9.2 HW z test

(1) $n = 50$ Claim $\mu > 30$
$\bar{x} = 31$ H_0 $\mu = 30$
$\sigma = 2.5$ H_1 $\mu > 30$
$\alpha = .01$

z test

$z = 2.83$

$p = .002 < \alpha = .01$

reject H_0

There is enough evidence to support the claim.

(2) Claim $\mu \le 10.3$ $n = 100$
H_0 $\mu = 10.3$ $\bar{x} = 9.9$
H_1 $\mu > 10.3$ $\sigma = 2.1$
$\alpha = .05$

z test

$z = -1.90$

$p = .972 > \alpha = .05$

do not reject H_0

There is not enough evidence to reject the claim.

(3) $n = 32$ Claim $\mu = 15$
$\sigma = 6.2$ H_0 $\mu = 15$
$\alpha = .05$ H_1 $\mu \ne 15$

z test

$z = -.15$

$p = .88 > \alpha = .05$

do not reject H_0

There is not enough evidence to reject the claim.

t test

9.3

1. claim $\mu > 45$ $\quad n = 25$

   H_0 $\mu = 45$ $\quad \bar{x} = 48$

   H_1 $\mu > 45$ $\quad s = 5.4$

   $\alpha = .10$

   t test

   $t = 2.78$

   $p = .005 < \alpha = .10$

   reject H_0

   There is enough evidence to support the claim.

2. claim $\mu = 115$ $\quad n = 34$

   H_0 $\mu = 115$ $\quad \bar{x} = 121.2$

   H_1 $\mu \neq 115$ $\quad s = 24.2$

   $\alpha = .10$

   t test

   $t = 1.49$

   $p = .145 > \alpha = .10$

   do not reject H_0

   There is not enough evidence to reject the claim.

3. claim $\mu < 32$ $\quad n = 18$

   H_0 $\mu = 32$ $\quad \alpha = .05$

   H_1 $\mu < 32$

   t test

   $t = -1.94$

   $p = .034 < \alpha = .05$

   reject H_0

   There is enough evidence to support the claim.

9.4   1 prop z test

① claim $p < .25$ $n = 200$

H_0 $p = .25$ $\hat{p} = .193$

H_1 $p < .25$ $\alpha = .05$

$x = \hat{p} \cdot n = 38.6 \uparrow = 39$

1 prop z test

$z = -1.80$

$p = .036 < \alpha = .05$

reject H_0

There is enough evidence to support the claim.

② claim $p \leq .75$ $n = 150$

H_0 $p = .75$ $\hat{p} = .77$

H_1 $p > .75$ $\alpha = .01$

$x = \hat{p} \cdot n = 115.5 \uparrow = 116$

1 prop z test

$z = .66$

$p = .25 > \alpha = .01$

do not reject H_0

There is not enough evidence to reject the claim.

③ claim $p > .80$ $n = 150$

H_0 $p = .80$ $\hat{p} = .79$

H_1 $p > .80$ $\alpha = .10$

$x = \hat{p} \cdot n = 118.5 \uparrow = 119$

1 prop z test

$z = -.20$

$p = .58 > \alpha = .10$

do not reject H_0

There is not enough evidence to support the claim.

④ claim $p < .35$ $n = 400$

H_0 $p = .35$ $x = 156$

H_1 $p < .35$ $\alpha = .10$

1 prop z test

$z = 1.68$

$p = .95 > \alpha = .10$

do not reject H_0

There is not enough evidence to support the claim.

2 Sample Z test & 2 Sample Z interval

11.1 & 10.1

① Claim $u_1 \neq u_2$
H_0 $u_1 = u_2$
H_1 $u_1 \neq u_2$

2 Samp Z test

$Z = -2.79$

$p = .0053 < \alpha = .10$

reject H_0

There is enough evidence to support the claim.

② Claim $u_1 < u_2$
H_0 $u_1 = u_2$
H_1 $u_1 < u_2$

A	B
$\bar{x}_1 = 14$	$\bar{x}_2 = 15.1$
$\sigma_1 = 2.9$	$\sigma_2 = 3.3$
$n_1 = 60$	$n_2 = 60$

$\alpha = .05$

2 Samp Z test

$Z = -1.94$

$p = .026 < \alpha = .05$

reject H_0

there is enough evidence to support the claim.

③ Claim $u_1 = u_2$
H_0 $u_1 = u_2$
H_1 $u_1 \neq u_2$

2 Samp Z test

$Z = .20$

$p = .8396 > \alpha = .01$

do not reject H_0

There is not enough evidence to reject the claim.

④ $\sigma_1 = .6$
$\sigma_2 = .5$

Claim $u_1 > u_2$
H_0 $u_1 = u_2$
H_1 $u_1 > u_2$

2 Samp Z test

$Z = 2.62$

$p = .004 < \alpha = .05$

reject H_0

There is enough evidence to support the claim.

⑤ $\sigma_1 = 8795$ $\quad \sigma_2 = 9250$
$\bar{x}_1 = 102650$ $\quad \bar{x}_2 = 85430$
$n_1 = 42$ $\quad n_2 = 38$

2 Samp Z int

(13255, 21185)

2 Sample t test & 2 Sample t interval

(1) $n_1 = 26$ $n_2 = 31$
$\bar{x}_1 = 127$ $\bar{x}_2 = 117$
$S_1 = 14$ $S_2 = 9$
$\alpha = .01$
Claim $u_1 \neq u_2$
H_0 $u_1 = u_2$
H_1 $u_1 \neq u_2$

variances equal → pool

2 Samp t test
$t = 3.26$
$p = .002 < \alpha = .01$
reject H_0
There is enough evidence to support the claim.

(2) Alleg (#1) Erie (#2)
$n = 19$ $n = 15$
$\bar{x} = 49700$ $\bar{x} = 42000$
$S = 8800$ $S = 5100$
$\alpha = .05$

variances unequal → do not pool

Claim $u_1 > u_2$
H_0 $u_1 = u_2$
H_1 $u_1 > u_2$

2 Samp t test
$t = 3.19$
$p = .002 < \alpha = .05$
reject H_0
There is enough evidence to support the claim.

(3) Claim $u_1 \neq u_2$
H_0 $u_1 = u_2$
H_1 $u_1 \neq u_2$

variance equal → pool

2 Samp t test
$t = -2.76$
$p = .0116 > \alpha = .01$
do not reject H_0
There is not enough evidence to support the claim.

(4) pool ✓

2 Samp t interval
(39.511, 312.49)

11.2 & 10.2 Dependent Samples (Before/After)

① claim $u_d > 0$ (before - after)

$H_0 \; u_d = 0$

$H_1 \; u_d > 0$

t test

$t = 7.14$

$p = 2.71E-5 < \alpha = .01$

reject H_0

There is enough evidence to support the claim.

② claim $u_d < 0$ (before - after)

$H_0 \; u_d = 0$

$H_1 \; u_d < 0$

t test

$t = .785$

$p = .77 > \alpha = .05$

do not reject H_0

There is not enough evidence to support the claim.

③ claim $u_d \neq 0$ (before - after)

$H_0 \; u_d = 0$

$H_1 \; u_d \neq 0$

t test

$t = -2.16$

$p = .0676 > \alpha = .05$

do not reject H_0

There is not enough evidence to support the claim.

④ t interval (before - after)

$(-1.763, -1.287)$

11.3 & 10.3 2 prop z test & 2 prop z interval

① Claim $p_1 \neq p_2$
H_0 $p_1 = p_2$
H_1 $p_1 \neq p_2$

$x_1 = 17$
$n_1 = 54$

$x_2 = 18$
$n_2 = 41$

2 prop z test
$z = -1.24$
$p = .2138 > \alpha = .01$
do not reject H_0
There is not enough evidence to support the claim.

② males / females
$n_1 = 200$ / $n_2 = 220$
$\hat{p}_1 = .39$ / $\hat{p}_2 = .45$
$x_1 = \hat{p} \cdot n = 78$ / $x_2 = \hat{p} \cdot n = 99$

$\alpha = .05$
Claim $p_1 < p_2$
H_0 $p_1 = p_2$
H_1 $p_1 < p_2$

2 prop z test
$z = -1.24$
$p = .107 > \alpha = .05$
do not reject H_0
There is not enough evidence to support the claim.

③ $n_1 = 480$ $n_2 = 360$
$x_1 = 408$ $x_2 = 288$ $\alpha = .05$

2 prop z test
$z = 1.90$
$p = .029 < \alpha = .05$
reject H_0

claim $p_1 > p_2$
H_0 $p_1 = p_2$
H_1 $p_1 > p_2$

There is enough evidence to support the claim.

④ 2 prop z int
$(-.0378, -.022)$

$n_1 = 10000$
$\hat{p}_1 = .06$
$x_1 = 600$

$n_2 = 8000$
$\hat{p}_2 = .09$
$x_2 = 720$

11.4 2 Sample F test

(1)

* larger so Group 1 *
35-49
S = 7.96

18-34
S = 6.19
Group #2

Claim $\sigma_1^2 \neq \sigma_2^2$
H_0 $\sigma_1^2 = \sigma_2^2$
H_1 $\sigma_1^2 \neq \sigma_2^2$

2 Sample F test

T.S. F = 1.66
$p = .650 > \alpha = .05$
do not reject H_0
There is not enough evidence to support the claim.

District one = Group 1
District two = Group 2

(2) Claim = H_0: $\sigma_1^2 = \sigma_2^2$
H_1 $\sigma_1^2 \neq \sigma_2^2$

2 Sample F test

T.S. F = 1.282
$p = .6619 > \alpha = .10$
do not reject H_0
There is not enough evidence to reject the claim.

NY = Group 1
CA = Group 2

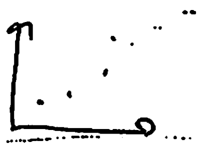
(3) Claim $\sigma_1^2 > \sigma_2^2$
H_0 $\sigma_1^2 = \sigma_2^2$
H_1 $\sigma_1^2 > \sigma_2^2$

2 Sample F test

T.S. F = 1.87
$p = .013 < \alpha = .05$
reject H_0
There is enough evidence to support the claim

4.1, 4.2, 13.1 Linear correlation and Regression

① increase
② decrease
③ −1 to 1 inclusive
④ The sample correlation coefficient r measures the strength & direction of a linear relationship between two variables;
$r = -.932$ indicates a stronger correlation because $|-.932| = .932$ is closer to 1 than $|.918| = .918$
⑤ r is the sample correlation coefficient
ρ is the population correlation coefficient
⑥ the fact that two variables have a linear relationship does not necessarily imply that one variable is the cause of the other.
⑦ strong negative linear correlation
(10 & 12) No linear correlation
⑪ perfect negative linear correlation
⑬ strong positive linear correlation
⑭ perfect positive linear correlation

⑮ 

* use linreg (ax+b) L_1, L_2 *
$r = .908$
strong positive linear correlation

⑯ 
$r = -.975$
strong negative linear correlation

⑰ 
$r = .967$
strong positive linear correlation

⑱ Claim $\rho \neq 0$

H_0: $\rho = 0$
H_a: $\rho \neq 0$

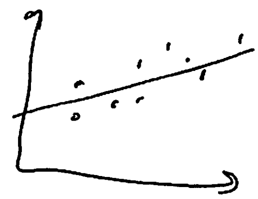
<u>linregttest</u>
$t = 1.95$
$p = .0991 > \alpha = .01$
do not reject H_0
There is not enough evidence to support the claim.

Regression

* Use linReg (ax+b) L_1, L_2, Y_1 *

Please also see my notes,

1. $\hat{y} = .065x + .465$

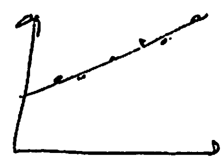


a) $\hat{y}(800) = 52.5$

b) $\hat{y}(750) = 49.3$

c) out of range of the original data

d) $\hat{y}(625) = 41.1$



2. $\hat{y} = 7.45x + 37.449$

a) $\hat{y}(3) = 59.8$

b) $\hat{y}(6.5) = 85.9$

c) $\hat{y}(13)$ out of range of the original data

d) $\hat{y}(4.5) = 70.974$

3. $\hat{y} = -2.04x + 520.668$

a) out of range of the original data

b) $\hat{y}(67) = 383.73$

c) $\hat{y}(90) = 336.72$

d) $\hat{y}(83) = 351.63$

① $r^2 = (.465)^2 = .216$
about 21.6% of the variation is explained &
about 78.4% of the variation is unexplained.

② $r^2 = (-.957)^2 = .916$
about 91.6% of the variation is explained &
about 8.4% of the variation is unexplained.

13.3 Multiple Regression

Substitute x_1 & x_2 into the equation

① a) 18832.7 pounds per acre

b) 18016.4

c) 17350.6

d) 16190.3

② a) 7.5 cubic feet

b) 16.8

c) 51.9

d) 62.1

12.1 Goodness of fit

① Claim = H_0: The distribution is as stated.
H_1: Distribution is not as stated.

GOF test

$\chi^2 = 18.7815$

$p = 8.8E-4 < \alpha = .10$

reject H_0

There is enough evidence to reject the claim.

② H_0: The distribution is as stated

Claim = H_1: The distribution is not as stated.

GOF Test

$\chi^2 = 17.595$

$p = .0073 < \alpha = .01$

reject H_0.

There is enough evidence to support the claim.

① Claim = H_0: Skill level in a subject is independent of location.

H_1: Skill level in a subject is dependent of location.

χ^2 test

T.S. $\chi^2 = .297$

$p = .8619 > \alpha = .01$

do not reject H_0.

There is not enough evidence to reject the claim.

② Claim = H_1: The number of times former smokers tried to quit is dependent of gender.

H_0: The number of times former smokers tried to quit is independent of gender.

χ^2 test

T.S. $\chi^2 = .002$

$p = .9991 > \alpha = .05$

do not reject H_0.

There is not enough evidence to support the claim.

1401 Anova

① $H_0: \mu_1 = \mu_2 = \mu_3$

claim = H_1: atleast one mean is different from the others

Anova (L_1, L_2, L_3)

T.S. $F = 4.080$

$p = .029 < \alpha = .05$

reject H_0

There is enough evidence to support the claim.

② Claim = $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_3$

H_1: atleast one mean is different from the others

Anova (L_1, L_2, L_3, L_4)

T.S. $F = 7.491$

$p = .00069 < \alpha = .01$

reject H_0

There is enough evidence to reject the claim

Chapter 15 Nonparametric Statistics HW Solutions

15.1 Sign Test

1. A nonparametric test is a hypothesis test that does not require any specific conditions concerning the shapes of populations of the values of any population parameters.
2. Median
3. When n is less than or equal to 25, the test statistic is equal to x (the smaller of the + or – signs). When n is greater than 25, the test statistic is equal to $z = \dfrac{(x+.5)-.5n}{\dfrac{\sqrt{n}}{2}}$
4. Answers will vary. *Sample answer*: It is called the sign test because each value in a sample is compared to the hypothesized median and assigned a + or – sign, based on whether the difference is positive or negative. The number of + signs and – signs are used to determine whether the null hypothesis should be rejected.
5. Verify that the sample is random. Identify the claim and state Ho and H1. Identify the level of significance and sample size. Find the critical value using the table. Calculate the test statistic. Make a decision and write a long conclusion.

6. A sample must be randomly selected from each population and the samples must be dependent.
7. a) claim: median > 300 Ho: median = 300 H1: median > 300
   b) critical value is 1.
   c) x= 5
   d) Because x > 1, we do not reject Ho
   e) There is not enough evidence to support the claim.
8. a) claim: median $\leq$ 193,000 Ho: median = 193,000 H1: median > 193,000
   b) critical value is 1.
   c) x= 4
   d) Because x > 1, we do not reject Ho
   e) There is not enough evidence to reject the claim.

9. a) claim= H1: scores will decrease after treatment
      Ho: scores will stay the same after treatment
   b) critical value is 1.
   c) x= 0
   d) Because x $\leq$ 1, reject Ho
   e) There is enough evidence to support the claim.

10. a) claim= H1: scores will improve
       Ho: scores will stay the same
    b) critical value is 1.
    c) x= 1
    d) Because x $\leq$ 1, reject Ho
    e) There is enough evidence to support the claim.

15.2&15.3 Wilcoxon Rank Sum and Wilcoxon Signed Rank

1. When the samples are dependent, use the Wilcoxon signed-rank test. When the samples are independent, use the Wilcoxon rank sum test.

2. a) claim: scores will increase or stay the same
    Ho: scores will stay the same
    H1: scores will decrease
   b) Wilcoxon signed rank
   c) critical value is 10.
   d)

Before	After	Difference	Absolute Difference	Rank	Signed Rank
108	99	9	9	8	8
109	115	-6	6	4.5	-4.5
120	105	15	15	12	12
129	116	13	13	10.5	10.5
112	115	-3	3	2	-2
111	117	-6	6	4.5	-4.5
117	108	9	9	8	8
135	122	13	13	10.5	10.5
124	120	4	4	3	3
118	126	-8	8	6	-6
130	128	2	2	1	1
115	106	9	9	8	8

   Sum of negative ranks= -17
   Sum of positive ranks= 61
   Ws= 17

   e) Because Ws> 10, do not reject Ho
   f) There is not enough evidence to reject the claim.

3. a) claim: There is difference between the 2 groups.
    Ho: There Is no difference between the 2 groups.
    H1: There is difference between the 2 groups.
   b) Wilcoxon rank sum test
   c) invnorm(.025, 0, 1) gives critical values are Z = $\pm$ 1.96
   d) next page

Ordered data	Sample	Rank
46	B	1
52	B	2.5
52	B	2.5
54	B	4
56	B	5.5
56	B	5.5
58	B	7
62	B	8
65	B	9
72	B	10
78	B	11
81	A	12
82	A	13
84	A	14
86	A	15.5
86	A	15.5
87	A	17
90	A	18
93	A	19.5
93	A	19.5
95	A	21

R = sum of ranks of bachelor's degrees = 165

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2}$$

$$\mu_R = \frac{10(10+11+1)}{2} = 110$$

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}$$

$$\sigma_R = \sqrt{\frac{10*11*(10+11+1)}{12}} = 14.201$$

$$z = \frac{R-\mu_R}{\sigma_R}$$

$$z = \frac{165-110}{14.201} = 3.87$$

e) Because z > 1.96, reject Ho
f) There is enough evidence to support the claim.

4. a) claim: There is difference between the 2 groups.
   Ho: There Is no difference between the 2 groups.
   H1: There is difference between the 2 groups.
   b) Wilcoxon rank sum test
   c) invnorm(.025, 0, 1) gives critical values are Z = $\pm$ 1.96
   d)

Ordered data	Sample	Rank
47	W	1
49	W	2
51	W	3
52	W	4
53	W	5
55	W	6.5
55	W	6.5
56	M	8.5
56	W	8.5
58	M	10
59	W	11
60	M	12
61	W	14
61	M	14
61	W	14
62	M	16
64	M	17.5
64	M	17.5
65	M	19
68	M	20
70	M	21.5
70	M	21.5
79	M	23

R = sum of ranks of bachelor's degrees = 75.5

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2}$$

$$\mu_R = \frac{11(11+12+1)}{2} = 132$$

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

$$\sigma_R = \sqrt{\frac{11*12*(11+12+1)}{12}} = 16.248$$

$$z = \frac{R - \mu_R}{\sigma_R}$$

$$z = \frac{75.5 - 132}{16.248} = -3.48$$

e) Because z < -1.96, reject Ho
f) There is enough evidence to support the claim.

Kruskal – Wallis Test

1. The conditions for using a Kruskal-Wallis test are that the samples must be random and independent, and the size of each sample must be at least 5.

2. a) claim= H1 is the distributions of the annual premiums in at least one state is different from the others.
   Ho: The distribution of the annual premiums is the same in all three states.
   b) $\chi_0^2 = 5.991$; rejection region $\chi^2 > 5.991$
   c) next page

Ordered data	Sample	Rank
605	VA	1
616	VA	2
688	VA	3
695	VA	4
784	MA	5
800	VA	6
848	CT	7
885	VA	8
916	MA	9
929	CT	10
982	VA	11
1007	MA	12
1013	CT	13
1052	MA	14
1053	CT	15
1070	CT	16
1132	MA	17
1137	MA	18
1163	CT	19
1288	CT	20
1322	MA	21

R1 = 100 R2 = 96 R3 = 35

$$H = \frac{12}{(N(N+1))}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right) - 3(N+1)$$

$$H = \frac{12}{21(21+1)}\left(\frac{100^2}{7} + \frac{96^2}{7} + \frac{35^2}{7}\right) - 3(21+1) = 9.848$$

d) Because H > 5.991, we reject Ho

e) There is enough evidence to support the claim.

3. a) claim= H1 is the distributions of the annual salaries in at least one state is different from the others.
   Ho: The distribution of the annual salaries is the same in all four states.
   b) $\chi_0^2 = 6.251$; rejection region $\chi^2 > 6.251$
   c) next page

Ordered data	Sample	Rank
28.3	KY	1
29.8	SC	2.5
29.8	SC	2.5
31.6	WV	4.5
31.6	WV	4.5
33.4	WV	6
33.7	KY	7
34.7	SC	8
34.9	WV	9
35.3	KY	10.5
35.3	KY	10.5
35.5	NC	12
36.1	SC	13
36.6	NC	14
37.0	KY	15
37.4	SC	16
39.6	NC	17
41.9	NC	18.5
41.9	WV	18.5
42.7	WV	20
42.9	SC	21
43.5	NC	23
43.5	NC	23
43.5	SC	23
45.9	KY	25
47.1	WV	26
54.3	NC	27
57.5	KY	28

R1 = 97   R2 = 134.5   R3 = 86   R4 = 88.5

$$H = \frac{12}{(N(N+1))}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} + \frac{R_4^2}{n_4}\right) - 3(N+1)$$

$$H = \frac{12}{28(28+1)}\left(\frac{97^2}{7} + \frac{134.5^2}{7} + \frac{86^2}{7} + \frac{88.5^2}{7}\right) - 3(28+1) = 3.206$$

d) Because H < 6.251, we do not reject Ho

e) There is not enough evidence to support the claim.