Announcements

Poisson Distribution

A discrete RV X is said to have a Poisson distribution with parameter μ ($\mu > 0$) if the pmf of X is:

$$\boxed{P[X=x] = \frac{\mu^{x} \cdot e^{-\mu}}{x!}} \quad , X = 0, 1, 2, 3, \ldots$$

Example: X = the number of customers arriving at a bank per unit time on Monday morning

$$\left.\begin{aligned} E[X] &= \mu \\ V[X] &= \mu \end{aligned}\right\} \quad \mu \text{ is also called the mean}$$

Problem 87 (page 136)

requests ~ Poisson : $\mu = 4/\text{hour}$

(a) P(10 requests during 2 hours)

$$P[X=x] = \frac{\mu^x \cdot e^{-\mu}}{x!} \qquad P[X_t=x] = \frac{(\mu t)^x \left(e^{-\mu t}\right)}{x!}$$

$$\left.\begin{array}{l} \mu = 4 \dfrac{\text{requests}}{\text{hour}} \\ t = 2 \text{ hours} \end{array}\right\} \mu t = 8 \text{ requests}$$

$$P[X_t = 10] = \frac{8^{10}\left(e^{-8}\right)}{10!} = \boxed{0.099}$$

(b) 30 min-break for lunch

$$P(X_t = 0) = \frac{2^0 \cdot e^{-2}}{0!} = \boxed{0.135}$$

$$\left.\begin{array}{l} t = 30\text{min} = 0.5 \text{ hours} \\ \mu = 4 \text{req}/\text{hour} \end{array}\right\} \mu t = \underline{2 \text{ requests}}$$

(c) $E[X] = \mu t = 2$

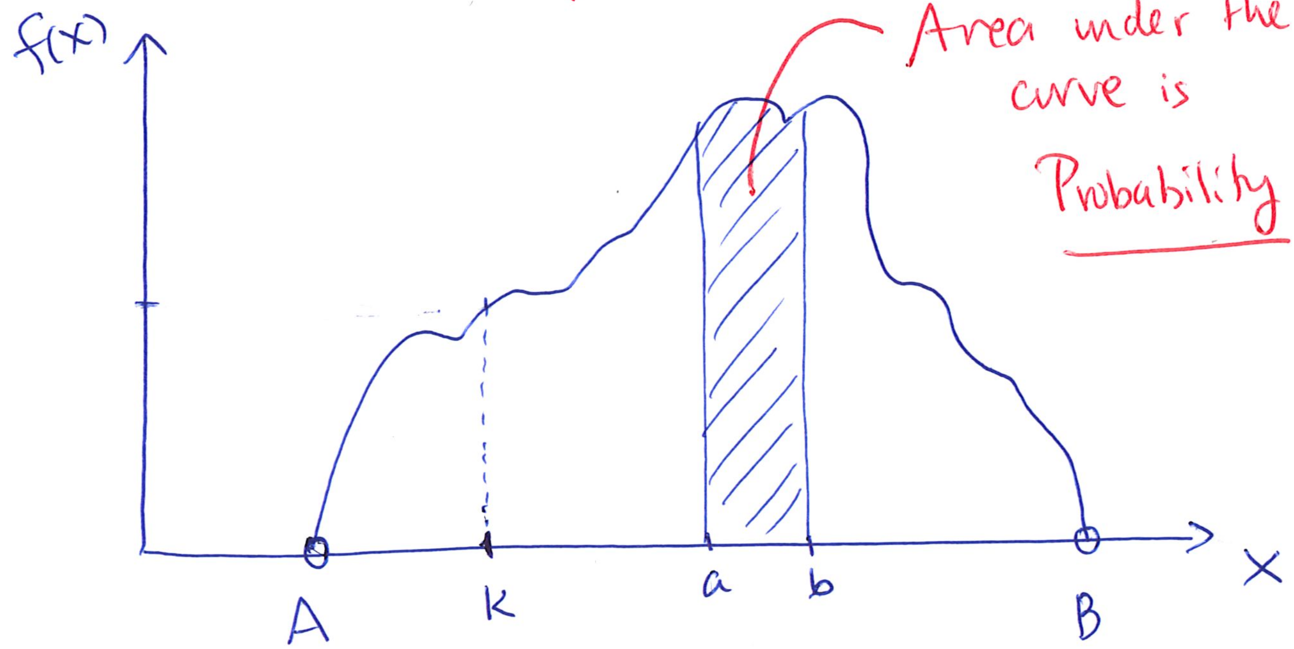
CHAPTER 4

Continuous RV & Probability Distributions

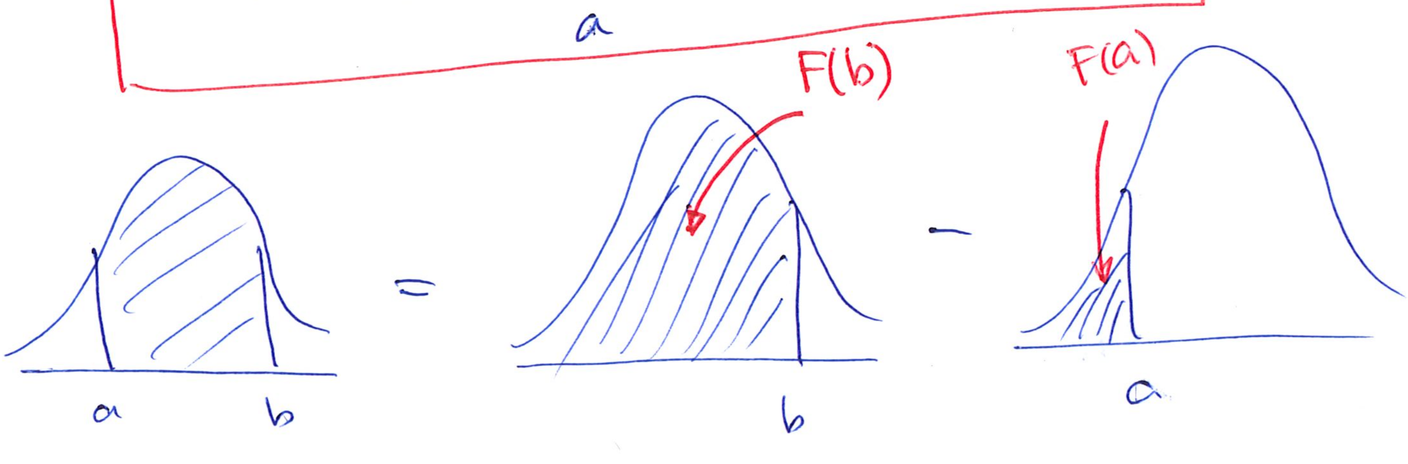
X = continuous RV with pdf : $f(x)$

probability
density
function

Area under the curve is **Probability**



$$\boxed{\begin{aligned} &P[X=k] = 0 \\ &P(a \le x \le b) = \int_a^b f(x)\,dx = F(b) - F(a) \end{aligned}}$$

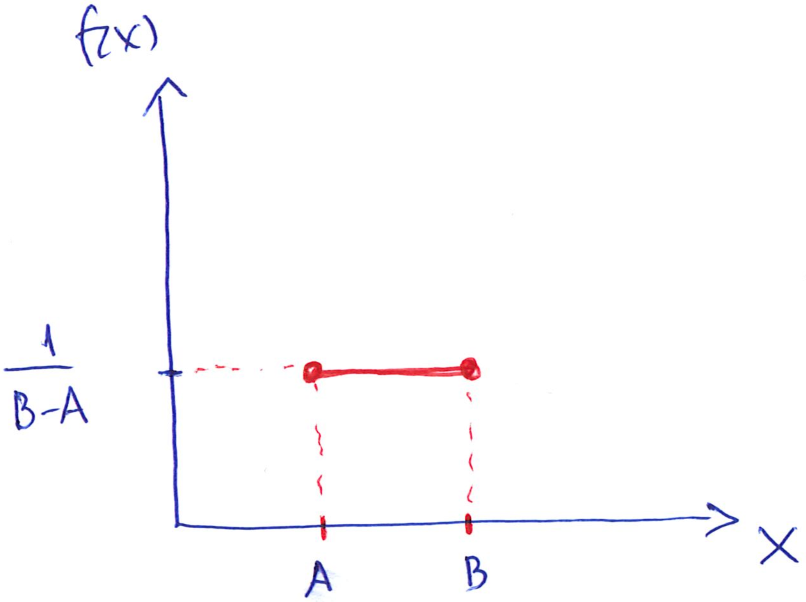


$$\left.\begin{aligned} P(a < x < b) &= \\ P(a \leq x < b) &= \\ P(a < x \leq b) &= \end{aligned}\right\} = P(a \leq x \leq b)$$

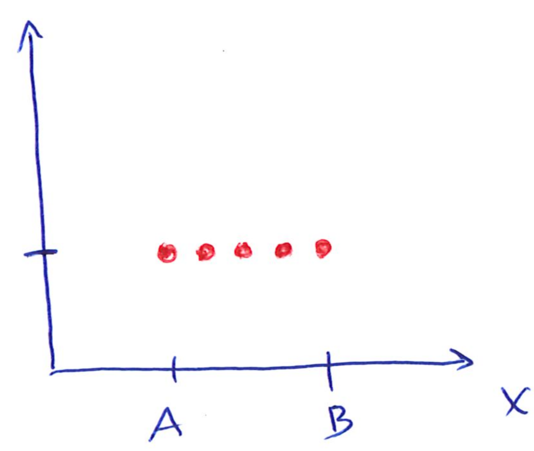
1 Uniform Distribution

RV X is said to be uniformly distributed on the interval [A, B] if the pdf is:

$$f(x; A, B) = \begin{cases} \dfrac{1}{B-A} & A \leq x \leq B \\ 0 & \text{otherwise} \end{cases}$$



if X is a discrete RV



Discrete RV

pmf $\to p(x)$

cdf $\to F(x)$

$$F(x) = \sum_{y \le x} p(x)$$

Continuous RV

pdf $\to f(x)$

cdf $\to F(x)$

$$P(a \le x \le b) = \int_a^b f(x)\,dx = F(b) - F(a)$$

$$P(x \le a) = \int_{-\infty}^{a} f(x)\,dx = F(a)$$

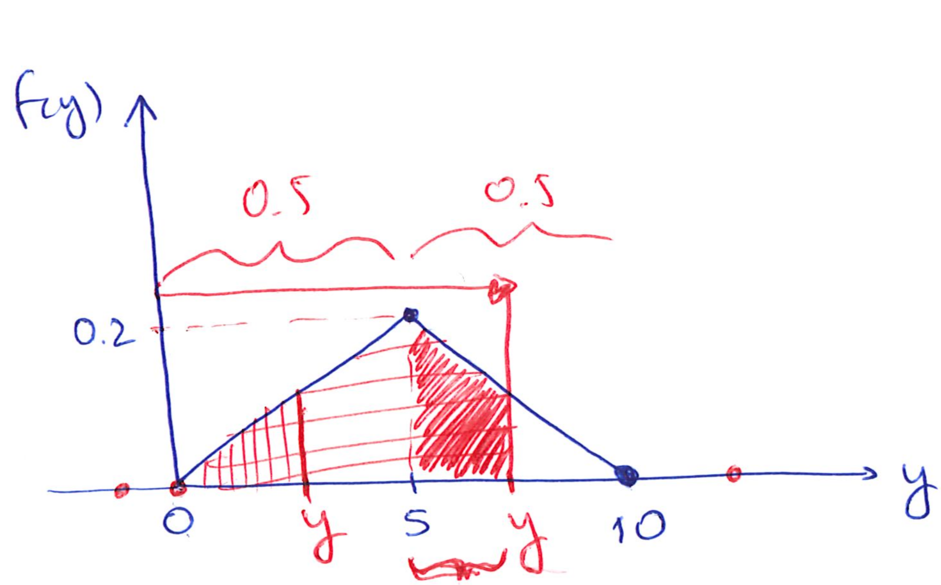
$$F(x) = \int_{-\infty}^{x} f(x)\,dx \Longrightarrow \boxed{F'(x) = f(x)}$$

Example:

$$f(y) = \begin{cases} \frac{1}{25}y & 0 \le y < 5 \\ \frac{2}{5} - \frac{1}{25}y & 5 \le y \le 10 \\ 0 & (y < 0 \text{ or } y > 10) \end{cases}$$

Find the CDF: $F(y)$

$$F(y) = \begin{cases} \boxed{0} & , y < 0 \\ \int_0^y \frac{1}{25}k\,dk = \frac{1}{25} \cdot \frac{y^2}{2} = \boxed{\frac{y^2}{50}} & , 0 \le y < 5 \\ \underbrace{\int_5^y \left(\frac{2}{5} - \frac{1}{25}k\right)dk}_{5 \to y} + \underbrace{\frac{5^2}{50}}_{0 \to 5} & , 5 \le y \le 10 \\ \boxed{1} & , y > 10 \end{cases}$$



$$F(y)\begin{cases} 0 & , \ y<0 \\ y^2/50 & , \ 0\le y<5 \\ 0.5+\frac{2}{5}(y-5)-\frac{1}{50}(y^2-25) & , \ 5\le y\le 10 \\ 1 & , \ y>10 \end{cases}$$