

Announcements

Do Writing Assignment 3 Calculations

WAB due April 7<sup>th</sup> (3.3)WAB peer review due April ~~10~~ 14<sup>th</sup>

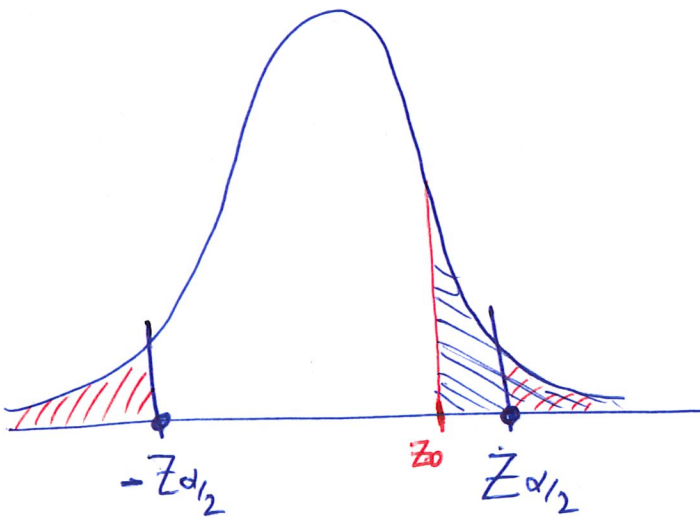
Testing:  $H_0: \mu = \mu_0$   
 $H_1: \mu \neq \mu_0$

$\sigma$  is known

Then we calculate a test statistic:

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Note: if  $H_0$  is true  
 then  $Z_0 \sim N(0, 1)$

Rules:Reject  $H_0$  if  $|Z_0| > Z_{\alpha/2}$ Fail to reject  $H_0$  if:

$$-Z_{\alpha/2} \leq Z_0 \leq Z_{\alpha/2}$$

Rejection  $H_0$   
areaFail to reject  $H_0$   
areaRejection  $H_0$   
area

Example 8.5 (page 319)

(2)

$H_0: \mu = 75$

Population is normal with  $\sigma = 9$

$H_a: \mu < 75$

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$n = 25$

$\bar{X} = 70.8$

$\sigma = 9$

$\mu_0 = 75$

$$Z_0 = \frac{70.8 - 75}{9 / \sqrt{25}} = -2.333$$

$\alpha = 5\%$



$Z_0 = -2.333$   
← Reject  $H_0$  | → Fail to reject  $H_0$

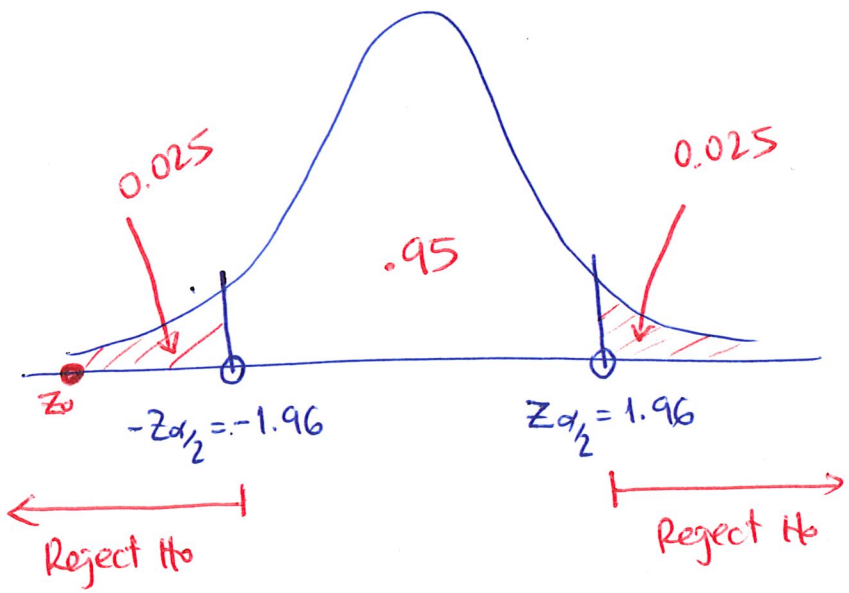
Then Reject  $H_0$  .. at 95% confidence level.

Now, if :  $H_0: \mu = 75$

$H_a: \mu \neq 75$  ( two-sided test )

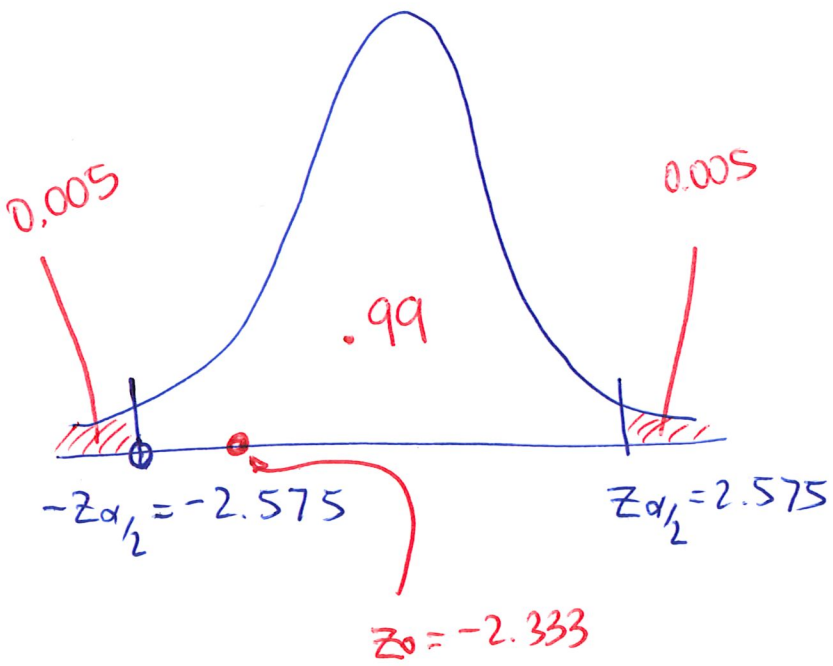
$\alpha = 5\%$

$$Z_0 = \frac{70.8 - 75}{9/\sqrt{25}} = -2.333$$



Reject  $H_0$   
 There is significant difference between 75 and the real  $\mu$

However, if  $\alpha = 0.01$



Fail to Reject  $H_0$   
 There is no significant difference between 75 and the real  $\mu$

- So, decision depends on the choice of  $\alpha$
- Hence, we could have a different decision when we change the value of  $\alpha$ .
- That is problematic, as the choice of value of  $\alpha$  is very subjective.

What are the cost-implications of  $\alpha$ -error?

$$\alpha = P(\text{Rejecting } H_0 / H_0 \text{ is true})$$

- Thus, instead of using a subjective  $\alpha$ -value, we present hypothesis result in terms of p-value

$$0 \leq \text{p-value} \leq 1$$

p-value represents an Area

p-value is related to  $Z_0$

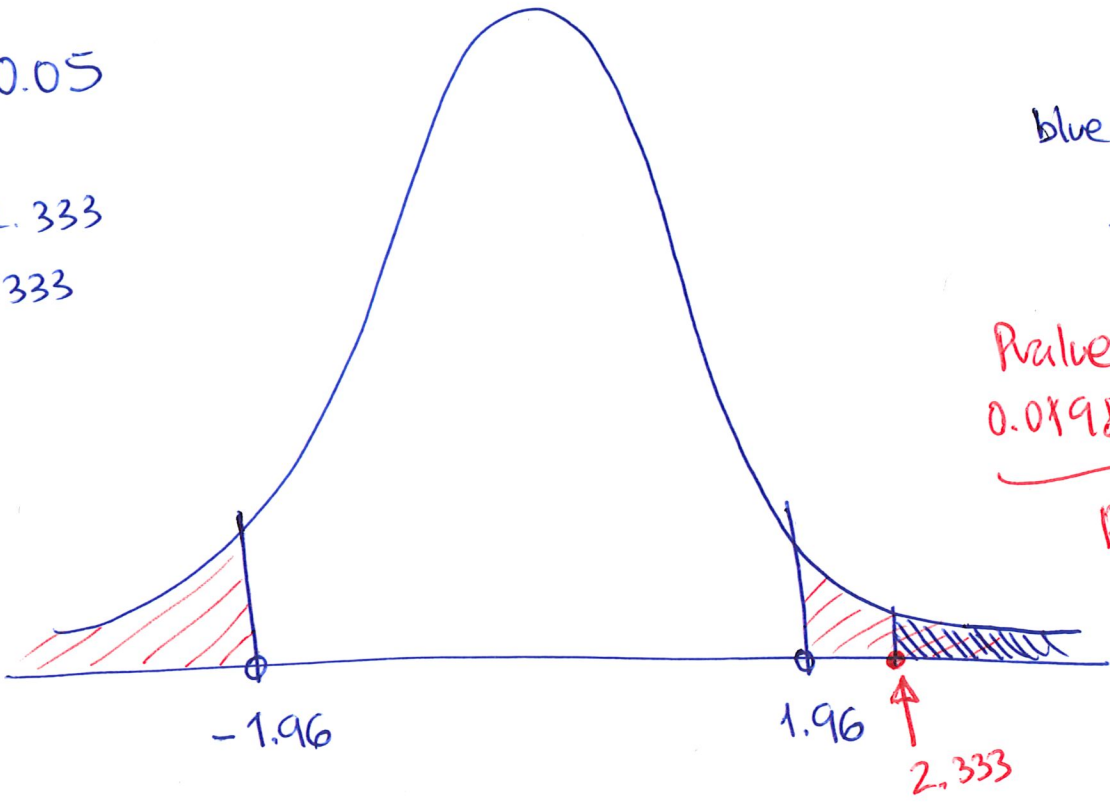
$\alpha = 0.05$

$Z_0 = -2.333$

$|Z_0| = 2.333$

blue shaded - area  
= Pvalue / 2

Pvalue  $\leq \alpha$   
 $0.0198 \leq 0.05$   
Reject  $H_0$

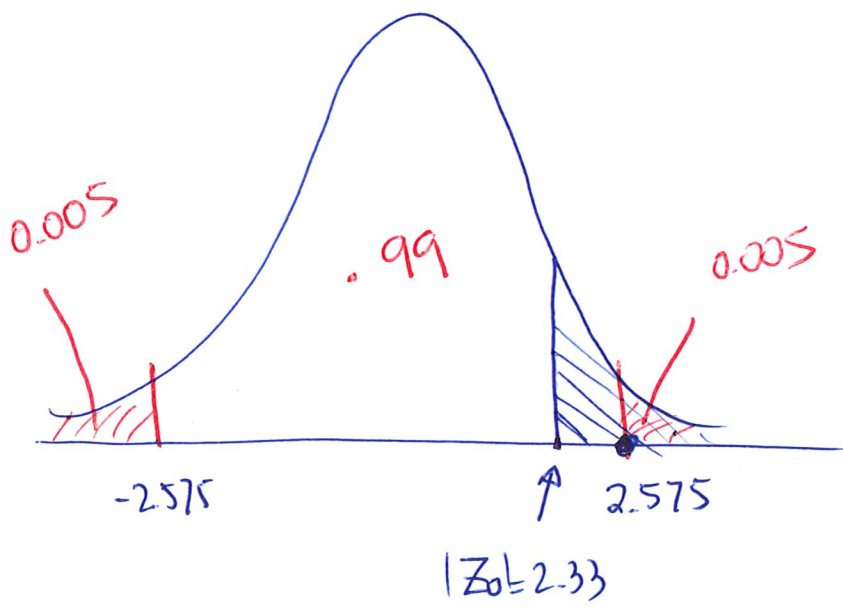


Then: Pvalue = 2 [shaded - Area] = 2 (0.0099) = 0.0198

If Pvalue  $\leq \alpha \Rightarrow$  REJECT  $H_0$

if  $\alpha = 0.01$

In this case Pvalue  $> \alpha \Rightarrow$  Fail to reject  $H_0$   
(0.0198  $>$  0.01)



Pvalue = minimum  $\alpha$ -error needed to reject  $H_0$

$$\text{Reject } H_0 \left\{ \begin{array}{l} \mu \notin \text{CI of } \mu \quad (\text{from ch7}) \\ |z_0| > z_{\alpha/2} \quad \checkmark \\ P\text{value} \leq \alpha \quad \checkmark \end{array} \right.$$

### Writing Assignment 3 - Calculations

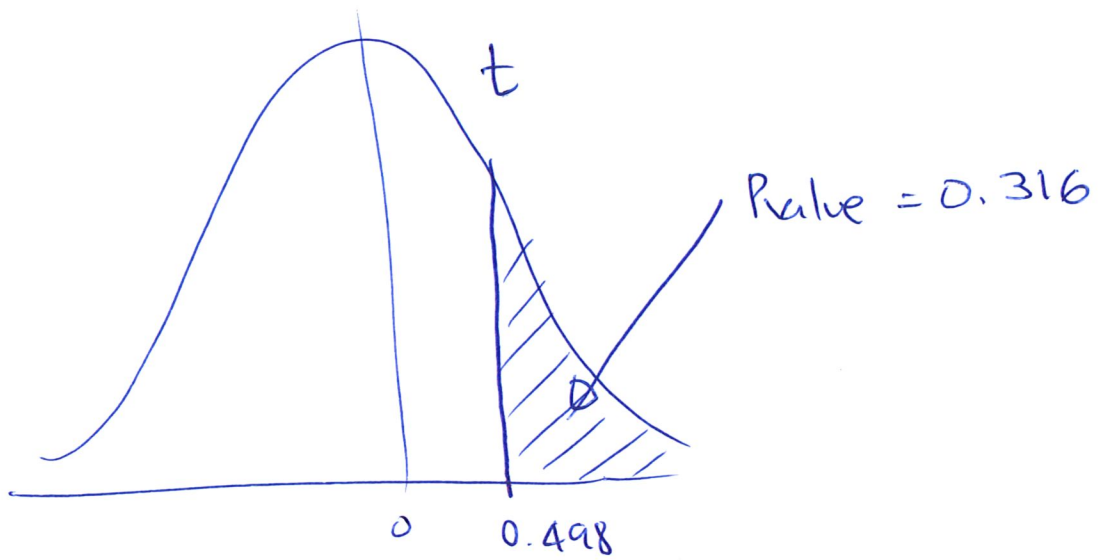
$$\left. \begin{array}{l} n = 8 \\ \bar{x} = 3.72 \\ s = 1.25 \\ \alpha = 0.05 \\ \mu_0 = 3.50 \end{array} \right\} \begin{array}{l} H_0: \mu = 3.50 \\ H_a: \mu > 3.50 \\ \\ t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1} \end{array}$$

Pvalue = area under the  $t_{n-1}$  curve to the right of  $t$

$$t = \frac{3.72 - 3.50}{1.25/\sqrt{8}} \sim t_7 \qquad t = 0.498$$

t = 0.50

7



$$P_{\text{value}} = 0.316 > 0.05 = \alpha$$

$\Rightarrow$  Fail to reject  $H_0$

There is not significant difference between the real  $\mu$  and 3.50