$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{\Delta t},$$
$$u = e^{\sigma\sqrt{\Delta t}},$$
$$d = \frac{1}{u} = e^{-\sigma\sqrt{\Delta t}}. \quad (13.9)$$

With this choice, the binomial model will closely match the values of the expected growth rate of the stock μ and its variance σ^2. The closeness of the match improves if Δt is made smaller, becoming exact as Δt goes to zero.

13.3.7 Black–Scholes Option Model

As shown in Figure 13.9(c), if we take a sufficiently small time interval and the expiration time becomes long, we can approximate the resulting share distribution by a lognormal function, and the option value at the current point can be calculated with the Black–Scholes option model. In 1973, Black and Scholes developed an option-pricing model based on *risk-free arbitrage,* which means that, over a short time interval, an investor is able to replicate the future payoff of the stock option by constructing a portfolio involving the stock and a risk-free asset. The model provides a trading strategy in which the investor is able to profit with a portfolio return equal to the risk-free rate. The Black–Scholes model is a continuous-time model and assumes that the resulting share distribution at expiration would be distributed lognormally.

- **Call Option.** A standard call option gives its holder the right, but not the obligation, to buy a fixed number of shares at the exercise price (K) on the maturity date. If the current price of the stock is S_0, the Black–Scholes formula for the price of the call is

$$C_{\text{call}} = S_0 N(d_1) - Ke^{-r_f T} N(d_2), \quad (13.10)$$

where

$$d_1 = \frac{\ln(S_0/K) + \left(r_f + \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln(S_0/K) + \left(r_f - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

$N(.)$ is the standard cumulative normal distribution function, T is the time to maturity, r_f is the risk-free rate of return, and σ^2 is the volatility of the stock return. The model is independent of the expected rate of return and the risk preference of investors. The advantage of the model is that all the input variables are observable except the variance of the return, which can easily be estimated from historical stock price data.

- **Put Option.** We can value a put option in a similar fashion. The new option formula is

$$C_{\text{put}} = Ke^{-r_f T} N(-d_2) - S_0 N(-d_1), \quad (13.11)$$

where

$$d_1 = \frac{\ln(S_0/K) + \left(r_f + \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln(S_0/K) + \left(r_f - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

Note that, in developing the preceding continuous-time model, Black and Scholes address only the valuation of a European option, which pays no dividend. For an American option, we still need to use the *discrete-time valuation* given in the binomial lattice approach. Of course, we can consider the effect of paying a cash dividend in the model, but we will not address this embellishment or others, as our focus is not the evaluation of financial options.

EXAMPLE 13.4 Option Valuation under a Continuous-Time Process

Consider a stock currently trading at $40. For a strike price of $44, you want to price both a call and a put option that mature two years from now. The volatility of the stock (σ^2) is 0.40^2 and the risk-free interest rate is 6%.

SOLUTION

Given: $S_0 = 40$, $K = \$44$, $r = 6\%$, $T = 2$ years, and $\sigma = 40\%$.

Find: C_{call} and C_{put}.

- The call-option value is calculated as follows:

$$d_1 = \frac{\ln(40/44) + \left(0.06 + 0.4^2/2\right)2}{0.4\sqrt{2}} = 0.3265,$$

$$d_2 = 0.3265 - 0.4\sqrt{2} = -0.2392,$$

$$N(d_1) = N(0.3265) = 0.628,$$

$$N(d_2) = N(-0.2392) = 0.405,$$

$$C_{\text{call}} = 40(0.628) - 44e^{-0.06(2)}(0.405)$$
$$= \$9.32.$$

- The put-option value is calculated as follows:

$$N(-d_1) = N(-0.3265) = 0.372,$$

$$N(-d_2) = N(0.2392) = 0.595,$$

$$C_{\text{put}} = 44e^{-0.06(2)}(0.595) - 40(0.372)$$
$$= \$8.34.$$

COMMENTS: Note that the put-option premium is smaller than the call-option premium, indicating that the upside potential is higher than the downside risk.

13.4 Real-Options Analysis

So far, we have discussed the conceptual foundation for pricing financial options. The idea is, "Can we apply the same logic to value the real assets?" To examine this possibility, we will explore a new way of thinking about corporate investment decisions. The idea is based on the premise that any corporate decision to invest or divest real assets is simply an option, giving the option holder a right to make an investment without any obligation to act. The decision maker therefore has more flexibility, and the value of this operating flexibility should be taken into consideration. Let's consider the following scenario as a starting point for our discussion:

Current Practices—the New Math in Action.[3] Let's say a company is deciding whether to fund a large Internet project that could make or lose lots of money—most likely, lose it. A traditional calculation of net present value, which discounts projected costs and revenues into today's dollars, examines the project as a whole and concludes that it's a no go. But a real-options analysis breaks the project into stages and concludes that it makes sense to fund at least the value of the first stage. Here is how it works:

Step 1: Evaluate each stage of the project separately. Say the first stage, setting up a website, has a net present value of −\$50 million. The second stage, an e-commerce venture to be launched in one year, is tough to value, but let's say that the best guess of its net present value is −\$300 million.

Step 2: Understand your options. Setting up the website gives you the opportunity—but not the obligation—to launch the e-commerce venture later. In a year, you will know better whether that opportunity is worth pursuing. If it's not, all you have lost is the investment in the website. However, the second stage could be immensely valuable.

Step 3: Reevaluate the project, using an options mind-set. In the stock market, formulas such as Black and Scholes calculate how much you should pay for an option to buy, say, IBM at \$90 a share by June 30 if its current price is \$82. Think of the first stage of your Internet project as buying such an option—risky and out of the money, but cheap.

Step 4: Go figure. Taking into account the limited downside of building a website and the huge, albeit iffy, opportunities it creates, we see that a real-options analysis could give the overall project a present value of, say, \$70 million. So the no go changes to a go.

Now we will see how the logical procedure just outlined can be applied to address the investment risk inherent in strategic business decisions.

[3] From "Exploiting Uncertainty—The 'Real-Options' Revolution in Decision-Making," *BusinessWeek*, June 7, 1999, p. 119.

13.4.1 A Conceptual Framework for Real Options in Engineering Economics

In this section, we will conceptualize how the financial options approach can be used to value the flexibility associated with a real investment opportunity. A decision maker with an opportunity to invest in real assets can be viewed as having a *right, but not an obligation*, to invest. He or she therefore owns a real option similar to a simple call option on a stock:

- An investment opportunity can be compared to a call option on the present value of the cash flows arising from that investment (V).
- The investment outlay that is required to acquire the assets is the exercise price (I).
- The time to maturity is the time it takes to make the investment decision or the time until the opportunity disappears.

The analogy between a call option on stocks and a real option in capital budgeting is shown in Figure 13.10. The value of the real option at expiration depends on the value of the asset and would influence the decision as to whether to exercise the option. The decision maker would exercise the real option only if doing so were favorable.

- **Concept of Real Calls.** The NPW of an investment obtained with the options framework in order to capture strategic concerns is called the *strategic* NPW (SNPW) and is equivalent to the option value calculated by the Black–Scholes model in Eq. (13.10). The value of the right to undertake the investment now is (R_{min}), which is the payoff if the option is exercised immediately. If $V > I$, the payoff is $V - I$, and if $V < I$, the payoff is 0. The true value of the option (R) that one needs to find is the SNPW of the real option. Since one would undertake the investment later only if the outcome is favorable, the SNPW is greater than the conventional NPW. The value of the flexibility associated with the option to postpone the investment is the difference

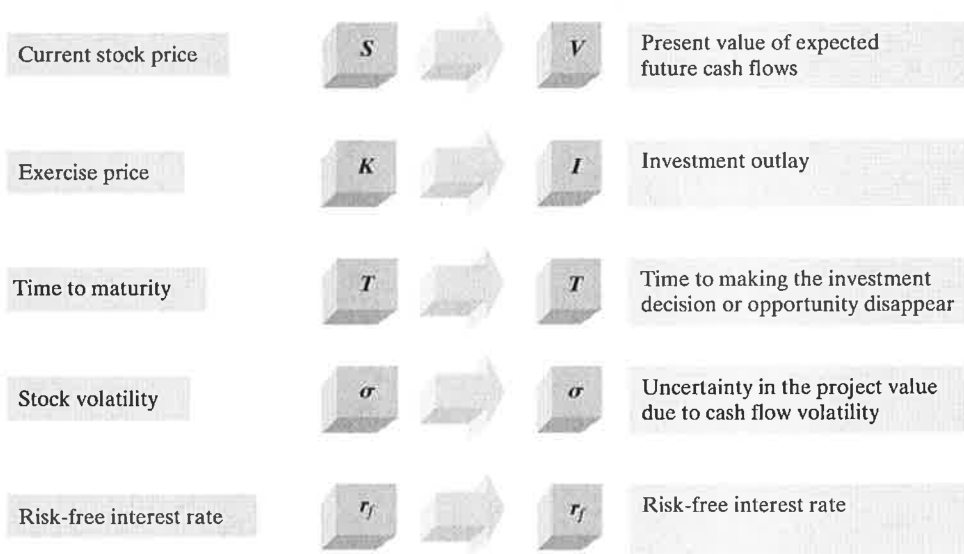

Figure 13.10 The analogy between a call option on a stock and a real option.

between the SNPW and the conventional NPW. This is the real-option premium (ROP), or the value of flexibility, defined as

$$\text{Value of flexibility (ROP)} = \text{SNPW} - \text{Conventional NPW}. \tag{13.12}$$

- **Concept of a Real Put Option.** The concept of a real put option is important from a strategic perspective. A put option gives its owner the right to dispose of an asset when it is favorable to do so. The put works like a guarantee or insurance when things go bad. An early-abandonment decision can be viewed as a simple put option. The option to abandon a project early may have value, as when an asset has a higher resale value in a secondary market than its use value. The put guarantees that the use value (V) of an asset does not fall below its market resale value (I). If it does, the option holder will exercise the put. In most instances, it is not possible to make an exact comparison between a standard put option on a stock and a real put option in capital budgeting.

13.4.2 Types of Real-Option Models

Most common types of real options can be classified into three categories as summarized in Table 13.1. Some of the unique features of each option will be examined with numerical examples.

TABLE 13.1 Types of Real Options

Common Real Options

Real Option Category	Real Option Type	Description	Examples
Invest/ grow	*Scale up*	Well-positioned businesses can scale up later through cost-effective sequential investments as the market grows.	• High technology • R&D intensive • Multinational • Strategic acquisition
	Switch up	A flexibility option to switch products or processes on plants, given a shift in the underlying price or demand of inputs or outputs.	• Small-batch goods producers • Utilities • Farming
	Scope up	Investments in proprietary assets in one industry enables a company to enter another industry cost effectively. Link and leverage.	• Companies with lock-in • De facto standard bearers
Defer/ learn	*Study/ start*	Delay the investment until more information or skill is acquired.	• Natural-resource companies • Real-estate development
Disinvest/ shrink	*Scale down*	Shrink or shut down a project partway through if new information changes the expected payoffs.	• Capital-intensive industries • Financial services • New-product introduction • Airframe order cancellations
	Switch down	Switch to more cost-effective and flexible assets as new information is obtained.	• Small-batch goods producers • Utilities
	Scope down	Limit the scope of (or abandon) operations in a related industry when there is no further potential in a business opportunity.	• Conglomerates

Source: "Get Real—Using Real Options in Security Analysis," *Frontiers of Finance*, Volume 10, by Michael J. Mauboussin, Credit Suisse First Boston, June 23, 1999.

Option to Defer Investment

The option to defer an investment is similar to a call option on stock. Suppose that you have a new product that is currently selling well in the United States and your firm is considering expanding the market to China. Because of many uncertainties and risks in the Chinese market (pricing, competitive pressures, market size, and logistics), the firm is thinking about hiring a marketing firm in China who can conduct a test market for the product. Any new or credible information obtained through the test market will determine whether or not the firm will launch the product. If the market is ready for the product, the firm will execute the expansion. If the market is not ready for the product, then the firm may wait another year or walk away and abandon the expansion plan altogether. In this case, the firm will lose only the cost associated with testing the market. An American expansion option will provide the firm with an estimate of the value to spend in the market research phase.

EXAMPLE 13.5 Delaying Investment: Value of Waiting

A firm is preparing to manufacture and sell a new brand of digital phone. Consider the following financial information regarding the digital phone project:

- The investment costs are estimated at $50M today, and a "most likely" estimate for net cash inflows is $12M per year for the next five years.
- Due to the high uncertainty in the demand for this new type of digital phone, the volatility of cash inflows is estimated at 50%.
- It is assumed that a two-year "window of opportunity" exists for the investment decision.
- If the firm delays the investment decision, the investment costs are expected to increase 10% per year.
- The firm's risk-adjusted discount rate (MARR) is 12% and the risk-free rate is 6%.

Should the firm invest in this project today? If not, is there value associated with delaying the investment decision?

SOLUTION

- **Conventional Approach: Should the Firm Invest Today?** If the traditional NPW criterion is used, the decision is not to undertake the investment today, as it has a negative NPW of $6.74 million:

$$\begin{aligned} PW(12\%) &= -\$50M + \$12M(P/A, 12\%, 5) \\ &= -\$6.74M \end{aligned}$$

- **Real-Options Approach.** Is there value to waiting two years? From an options approach, the investment will be treated as an opportunity to wait and then to undertake the investment two years later if events are favorable. As shown in Figure 13.11, this delay option can be viewed as a call option. If the value V of the project is greater than the investment cost I two years from now,

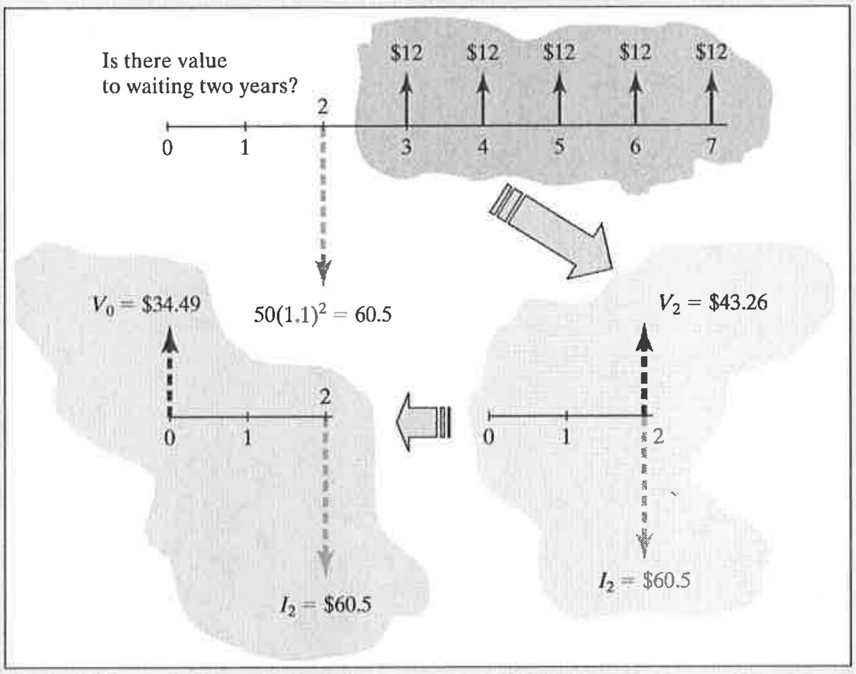

Figure 13.11 Transforming cash flow data to obtain input parameters V_0 and I_2 for option valuation.

then the option to undertake the project should be exercised. If not, it expires worthless. Therefore,

$$\text{Delay option} = \max[V_2 - I_2, 0],$$

where V_2 is a random variable that is dependent upon market demand.

Now, assume that one can use the Black–Scholes formula to value[4] the opportunity to undertake this investment in year 2. The opportunity to wait can be considered as a European call option on the present value of the future cash flows $V_0 = \$34.49$ million, with an exercise price of $I = \$60.5$ million, expiring two years from now ($T = 2$). Since $r_f = 6\%$ and $\sigma = 50\%$, using the Black–Scholes formula, we obtain $d_1 = -0.2715$, $d_2 = -0.976$, $N(d_1) = 0.3930$, and $N(d_2) = 0.1639$. The value of the real option is therefore

$$\begin{aligned} C = \text{SNPW} &= 34.49(0.3930) - 60.5e^{-(0.06)2}(0.1639) \\ &= 13.5546 - 8.7947 \\ &= \$4.76, \\ \text{ROP} &= \$4.76 - (-\$6.74) \\ &= \$11.50\text{ M}. \end{aligned}$$

[4] One can arrive at approximately the same value with the binomial model by dividing the one-year time to maturity into sufficiently small time intervals.

Thus, the value of retaining the flexibility of having the delay option is worth \$11.50M. The ability to wait provides a decision maker a strategic NPW of \$4.61M, instead of a negative NPW of \$6.74M. On the one hand, if the market for the product turns out to be favorable, the company will exercise the real option in the money and undertake the investment in year 2. On the other hand, if the market two years later turns out to be unfavorable, the decision will be not to undertake the investment. The opportunity cost is only \$4.61M, compared with the actual investment cost of \$50 million.

COMMENTS: What exactly does the \$4.76M delay value imply? Suppose the firm's project portfolio consists of 10 projects, including this digital phone project. Then if the other 9 projects have a net present value of \$100M, the value of the firm using standard NPW would still be \$100M, because the digital phone project would not be accepted, since it has a negative NPW. If the option to invest in the digital phones is included in the valuation, then the value of the firm's portfolio is \$104.76M. Therefore, the delay option that the firm possesses is worth an additional \$4.76M.

Patent and License Valuation

A patent or license provides a firm the right, but not the obligation, to develop a product (or land) over some prescribed time interval. The right to use the patent has value if and only if the expected benefits (V) exceed the projected development costs (I), or $\max\{V - I, 0\}$. This right is considered as a real option that can be used to value the worth of a license to a firm.

EXAMPLE 13.6 Option Valuation for Patent Licensing

A technology firm is contemplating purchasing a patent on a new type of digital phone. The patent would provide the firm with three years of exclusive rights to the digital phone technology. Estimates of market demand show net revenues for seven years of \$50M per annum. The estimated cost of production is \$200M. Assume that MARR = 12%, $r = 6\%$, and the volatility due to market uncertainty related to product demand is 35%. Determine the value of the patent.

SOLUTION

The patent provides the right to use the digital phone technology anytime over the next three years. Therefore,

$$V_3 = \$50\text{M}(P/A, 12\%, 7) = \$228.19\text{M},$$

$$V_0 = \$228.19\text{M}(P/F, 12\%, 3) = \$162.42\text{M},$$

$$I_3 = \$200\text{M},$$

$$T = 3,$$

$$r_f = 6\%,$$
$$\sigma = 35\%.$$

Substituting these values into Eq. (13.10), we obtain the Black–Scholes value of this call option: \$36.95M. With the real-options analysis, the firm now has an upper limit on the value of the patent. At the most, the firm should pay \$36.95M for it, and the firm can use this value as part of the negotiation process.

Growth Option

A growth option occurs when an initial investment is required to support follow-on investments, such as (1) an investment in phased expansion, (2) a Web-based technology investment, and (3) an investment in market positioning. The amount of loss from the initial investment represents the call-option premium. Making the initial investment provides the option to invest in any follow-on opportunity. For example, suppose a firm plans on investing in an initial small-scale project. If events are favorable, then the firm will invest in a large-scale project. The growth option values the flexibility to invest in the large-scale project if events are favorable. The loss on the initial project is viewed as the option premium.

EXAMPLE 13.7 Valuation of a Growth Option

A firm plans to market and sell its product in two markets: locally and regionally. The local market will require an initial investment, followed by some estimated cash inflows for three years. If events are favorable, the firm will invest and sell the product regionally, expecting benefits for four years. Figure 13.12 details the investment opportunities related to the growth options of this project.

Assume that the firm's MARR is 12% and the risk-free interest rate is 6%. Determine the value of this growth option.

SOLUTION

The NPW of each phase is as follows:

$$\begin{aligned} \text{PW}(12\%)_{\text{Small},\,0} &= -\$30 + \$10(P/F, 12\%, 1) + \$12(P/F, 12\%, 2) \\ &\quad + \$14(P/F, 12\%, 3) \\ &= -\$1.54\text{M [in year 0]}. \\ \text{PW}(12\%)_{\text{Large},\,3} &= -\$60 + \$16(P/F, 12\%, 1) + \$18(P/F, 12\%, 2) \\ &\quad + \$20(P/F, 12\%, 3) + \$20(P/F, 12\%, 4) \\ &= -\$4.42\text{M [in year 3]}. \end{aligned}$$