**SOLUTION** For $w$ in the interval $(0, 2]$, the shipping cost is $y = 25$. For $w$ in $(2, 3]$, the shipping cost is $y = 25 + 3 = 28$. For $w$ in $(3, 4]$, the shipping cost is $y = 28 + 3 = 31$, and so on. The graph is shown in Figure 11.

The function discussed in Example 7 is called a **step function**. Many real-life situations are best modeled by step functions. Additional examples are given in the exercises.

In Chapter 1 you saw several examples of linear models. In Example 8, we use a quadratic equation to model the area of a lot.

## EXAMPLE 8 Area

A fence is to be built against a brick wall to form a rectangular lot, as shown in Figure 12. Only three sides of the fence need to be built, because the wall forms the fourth side. The contractor will use 200 m of fencing. Let the length of the wall be $l$ and the width $w$, as shown in Figure 12.

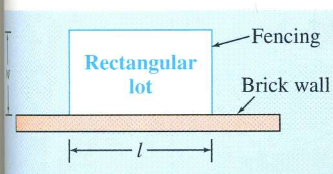


FIGURE 12

**(a)** Find the area of the lot as a function of the length $l$.

**SOLUTION** The area formula for a rectangle is area = length × width, or

$$A = lw.$$

We want the area as a function of the length only, so we must eliminate the width. We use the fact that the total amount of fencing is the sum of the three sections, one length and two widths, so $200 = l + 2w$. Solve this for $w$:

$$200 = l + 2w$$
$$200 - l = 2w \qquad \text{Subtract } l \text{ from both sides.}$$
$$100 - l/2 = w. \qquad \text{Divide both sides by 2.}$$

Substituting this into the formula for area gives

$$A = l(100 - l/2).$$

**(b)** Find the domain of the function in part (a).

**SOLUTION** The length cannot be negative, so $l \geq 0$. Similarly, the width cannot be negative, so $100 - l/2 \geq 0$, from which we find $l \leq 200$. Therefore, the domain is $[0, 200]$.

**(c)** Sketch a graph of the function in part (a).

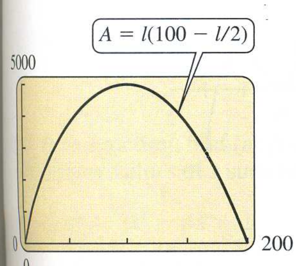


FIGURE 13

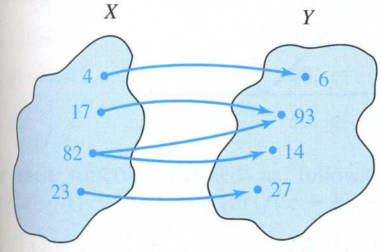
**SOLUTION** The result from a graphing calculator is shown in Figure 13. Notice that at the endpoints of the domain, when $l = 0$ and $l = 200$, the area is 0. This makes sense: If the length or width is 0, the area will be 0 as well. In between, as the length increases from 0 to 100 m, the area gets larger, and seems to reach a peak of 5000 $m^2$ when $l = 100$ m. After that, the area gets smaller as the length continues to increase because the width is becoming smaller.

In the next section, we will study this type of function in more detail and determine exactly where the maximum occurs.

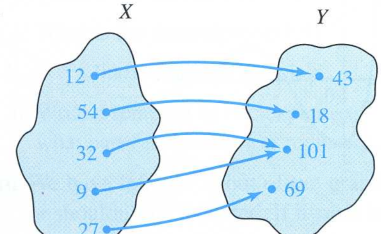
## 10.1 EXERCISES

Which of the following rules define $y$ as a function of $x$?

1.




2.



3.

| $x$ | $y$ |
|---|---|
| 3 | 9 |
| 2 | 4 |
| 1 | 1 |
| 0 | 0 |
| −1 | 1 |
| −2 | 4 |
| −3 | 9 |

4.

| $x$ | $y$ |
|---|---|
| 9 | 3 |
| 4 | 2 |
| 1 | 1 |
| 0 | 0 |
| 1 | −1 |
| 4 | −2 |
| 9 | −3 |

5. $y = x^3 + 2$

6. $y = \sqrt{x}$

7. $x = |y|$

8. $x = y^2 + 4$

**List the ordered pairs obtained from each equation, given $\{-2, -1, 0, 1, 2, 3\}$ as the domain. Graph each set of ordered pairs. Give the range.**

9. $y = 2x + 3$

10. $y = -3x + 9$

11. $2y - x = 5$

12. $6x - y = -1$

13. $y = x(x + 2)$

14. $y = (x - 2)(x + 2)$

15. $y = x^2$

16. $y = -4x^2$

**Give the domain of each function defined as follows.**

17. $f(x) = 2x$

18. $f(x) = 2x + 3$

19. $f(x) = x^4$

20. $f(x) = (x + 3)^2$

21. $f(x) = \sqrt{4 - x^2}$

22. $f(x) = |3x - 6|$

23. $f(x) = (x - 3)^{1/2}$

24. $f(x) = (3x + 5)^{1/2}$

25. $f(x) = \dfrac{2}{1 - x^2}$

26. $f(x) = \dfrac{-8}{x^2 - 36}$

27. $f(x) = -\sqrt{\dfrac{2}{x^2 - 16}}$

28. $f(x) = -\sqrt{\dfrac{5}{x^2 + 36}}$

29. $f(x) = \sqrt{x^2 - 4x - 5}$

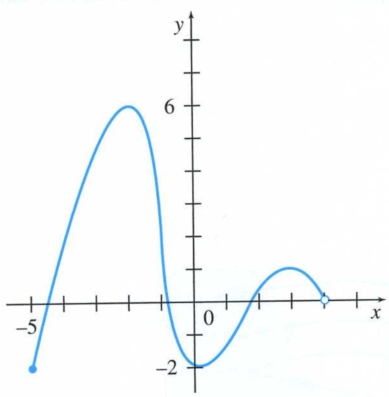
30. $f(x) = \sqrt{15x^2 + x - 2}$

31. $f(x) = \dfrac{1}{\sqrt{3x^2 + 2x - 1}}$

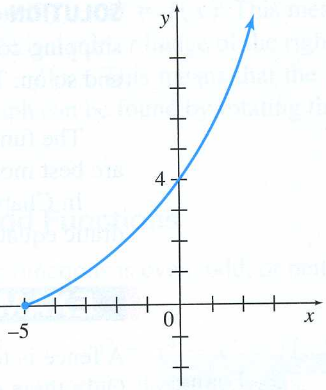
32. $f(x) = \sqrt{\dfrac{x^2}{3 - x}}$

**Give the domain and the range of each function. Where arrows are drawn, assume the function continues in the indicated direction.**

33.




34.

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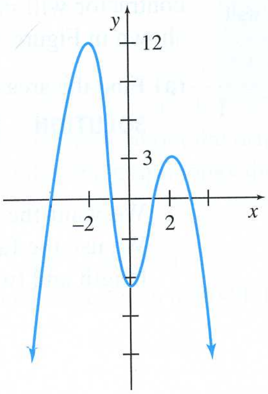


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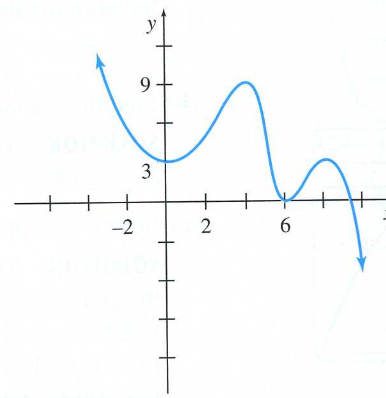
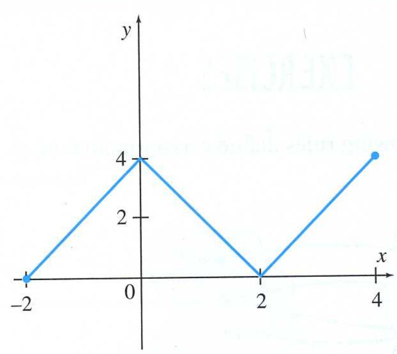



**In Exercises 37–40, give the domain and range. Then, use the graph to find (a) $f(-2)$, (b) $f(0)$, (c) $f(1/2)$, and (d) any value of $x$ such that $f(x) = 1$.**

37.

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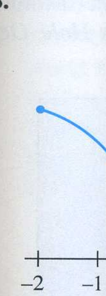
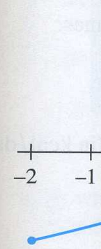
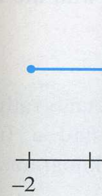



39.




40.




**For each func... and (e) any va...**

41. $f(x) = 3x$...

43. $f(x) = \{$...

44. $f(x) = \{$...

**Let $f(x) = 6$... following valu...**

45. $f(t + 1)$...

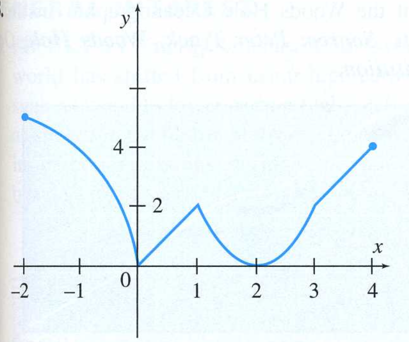
47. $g(r + h)$...

49. $g\left(\dfrac{3}{q}\right)$...

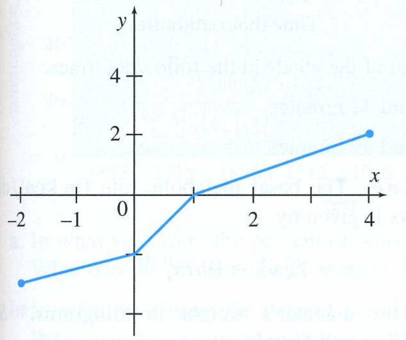
**For each fun... (b) $f(x + h)$...**

51. $f(x) = 2x$...

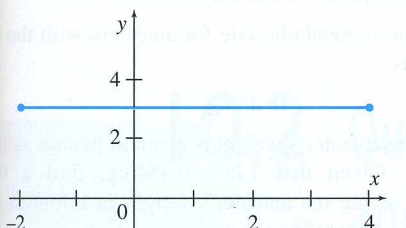
8.




9.




0.




or each function, find (a) $f(4)$, (b) $f(-1/2)$, (c) $f(a)$, (d) $f(2/m)$, nd (e) any values of $x$ such that $f(x) = 1$.

1. $f(x) = 3x^2 - 4x + 1$ **42.** $f(x) = (x + 3)(x - 4)$

3. $f(x) = \begin{cases} \dfrac{2x+1}{x-4} & \text{if } x \neq 4 \\ 7 & \text{if } x = 4 \end{cases}$

4. $f(x) = \begin{cases} \dfrac{x-4}{2x+1} & \text{if } x \neq -\dfrac{1}{2} \\ 10 & \text{if } x = -\dfrac{1}{2} \end{cases}$

nge. Then, use eac
nd (d) any values c

et $f(x) = 6x^2 - 2$ and $g(x) = x^2 - 2x + 5$ to find the llowing values.

5. $f(t + 1)$ **46.** $f(2 - r)$

7. $g(r + h)$ **48.** $g(z - p)$

9. $g\left(\dfrac{3}{q}\right)$ **50.** $g\left(-\dfrac{5}{z}\right)$

or each function defined as follows, find (a) $f(x + h)$, ) $f(x + h) - f(x)$, and (c) $[f(x + h) - f(x)]/h$.

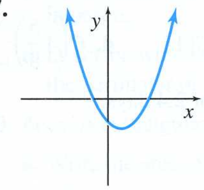
1. $f(x) = 2x + 1$ **52.** $f(x) = x^2 - 3$

**53.** $f(x) = 2x^2 - 4x - 5$ **54.** $f(x) = -4x^2 + 3x + 2$

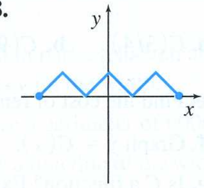
**55.** $f(x) = \dfrac{1}{x}$ **56.** $f(x) = -\dfrac{1}{x^2}$

**Decide whether each graph represents a function.**

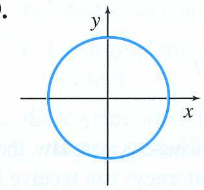
**57.**



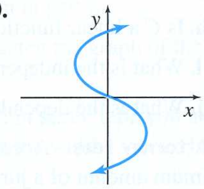

**58.**



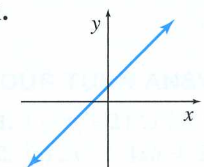

**59.**



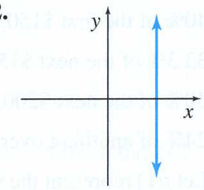

**60.**




**61.**




**62.**




**Classify each of the functions in Exercises 63–70 as even, odd, or neither.**

**63.** $f(x) = 3x$ **64.** $f(x) = 5x$

**65.** $f(x) = 2x^2$ **66.** $f(x) = x^2 - 3$

**67.** $f(x) = \dfrac{1}{x^2 + 4}$ **68.** $f(x) = x^3 + x$

**69.** $f(x) = \dfrac{x}{x^2 - 9}$ **70.** $f(x) = |x - 2|$

## APPLICATIONS

### Business and Economics

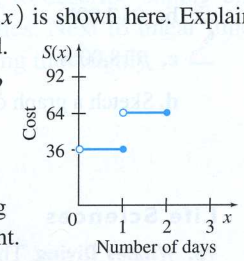
**71. Saw Rental** A chain-saw rental firm charges $28 per day or fraction of a day to rent a saw, plus a fixed fee of $8 for resharpening the blade. Let $S(x)$ represent the cost of renting a saw for $x$ days. Find the following.

**a.** $S\left(\dfrac{1}{2}\right)$ **b.** $S(1)$ **c.** $S\left(1\dfrac{1}{4}\right)$

**d.** $S\left(3\dfrac{1}{2}\right)$ **e.** $S(4)$ **f.** $S\left(4\dfrac{1}{10}\right)$

**g.** What does it cost to rent a saw for $4\frac{9}{10}$ days?

**h.** A portion of the graph of $y = S(x)$ is shown here. Explain how the graph could be continued.




**i.** What is the independent variable?

**j.** What is the dependent variable?

**k.** Is $S$ a linear function? Explain.

**l.** Write a sentence or two explaining what part f and its answer represent.

**m.** We have left $x = 0$ out of the graph. Discuss why it should or shouldn't be included. If it were included, how would you define $S(0)$?