long will it take for his investment to grow to \$8000?

50. **Compound Interest** Ping invests \$4000 in a saving certificate that has an interest rate of 2.75% per year, compounded semiannually.
   (a) Find a formula for the amount A of the certificate after t years.
   (b) What is the amount after 4 years?
   (c) How long will it take for her investment to grow to \$5000?

51. **Compound Interest** Suzanne is planning to invest \$2000 in a certificate of deposit. How long does it take for the investment to grow to \$3000 under the given conditions?
   (a) The certificate of deposit pays $3\frac{1}{2}\%$ interest annually, compounded every month.
   (b) The certificate of deposit pays $2\frac{7}{8}\%$ interest annually, compounded continuously.

52. **Compound Interest** Masako is planning to invest \$5000 in a certificate of deposit. How long does it take for the investment to grow to \$8000 under the given conditions?
   (a) The certificate of deposit pays 3.55% interest annually, compounded every month.
   (b) The certificate of deposit pays 3.05% interest annually, compounded continuously.

53. ***Salmonella* Bacteria Count** Although cooking meat kills the microorganisms in it, they may survive gentle frying and roasting, especially if the meat was not properly defrosted before preparation. Inspectors for the U.S. Department of Agriculture test for *Salmonella* in a sample of chicken at a meat-packing plant. The sample is found to have a bacteria count of 15 colony-forming units per milliliter (CFU/mL). The sample is kept at a temperature of 80°F, and 6 hours later the count is 20,000 CFU/mL.
   (a) Find the instantaneous growth rate r for the bacteria count in the sample.
   (b) Find an exponential model $f(t) = Ce^{rt}$ for the bacteria count in the sample, where t is measured in hours.
   (c) What does the model predict the bacteria count will be after 4 hours?
   (d) How long will it take for the bacteria count to reach 10 million CFU/mL?
   (e) Find the doubling time for the *Salmonella* bacteria.

54. ***E. coli* Bacteria Count** Inspectors for the U.S. Department of Agriculture test a sample of ground beef for the bacterium *E. coli*. The sample is found to have a bacteria count of 100 colony-forming units per milliliter (CFU/mL). The sample is kept at a temperature of 100°F, and 2 hours later the meat has a count of 13,300 CFU/mL.
   (a) Find the instantaneous growth rate r (per hour) for the bacteria count in the sample.
   (b) Find an exponential model $f(t) = Ce^{rt}$ for the bacteria count in the sample, where t is measured in hours.
   (c) What does the model predict the bacteria count will be after 3 hours?
   (d) How long will it take for the bacteria count to reach one million CFU/mL?
   (e) Find the doubling time for the population of *E. coli* bacteria.

55. **Population of Ethiopia** In 2003 the United Nations estimated that the population of Ethiopia was about 70.7 million, with an annual growth rate of 2.9%. Assume that this rate of growth continues.

(a) Find the yearly growth factor a.

(b) Find an exponential growth model $f(t) = Ca^t$ for the population t years since 2003.

(c) How long will it take for the population to double?

(d) Use the model found in part (b) to predict the year in which the population will reach 90 million.

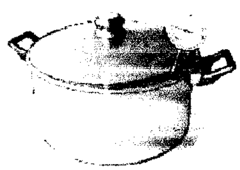
56. Population of Germany In 2004 the population of Germany was about 82.5 million, with an annual growth rate of 0.02%. Assume that this rate of growth continues.

(a) Find the yearly growth factor a.

(b) Find an exponential growth model $f(t) = Ca^t$ for the population t years since 2004.

(c) How long will it take for the population to double?

(d) Use the model found in part (b) to predict the year in which the population will reach 83 million.



57. Pot of Chili Angela prepares a large pot of chili the night before a church potluck. The temperature of the chili is 212°F, and it must cool down to 70°F before it can be stored in the refrigerator. Assume that the ambient temperature is 65°F and the heat transfer coefficient is $k = 2.895$.

(a) Find a model for the temperature T of the pot of chili t hours after cooling.

(b) How long will it take for the pot of chili to cool down to the desired temperature of 70°F?

(c) Graph the function T to confirm your answers to parts (a) and (b).

58. Time of Death Newton's Law of Cooling is used in homicide investigations to determine the time of death. Suppose that a body is discovered in a location whose ambient temperature is 60°F. The police determine that the heat transfer coefficient in this case is $k = 0.1947$. (The heat transfer coefficient depends on many factors, including the size of the body and the amount of clothing.) Normal body temperature is 98.6°F.

(a) Find a model for the temperature T of the body t hours after death.

(b) When the body was found, it had a temperature of 72°F. Find the length of time the victim has been dead.

(c) Graph the function T to confirm your answers to parts (a) and (b).

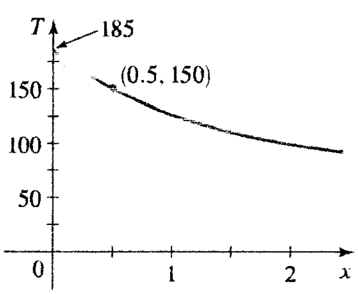
59. Boiling Water A kettle of water is brought to a boil in a room where the temperature is 20°C. After 15 minutes the temperature of the water has decreased from 100°C to 75°C.

(a) Find the heat transfer coefficient k, and find a model for the temperature T of the water t hours after it is brought to a boil.

(b) Use the model to predict the temperature of the water after 25 minutes. Illustrate by graphing the temperature function.

(c) How long will it take the water to cool to 40°C?

(d) Graph the function T to confirm your answers to parts (b) and (c).



60. Cooling Turkey A roasted turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F. The graph shows the temperature of the turkey after x hours.

(a) Find a model for the temperature T of the turkey t hours after it is taken out of the oven.

(b) Use the model to predict the temperature of the turkey after 45 minutes.

(c) How long will it take the turkey to cool to 100°F?

61. Radioactive Radium The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample.

(a) Find the yearly growth factor a.

(b) Find an exponential model $m(t) = Ca^t$ for the mass remaining after t years.

(c) How much of the sample will remain after 4000 years?

(d) After how long will only 18 mg of the sample remain?

62. **Radioactive Cesium** The half-life of cesium-137 is 30 years. Suppose we have a 10-gram sample.

(a) Find the yearly growth factor a.

(b) Find an exponential model $m(t) = Ca^t$ for the mass remaining after t years.

(c) How much of the sample will remain after 80 years?

(d) After how long will only 2 mg of the sample remain?

63. **Plutonium-239** A nuclear power plant produces radioactive plutonium-239, which has a half-life of 24,110 years. Because of its long half-life, plutonium-239 must be disposed of safely. How long does it take 100 grams of radioactive waste from the nuclear power plant to decay to 10 grams?

64. **Strontium-90** One radioactive material that is produced in atomic bombs is the isotope strontium-90, which has a half-life of 28 years. If a person is exposed to strontium-90, it can collect in human bone tissue, where it can cause leukemia and other cancers. If an atomic bomb test site is contaminated by strontium-90, how long will it take for the radioactive material to decay to 10% of the original amount?

65. **Carbon-14 Dating** Archeologists find an ancient shard of pottery and use some burnt olive pits found in the same layer of the site to determine the age of the shard. The archeologists determine that the olive pits contain 69.32% of the carbon-14 that is present in a living olive. (The half-life of carbon-14 is 5730 years.) How old is the shard of pottery?

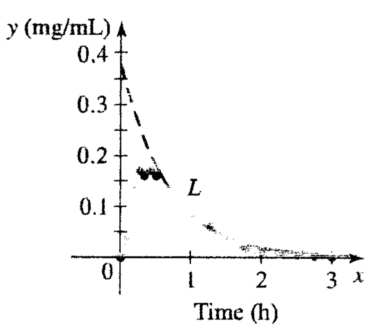
66. **Carbon-14 Dating** A donkey bone is estimated to contain 73% of the carbon-14 that it contained originally. How old is the donkey bone? (The half-life of carbon-14 is 5730 years.)

67. **Dead Sea Scrolls** The Dead Sea Scrolls are documents that contain some of the oldest known texts of parts of the Hebrew bible. They were discovered between 1947 and 1956 in several caves near the ruins of the ancient settlement of Khirbet Qumran on the northwest shore of the Dead Sea. Archeologists determine that a sample taken from the scrolls contains 79.10% of the carbon-14 that it originally contained. (The half-life of carbon-14 is 5730 years.)

(a) Estimate the age of the Dead Sea Scrolls.

(b) Search the Internet for the historical date of the writing of the Dead Sea Scrolls. How does this date compare with your estimate?

Donald Parry



68. **Algebra and Alcohol** The table in the Prologue (page P2) gives the alcohol concentration at different times following the consumption of 15 mL of alcohol. A scatter plot of the data (see the margin) shows that the alcohol concentration reaches a maximum in about half an hour and then begins to decay exponentially. The function

$$L(t) = 0.38e^{-1.5t}$$

closely models the decay part of the alcohol concentration. Use this function to determine the time at which the concentration decays to 0.05 mg/mL.

Solution

We need to check that $f(g(x)) = x$ and $g(f(x)) = x$. We have

$$\begin{aligned} f(g(x)) &= f\left(10^{x/2}\right) && \text{Definition of } g \\ &= \log\left(10^{x/2}\right)^2 && \text{Definition of } f \\ &= \log 10^x \\ &= x \end{aligned}$$

Also

$$\begin{aligned} g(f(x)) &= g\left(\log x^2\right) && \text{Definition of } f \\ &= 10^{(\log x^2)/2} \\ &= 10^{(2\log x)/2} \\ &= 10^{\log x} && \text{Simplify} \\ &= x && \log x \text{ and } 10^x \text{ are inverse functions} \end{aligned}$$

This shows that f and g are inverses of each other.

■ **NOW TRY EXERCISE 41** ■

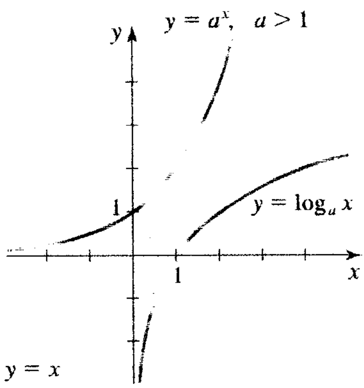


figure **9** Graphs of $f(x) = a^x$ and $g(x) = \log_a x$

You may have noticed that the graphs of the exponential and logarithm functions have similar shapes. In fact, the graph of $g(x) = \log_a x$ is the mirror image of the graph of $f(x) = a^x$, where the "mirror" is the line $y = x$ (Figure 9).

In general, the graph of f^{-1} is the reflection of the graph of f through the line $y = x$. This is because if $f(s) = t$, then $f^{-1}(t) = s$, so if (s, t) is on the graph of f, then (t, s) is on the graph of f^{-1}.

4.6 Exercises

CONCEPTS

Fundamentals

1. If $g(2) = 5$ and $f(5) = 12$, then $f(g(2)) =$ ________.
2. If the rule of the function f is "Add 1" and the rule of the function g is "Multiply by 2," then the rule of the function $M(x) = f(g(x))$ is "________________," and the rule of the function $N(x) = g(f(x))$ is "________________." Now express these functions algebraically:

   $f(x) =$ ________ $g(x) =$ ________

   $M(x) =$ ________ $N(x) =$ ________
3. A function f is one-to-one if different inputs produce ________ outputs. You can tell from the graph that a function is one-to-one by using the ________ ________ Test.
4. **(a)** For a function to have an inverse, it must be ________. So which one of the following functions has an inverse? $f(x) = x^2$, $g(x) = x + 2$

   (b) What is the inverse of the function that you chose in part (a)?

5. If the rule for the function f is "Multiply by 3, add 5, and then take the third power," then the rule for f^{-1} is "____________." We can express these functions algebraically as:

$$f(x) = \_\_\_\_\_\_ \qquad f^{-1}(x) = \_\_\_\_\_\_$$

6. The inverse of $f(x) = e^x$ is the function $f^{-1}(x) = $ ______.

Think About It

7. If f and g are linear functions, what kind of function is f composed with g? What kind of function is f^{-1}?

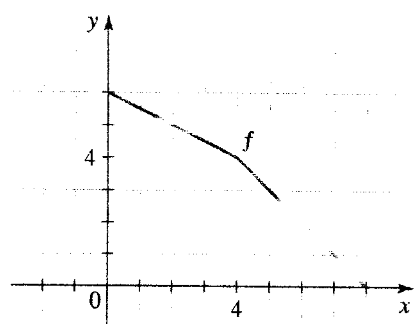
8. True or false?

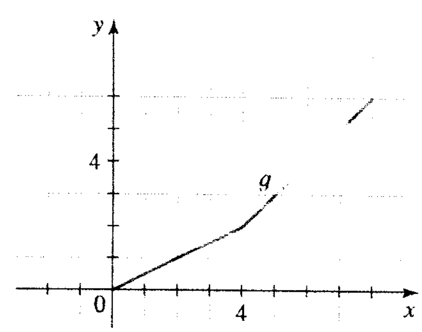
(a) If f has an inverse, then $f^{-1}(x)$ is the same as $\dfrac{1}{f(x)}$.

(b) If f has an inverse, then $f^{-1}(f(x)) = x$.

SKILLS

9–12 ■ The graphs of two functions f and g are given. Use the graphs to evaluate the indicated expressions.





9. (a) $f(g(4))$ (b) $g(f(4))$

10. (a) $f(g(0))$ (b) $g(f(0))$

11. (a) $f(f(6))$ (b) $g(g(6))$

12. (a) $f(g(f(8)))$ (b) $g(f(g(8)))$

13–14 ■ Two functions f and g are given by tables. Complete the tables for the functions "f composed with g" and "g composed with f."

13.

x	$f(x)$
1	5
2	1
3	2
4	2
5	4

x	$g(x)$
1	3
2	5
3	4
4	1
5	3

x	$f(g(x))$
1	2
2	
3	
4	
5	

x	$g(f(x))$
1	3
2	
3	
4	
5	

14.

x	$f(x)$	x	$g(x)$	x	$f(g(x))$	x	$g(f(x))$
−3	0	−3	3	−3	−3	−3	0
−2	1	−2	3	−2		−2	
−1	−1	−1	1	−1		−1	
0	2	0	0	0		0	
1	−2	1	−1	1		1	
2	3	2	−3	2		2	
3	−3	3	−3	3		3	

15–22 ■ Two functions f and g are given.

(a) Give a verbal description of the functions $M(x) = f(g(x))$ and $N(x) = g(f(x))$.

(b) Find algebraic expressions for the functions $M(x)$ and $N(x)$.

(c) Evaluate $M(3)$ and $N(-2)$.

15. $f(x) = x + 2, \quad g(x) = 3x$

16. $f(x) = 5x, \quad g(x) = x - 1$

17. $f(x) = 2x^2, \quad g(x) = x - 1$

18. $f(x) = x - 2, \quad g(x) = x^2$

19. $f(x) = e^x, \quad g(x) = x + 3$

20. $f(x) = 2x - 4, \quad g(x) = 10^x$

21. $f(x) = 3x + 5, \quad g(x) = \ln x$

22. $f(x) = \log x, \quad g(x) = x^4 + 1$

23–28 A verbal description of the rule of a function f is given.

(a) Give a verbal description of the inverse function f^{-1}.

(b) Find algebraic expressions for $f(x)$ and $f^{-1}(x)$.

(c) Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

23. Multiply the input by 3, then subtract 6.

24. Multiply the input by 5, then add 2.

25. Add 4 to the input, then multiply by $\frac{1}{2}$.

26. Subtract 2 from the input, then divide by 4.

27. Square the input, then multiply by −2 and add 3.

28. Square the input, add 1, then divide by 7.

29–32 Assume that f is a one-to-one function.

29. If $f(2) = 7$, find $f^{-1}(7)$.

30. If $f(-5) = 18$, find $f^{-1}(18)$.

31. If $f^{-1}(3) = -1$, find $f(-1)$.

32. If $f^{-1}(4) = 2$, find $f(2)$.

33–34 ■ A table of values for a one-to-one function f is given. Find

(a) $f(3)$ **(b)** $f^{-1}(5)$ **(c)** $f^{-1}(f(4))$ **(d)** $f^{-1}(10)$

33.

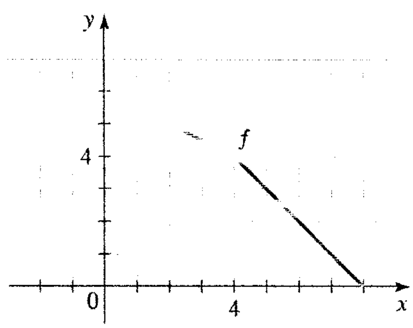
x	$f(x)$
0	7
1	−3
2	5
3	12
4	−1
5	10
6	6

34.

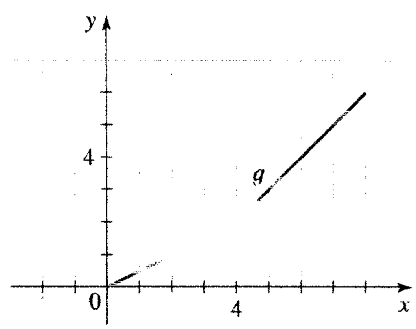
x	$f(x)$
−1	3
0	5
1	−2
2	0
3	10
4	1
5	6

35–36 ■ The graph of a function is given. Use the graph to find the indicated values.

35. **(a)** $f^{-1}(2)$, **(b)** $f^{-1}(4)$, **(c)** $f^{-1}(6)$



36. **(a)** $g^{-1}(2)$, **(b)** $g^{-1}(5)$, **(c)** $g^{-1}(6)$



37–44 ■ Show that f and g are inverses of each other (as in Example 6).

37. $f(x) = 2x - 5$; $g(x) = \dfrac{x+5}{2}$

38. $f(x) = \dfrac{3-x}{4}$; $g(x) = 3 - 4x$

39. $f(x) = x^3 + 1$; $g(x) = \sqrt[3]{x-1}$

40. $f(x) = \dfrac{1}{x-1}$; $g(x) = \dfrac{1}{x} + 1$

41. $f(x) = 10 \cdot 4^x$; $g(x) = \log_4\left(\dfrac{x}{10}\right)$

42. $f(x) = 10^{x/3}$; $g(x) = \log x^3$

43. $f(x) = \log_5 x^2$; $g(x) = 5^{x/2}$

44. $f(x) = \ln x + 2$; $g(x) = e^{x-2}$

45–56 ■ A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether the function is one-to-one.

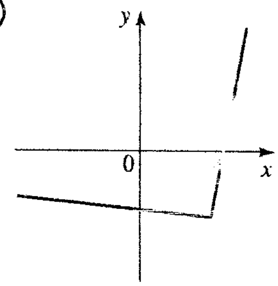
45.

x	0	1	2	3	4	5
$f(x)$	1	$\frac{1}{3}$	3	9	27	81

46.

x	0	1	2	3	4	5
$f(x)$	1.1	2.4	5.8	3.6	2.1	2.4

47.



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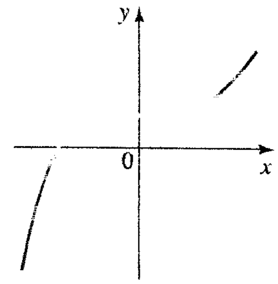

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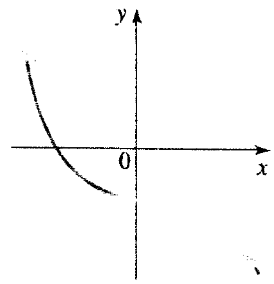

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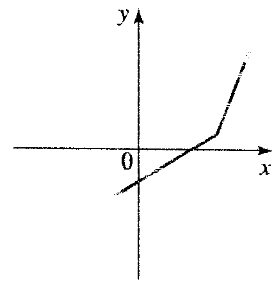


51. $f(x) = -2x + 4$

52. $f(x) = 3x - 2$

53. $g(x) = x^2 - 2x$

54. $g(x) = x^4 + 5$

55. $h(t)$ is the height of a basketball t seconds after it is thrown in a free-throw shot.

56. $S(x)$ is your shoe size at age x.

57–70 ■ Find the inverse function of f.

57. $f(x) = 4x + 7$

58. $f(x) = 3 - 5x$

59. $f(x) = \dfrac{x}{2}$

60. $f(x) = \dfrac{1}{x}$

61. $f(x) = x^3 - 4$

62. $f(x) = \sqrt[3]{x + 2}$

63. $f(x) = \dfrac{x - 2}{x + 2}$

64. $f(x) = \dfrac{1 + x}{3 - x}$

65. $f(x) = \log_2(x + 1)$

66. $f(x) = 10^{3x}$

67. $f(x) = e^{0.5x}$

68. $f(x) = \log_3(2x)$

69. $f(x) = \ln(x - 3)$

70. $f(x) = 2^{x^3}$

71–76 ■ Draw the graph of f and use it to determine whether the function is one-to-one.

71. $f(x) = x^3 - x$

72. $f(x) = x^3 + x$

73. $f(x) = \dfrac{x + 12}{x - 6}$

74. $f(x) = \sqrt{x^3 - 4x + 1}$

75. $f(x) = |x| - |x - 6|$

76. $f(x) = x \cdot |x|$

77. **Multiple Discounts** You have a \$50 coupon from the manufacturer good for the purchase of a cell phone. The store where you are purchasing your cell phone is offering a 20% discount on all cell phones. Let x represent the sticker price of the cell phone.

(a) Suppose that only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the sticker price x.

(b) Suppose that only the \$50 coupon applies. Find a function g that models the purchase price of the cell phone as a function of the sticker price x.

(b) Again we start with the graph of $f(x) = \log_{10} x$, but here we reflect in the y-axis to get the graph of $h(x) = \log_{10}(-x)$ in Figure 13.

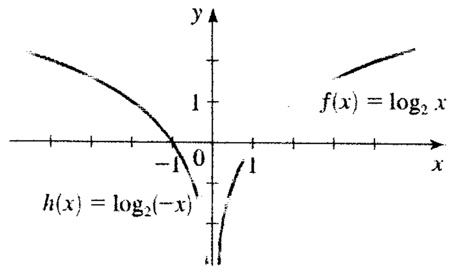


figure **13**

■ **NOW TRY EXERCISES 49 AND 51** ■

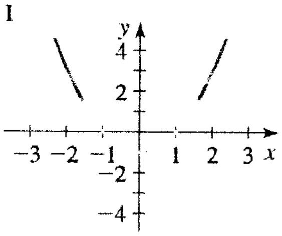
5.1 Exercises

CONCEPTS

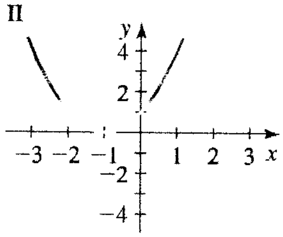
Fundamentals

1. Fill in the blank with the appropriate direction (left, right, up, or down).
   - **(a)** The graph of $y = f(x) + 3$ is obtained from the graph of $y = f(x)$ by shifting ________ 3 units.
   - **(b)** The graph of $y = f(x + 3)$ is obtained from the graph of $y = f(x)$ by shifting ________ 3 units.
2. Fill in the blank with the appropriate direction (left, right, up, or down).
   - **(a)** The graph of $y = f(x) - 3$ is obtained from the graph of $y = f(x)$ by shifting ________ 3 units.
   - **(b)** The graph of $y = f(x - 3)$ is obtained from the graph of $y = f(x)$ by shifting ________ 3 units.
3. Fill in the blank with the appropriate axis (x-axis or y-axis).
   - **(a)** The graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by reflecting in the ________.
   - **(b)** The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$ by reflecting in the ________.
4. Match the graph with the function.
   (a) $y = (x + 1)^2$ **(b)** $y = (x - 1)^2$ **(c)** $y = x^2 - 1$ **(d)** $y = -x^2$

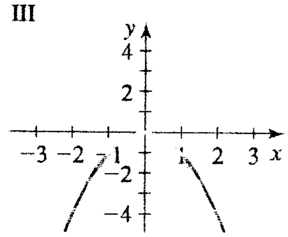
I



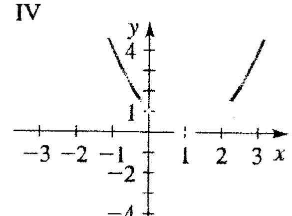
II

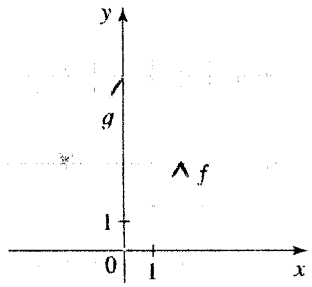


III



IV





Think About It

5–6 ■ Can the function g be obtained from f by transformations? If so, describe the transformations needed.

5. f and g are described algebraically: $f(x) = (x + 2)^2$, $g(x) = (x - 2)^2 + 5$

6. f and g are described graphically in the figure in the margin.

SKILLS

7–12 ■ Use the graph of $f(x) = x^2$ to graph the following.

7. (a) $g(x) = x^2 - 4$ **(b)** $g(x) = x^2 + 2$

8. (a) $g(x) = x^2 - 1$ **(b)** $g(x) = x^2 + 6$

9. (a) $g(x) = (x + 2)^2$ **(b)** $g(x) = (x - 4)^2$

10. (a) $g(x) = (x - 7)^2$ **(b)** $g(x) = (x + 3)^2$

11. (a) $g(x) = (x + 2)^2 - 1$ **(b)** $g(x) = (x - 3)^2 + 5$

12. (a) $g(x) = (x - 5)^2 - 3$ **(b)** $g(x) = (x + 3)^2 + 2$

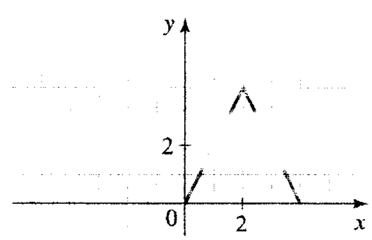
13. Use the graph of $f(x) = \sqrt{x}$ to graph the following.
(a) $g(x) = \sqrt{x + 4}$ **(b)** $g(x) = \sqrt{x} + 1$ **(c)** $g(x) = \sqrt{x + 2} + 2$

14. Use the graph of $f(x) = |x|$ (see Example 9 of Section 1.6, page 70) to graph the following.
(a) $g(x) = |x - 3|$ **(b)** $g(x) = |x| + 1$ **(c)** $g(x) = |x - 2| + 2$

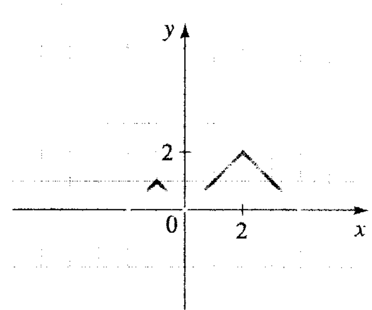
15. Use the graph of $f(x) = 3^x$ to graph the following.
(a) $g(x) = 3^x - 2$ **(b)** $g(x) = -3^x$ **(c)** $g(x) = 1 - 3^x$

16. Use the graph of $f(x) = \log_3 x$ to graph the following.
(a) $g(x) = \log_3(x + 1)$ **(b)** $g(x) = -\log_3 x$ **(c)** $g(x) = -\log_3(x - 4)$

17. The graph of $y = f(x)$ is given. Sketch graphs of the following functions.
(a) $y = f(x - 2)$ **(b)** $y = f(x) - 2$



18. The graph of $y = g(x)$ is given. Sketch graphs of the following functions.
(a) $y = g(x) - 2$ **(b)** $y = -g(x) - 1$



19–24 ■ A function f is given by the table. Complete the table for the given transformation.

19.

x	$f(x)$	$f(x) + 1$
−3	20	21
−2	27	
−1	53	
0	42	
1	39	
2	70	
3	21	

20.

x	$f(x)$	$f(x) - 1$
−3	9	8
−2	28	
−1	15	
0	20	
1	31	
2	27	
3	19	

21.

x	$f(x)$	$f(x + 2)$
−6	105	99
−4	99	
−2	82	
0	53	
2	20	
4	6	
6	2	—

22.

x	$f(x)$	$f(x - 2)$
−6	13	—
−4	29	13
−2	38	
0	49	
2	55	
4	62	
6	75	

23.

x	$f(x)$	$f(-x)$
−6	105	2
−4	99	
−2	82	
0	53	
2	20	
4	6	
6	2	

24.

x	$f(x)$	$1 - f(x)$
−6	105	−104
−4	99	
−2	82	
0	53	
2	20	
4	6	
6	2	

25–28 ■ Sketch the graph of the function, not by plotting points, but by starting with the graph of a basic function and applying a vertical shift.

25. $f(x) = x^2 - 1$

26. $f(x) = x^3 + 5$

27. $f(x) = \sqrt{x} + 1$

28. $f(x) = |x| - 1$

29–32 ■ Sketch the graph of the function, not by plotting points, but by starting with the graph of a basic function and applying a horizontal shift.

29. $f(x) = (x - 5)^2$

30. $f(x) = (x + 1)^2$

31. $f(x) = |x - 3|$

32. $f(x) = \sqrt{x + 4}$

33–36 ■ Sketch the graph of the function, not by plotting points, but by starting with the graph of a basic function and applying vertical stretching or shrinking.

33. $f(x) = \frac{1}{4}x^2$

34. $f(x) = 3|x|$

35. $f(x) = 2x^4$

36. $f(x) = \frac{1}{5}x^3$

37–52 ■ Sketch the graph of the function, not by plotting points, but by starting with the graph of a basic function and applying transformations.

37. $f(x) = 1 - x^2$

38. $f(x) = -2 - x^2$

39. $f(x) = -|x|$

40. $f(x) = -\sqrt{x}$

41. $f(x) = -(x - 3)^2 + 5$

42. $f(x) = -(x + 4)^2 - 3$

43. $f(x) = 6 - \sqrt{x + 4}$

44. $f(x) = 3 - |x - 2|$

45. $f(x) = 2^x - 3$

46. $f(x) = 3^x + 5$

47. $f(x) = -3^x$

48. $f(x) = 1 + 2^{-x}$

49. $f(x) = \log_2(x - 4)$

50. $f(x) = \log_2(x + 2)$

51. $f(x) = -\log_5(-x)$

52. $f(x) = -\log_{10}(x + 2)$

53–60 ■ A function f is given, and the indicated transformations are applied to its graph (in the given order). Write the equation for the final transformed graph.

53. $f(x) = x^2$; shift upward 3 units

54. $f(x) = x^2$; shift downward 1 unit

55. $f(x) = \sqrt{x}$; shift 2 units to the left

56. $f(x) = \sqrt{x}$; shift 1 unit to the right

57. $f(x) = |x|$; shift 3 units to the right and shift upward 1 unit

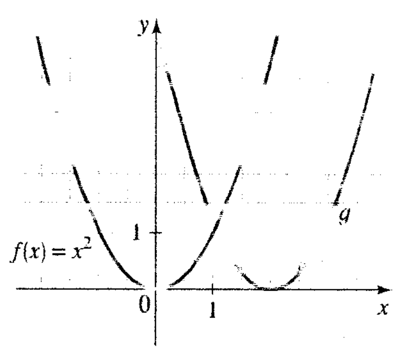
58. $f(x) = x^2$; shift 2 units to the left and reflect in the x-axis

59. $f(x) = 2^x$; shift downward 2 units

60. $f(x) = \log_3 x$; shift 2 units to the right and reflect in the y-axis

61–64 ■ The graphs of f and g are given. Find a formula for the function g.

61.



62.