

- The sum of  $x$  and  $y$  is 140 and the sum of the squares of  $x$  and  $y$  is minimized.
- $x + y = 90$  and  $x^2y$  is maximized.
- $x + y = 105$  and  $xy^2$  is maximized.

## APPLICATIONS

### Business and Economics

**Average Cost** In Exercises 5 and 6, determine the average cost function  $\bar{C}(x) = C(x)/x$ . To find where the average cost is smallest, first calculate  $\bar{C}'(x)$ , the derivative of the average cost function. Then use a graphing calculator to find where the derivative is 0. Check your work by finding the minimum from the graph of the function  $\bar{C}(x)$ .

$$5. C(x) = \frac{1}{2}x^3 + 2x^2 - 3x + 35$$

$$6. C(x) = 10 + 20x^{1/2} + 16x^{3/2}$$

**7. Revenue** If the price charged for a candy bar is  $p(x)$  cents, then  $x$  thousand candy bars will be sold in a certain city, where

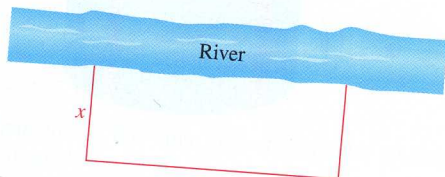
$$p(x) = 160 - \frac{x}{10}$$

- Find an expression for the total revenue from the sale of  $x$  thousand candy bars.
  - Find the value of  $x$  that leads to maximum revenue.
  - Find the maximum revenue.
- 8. Revenue** The sale of compact disks of "lesser" performers is very sensitive to price. If a CD manufacturer charges  $p(x)$  dollars per CD, where

$$p(x) = 12 - \frac{x}{8},$$

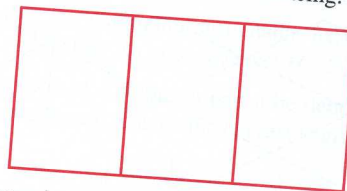
then  $x$  thousand CDs will be sold.

- Find an expression for the total revenue from the sale of  $x$  thousand CDs.
  - Find the value of  $x$  that leads to maximum revenue.
  - Find the maximum revenue.
- 9. Area** A campground owner has 1400 m of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river. (See the sketch.) Let  $x$  represent the width of the field.



- Write an expression for the length of the field.
  - Find the area of the field (area = length  $\times$  width).
  - Find the value of  $x$  leading to the maximum area.
  - Find the maximum area.
- 10. Area** Find the dimensions of the rectangular field of maximum area that can be made from 300 m of fencing material. (This fence has four sides.)

**11. Area** An ecologist is conducting a research project on breeding pheasants in captivity. She first must construct suitable pens. She wants a rectangular area with two additional fences across its width, as shown in the sketch. Find the maximum area she can enclose with 3600 m of fencing.



- Area** A farmer is constructing a rectangular pen with one additional fence across its width. Find the maximum area that can be enclosed with 2400 m of fencing.
- Cost with Fixed Area** A fence must be built in a large field to enclose a rectangular area of 25,600 m<sup>2</sup>. One side of the area is bounded by an existing fence; no fence is needed there. Material for the fence costs \$3 per meter for the two ends and \$1.50 per meter for the side opposite the existing fence. Find the cost of the least expensive fence.
- Cost with Fixed Area** A fence must be built to enclose a rectangular area of 20,000 ft<sup>2</sup>. Fencing material costs \$2.50 per foot for the two sides facing north and south and \$3.20 per foot for the other two sides. Find the cost of the least expensive fence.
- Revenue** A local club is arranging a charter flight to Hawaii. The cost of the trip is \$1600 each for 90 passengers, with a refund of \$10 per passenger for each passenger in excess of 90.
  - Find the number of passengers that will maximize the revenue received from the flight.
  - Find the maximum revenue.
- Profit** In planning a restaurant, it is estimated that a profit of \$8 per seat will be made if the number of seats is no more than 50, inclusive. On the other hand, the profit on each seat will decrease by 10¢ for each seat above 50.
  - Find the number of seats that will produce the maximum profit.
  - What is the maximum profit?
- Timing Income** A local group of scouts has been collecting aluminum cans for recycling. The group has already collected 12,000 lb of cans, for which they could currently receive \$7.50 per hundred pounds. The group can continue to collect cans at the rate of 400 lb per day. However, a glut in the aluminum market has caused the recycling company to announce that it will lower its price, starting immediately, by \$0.15 per hundred pounds per day. The scouts can make only one trip to the recycling center. Find the best time for the trip. What total income will be received?
- Pricing** Decide what you would do if your assistant presented the following contract for your signature:  
*Your firm offers to deliver 250 tables to a dealer, at \$160 per table, and to reduce the price per table on the entire order by 50¢ for each additional table over 250.*  
 Find the dollar total involved in the largest possible transaction between the manufacturer and the dealer; then find the smallest possible dollar amount.