UNIT 2
Construction Math and Application

TECHNICAL TERMS

arc
area
circumference
decimal fraction
denominator
diameter
improper fraction
mixed number
numerator
pi
proper fraction
radius

LEARNING OBJECTIVES

After completing this unit, you will be able to:

- Convert between improper fractions and mixed numbers.
- Add, subtract, multiply, and divide decimal fractions.
- Calculate dimensions.
- Calculate areas and volumes of objects.
- Relate math to construction problems.

Construction workers and estimators often need to make calculations when working with prints. This unit deals with construction-oriented calculations, including fractions and decimals in both customary and metric units.

Fractions

Fractions are written with one number over the other, such as $\frac{9}{16}$. The number on the bottom (*16*) is called the ***denominator***. It indicates the number of equal parts into which a unit is divided. The number on top (*9*) is called the ***numerator***. It indicates the number of equal parts taken, **Figure 2-1**. In the fraction shown (*9/16*), nine of the sixteen parts are taken.

A ***proper fraction*** is one whose numerator is less than its denominator, such as 7/16 or 3/4. An ***improper fraction*** is one whose numerator is greater than its denominator, such as 5/4 or 19/16. A ***mixed number*** is a number that consists of a whole number and a proper fraction, such as 2 3/4 or 5 1/8. Proper fractions represent numbers between zero and one. Improper fractions and mixed numbers represent numbers larger than one.

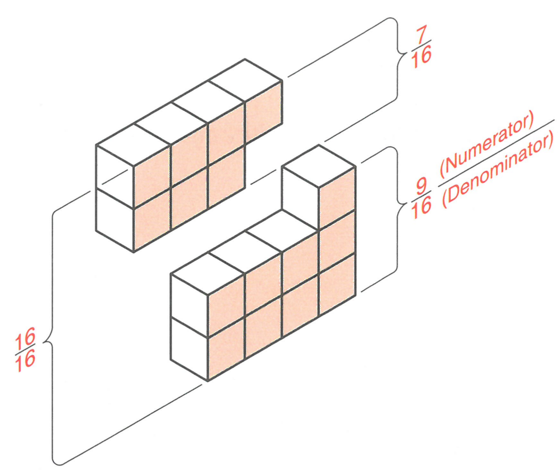

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Figure 2-1. The fraction 9/16 represents 9 pieces of a whole divided into 16 equal pieces.

Using Fractions

- Whole numbers can be changed to fractions by multiplying the numerator and denominator by the same number.
  Example: Change 6 (whole number) into fourths.

  $$\frac{6}{1} \times \frac{4}{4} = \frac{24}{4}$$

  Each whole unit contains 4 fourths.
  Six units contain 6×4 fourths, or 24 fourths.
  The value of the number has not changed.
- Mixed numbers can be changed to fractions by changing the whole number to a fraction with the same denominator as the fractional part and adding the two fractions.

  Example: Convert 3 5/8 to an improper fraction.

  $$3\frac{5}{8} = \left(\frac{3}{1} \times \frac{8}{8}\right) + \frac{5}{8} = \frac{24}{8} + \frac{5}{8} = \frac{29}{8}$$

  Three units contain 3×8 eighths, or 24 eighths. Adding the 5/8 part of the mixed number to 24/8 gives us 29/8.
- Improper fractions can be reduced to a whole or mixed number by dividing the numerator by the denominator:

  $$\frac{17}{4} = 17 \div 4 = 4\frac{1}{4}$$

- Fractions can be reduced to the lowest form by dividing the numerator and denominator by the same number:

  $$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

  The value of a fraction does not change if the numerator and denominator are divided by the same number, since this is the same as dividing by 1.
- Fractions can be changed to higher terms by multiplying the numerator and denominator by the same number:

  $$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$$

  The value of a fraction is not changed by multiplying the numerator and denominator by the same number.

ADDING FRACTIONS

To add fractions, the denominators must all be the same.

Example: $\frac{5}{16} + \frac{3}{8} + \frac{11}{32} = ?$

The *least common denominator (LCD)* into which these denominators can be divided is 32. Change all fractions to have 32 in the denominator:

$$\frac{5}{16} \times \frac{2}{2} = \frac{10}{32}$$

$$\frac{3}{8} \times \frac{4}{4} = \frac{12}{32}$$

Now that the fractions have the same denominator, add their numerators. The common denominator is used in the sum:

$$\frac{10}{32} + \frac{12}{32} + \frac{11}{32} = \frac{33}{32}$$

Convert to a mixed number:

$$\frac{33}{32} = 1\frac{1}{32}$$

PRACTICE PROBLEMS

Adding Fractions

Add the following fractions. Reduce answers to lowest form.

1. $\frac{3}{4} + \frac{1}{8} + \frac{1}{2} =$
2. $\frac{7}{8} + \frac{3}{16} =$
3. $\frac{5}{12} + \frac{3}{8} + \frac{3}{4} =$
4. $\frac{3}{10} + \frac{9}{10} + \frac{1}{20} =$
5. $\frac{7}{16} + \frac{3}{32} + \frac{1}{4} =$
6. $1\frac{3}{4} + \frac{7}{8} + 1\frac{1}{16} =$
7. $\frac{5}{32} + \frac{7}{64} + \frac{7}{8} =$
8. $1\frac{3}{8} + \frac{3}{32} + \frac{7}{16} =$
9. $3\frac{1}{16} + \frac{9}{16} + \frac{1}{2} =$
10. $5\frac{1}{5} + \frac{3}{10} + 8\frac{1}{2} =$
11. $4\frac{5}{8} + 20\frac{7}{32} =$
12. $\frac{3}{8} + \frac{7}{64} + \frac{9}{16} =$
13. $12\frac{7}{8} + 25\frac{3}{8} =$
14. $\frac{21}{32} + \frac{9}{64} + \frac{1}{4} =$
15. $\frac{3}{8} + 1\frac{1}{2} + \frac{7}{16} + \frac{7}{8} =$

SUBTRACTING FRACTIONS

To subtract fractions, the denominators must all be the same.

Example: $\frac{3}{4} - \frac{5}{16} = ?$

The least common denominator into which these denominators can be divided is 16.

Change 3/4 so that its denominator is 16:

$$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$$

Subtract the numerators and retain the common denominator:

$$\frac{12}{16} - \frac{5}{16} = \frac{7}{16}$$

PRACTICE PROBLEMS

Subtracting Fractions

Subtract the following fractions. Reduce answers to lowest form.

1. $\frac{3}{8} - \frac{1}{4} =$
2. $\frac{3}{4} - \frac{5}{16} =$
3. $1\frac{7}{8} - \frac{13}{16} =$
4. $3\frac{1}{2} - \frac{9}{16} =$ (*borrow* $\frac{16}{16}$ *from* 3)
5. $10\frac{3}{8} - 7\frac{3}{32} =$
6. $5 - 2\frac{3}{8} =$
7. $12\frac{1}{16} - 8\frac{1}{2} =$
8. $4\frac{1}{4} - 3\frac{1}{16} =$
9. $20\frac{7}{8} - 11\frac{3}{64} =$
10. $15\frac{5}{8} - 5\frac{1}{2} =$

MULTIPLYING FRACTIONS

Fractions can be multiplied as follows:

1. Change all mixed numbers to improper fractions.
2. Multiply all numerators to get the numerator of the answer.
3. Multiply all denominators to get the denominator of the answer.
4. Reduce the fraction to lowest terms.

Example:

$\frac{1}{2} \times 3\frac{1}{8} \times 4 = ?$

$\frac{1}{2} \times \frac{25}{8} \times \frac{4}{1} = \frac{100}{16}$

$\frac{100}{16} = 6\frac{4}{16} = 6\frac{1}{4}$

PRACTICE PROBLEMS

Multiplying Fractions

Multiply the following fractions. Reduce answers to lowest terms.

1. $\frac{3}{4} \times \frac{1}{2} =$
2. $2\frac{5}{8} \times \frac{1}{4} =$
3. $\frac{7}{8} \times 5 =$
4. $6\frac{3}{4} \times \frac{1}{3} =$
5. $12\frac{1}{2} \times \frac{1}{2} =$
6. $4\frac{3}{4} \times \frac{1}{2} \times \frac{1}{8} =$
7. $16 \times \frac{3}{4} =$
8. $9\frac{5}{8} \times \frac{1}{2} =$
9. $10 \times \frac{4}{5} =$
10. $\frac{14}{3} \times 6 =$

DIVIDING FRACTIONS

Fractions can be divided as follows:

1. Change all mixed numbers to improper fractions.
2. Invert (turn upside-down) the divisor and multiply.

Example:

$$5\frac{1}{4} \div 1\frac{1}{2} = ?$$

$$\frac{21}{4} \div \frac{3}{2} =$$

$$\frac{21}{4} \times \frac{2}{3} = \frac{42}{12}$$

$$\frac{42}{12} = 3\frac{6}{12} = 3\frac{1}{2}$$

PRACTICE PROBLEMS

Dividing Fractions

Divide the following fractions. Reduce answers to lowest terms.

1. $2\frac{3}{4} \div 6 =$
2. $12 \div \frac{3}{4} =$
3. $16\frac{1}{8} \div 2 =$
4. $8\frac{2}{3} \div \frac{1}{3} =$
5. $16\frac{1}{4} \div 20 =$
6. $\frac{7}{8} \div \frac{7}{16} =$
7. $15 \div 1\frac{1}{4} =$
8. $21 \div 3\frac{1}{8} =$
9. $5\frac{1}{4} \div \frac{3}{8} =$
10. $3\frac{5}{8} \div 2 =$

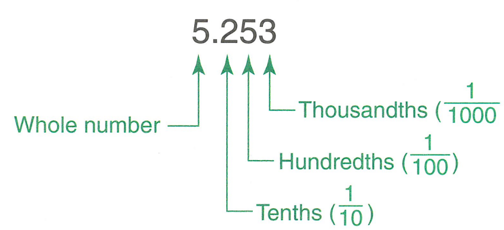
Decimal Fractions

The denominator in a ***decimal fraction*** is 10 or a multiple of 10 (100, 1000, etc.). When writing decimal fractions, omit the denominator and place a decimal point in the numerator.

3/10 is written 0.3 (three tenths)
87/100 is written 0.87 (eighty-seven hundredths)
375/1000 is written 0.375 (three hundred seventy-five thousandths)
4375/10000 is written 0.4375 (four thousand three hundred seventy-five ten thousandths)

Whole numbers are written to the left of the decimal point and fractional parts are to the right:

5253/1000 is written 5.253 (five and two hundred fifty-three thousandths)



ADDING AND SUBTRACTING DECIMALS

Decimals are added and subtracted in the same manner as whole numbers. With decimals, however, the decimal points must be aligned vertically.

Example:

Add:

$$\begin{array}{r} 7.3125 \\ 1.25 \\ 0.625 \\ +\ 3.375 \\ \hline 12.5625 \end{array}$$

Subtract:

$$\begin{array}{r} 8.625 \\ -\ 2.25 \\ \hline 6.375 \end{array}$$

The decimal point in the answer is directly below the decimal points in the problem.

PRACTICE PROBLEMS

Adding and Subtracting Decimals

Solve the following problems:

Add:

1. 4.5625 + 0.875 + 2.75 + 5.8137 =
2. 1.9375 + 3.25 + 0.375 =
3. 7.0625 + 0.125 + 8.0 =
4. 11.342 + 16.17 + 0.4207 =
5. 0.832 + 0.4375 + 0.27 =

Subtract:

6. 27.9375 – 16.937 =
7. 3.306 – 1.875 =
8. 4.0 – 0.0625 =
9. 10 – 0.75 =
10. 2.25 – 1.125 =

MULTIPLYING DECIMALS

Decimals are multiplied in the same manner as whole numbers. The decimal points are disregarded until the multiplication is completed. To find the position of the decimal point in the answer, count the total number of decimal places to the right of the decimal point in the numbers being multiplied; then set off this number of decimal places in the answer, starting at the right.

Example:

$$\begin{array}{r} 6.25 \\ \times\ 1.5 \\ \hline 9.375 \end{array}$$

(3 decimal places in the two numbers)

(3 decimal places)

PRACTICE PROBLEMS

Multiplying Decimals

Solve the following problems:

1. $\begin{array}{r} 4.825 \\ \times\ 1.75 \\ \hline \end{array}$
2. $\begin{array}{r} 12.05 \\ \times\ 4.124 \\ \hline \end{array}$
3. $\begin{array}{r} 167 \\ \times\ 0.25 \\ \hline \end{array}$
4. $\begin{array}{r} 0.838 \\ \times\ 5.9 \\ \hline \end{array}$
5. $\begin{array}{r} 65.96 \\ \times\ 0.37 \\ \hline \end{array}$
6. $\begin{array}{r} 0.375 \\ \times\ 6 \\ \hline \end{array}$
7. $\begin{array}{r} 4.95 \\ \times\ 1.35 \\ \hline \end{array}$
8. $\begin{array}{r} 3.75 \\ \times\ 100 \\ \hline \end{array}$
9. $\begin{array}{r} 93.18 \\ \times\ 0.07 \\ \hline \end{array}$
10. $\begin{array}{r} 5639.25 \\ \times\ 10 \\ \hline \end{array}$

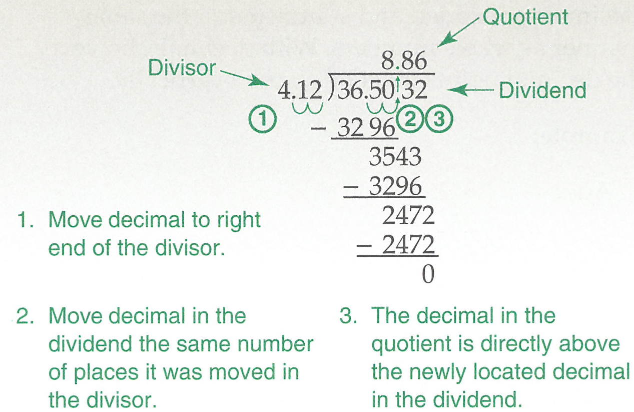
DIVIDING DECIMALS

Dividing decimals is identical to dividing whole numbers, except that the decimal point must be properly placed in the quotient (answer).

Example: 26 ÷ 6.5 = 4.0

To place the decimal point in the quotient, count the number of places to the right of the decimal point in the divisor. Add this number of places to the right of the decimal point in the dividend and place the decimal point directly above in the quotient.

Example: 36.5032 ÷ 4.12 = ?



PRACTICE PROBLEMS

Dividing Decimals

Solve the following problems:

1. 9.45 ÷ 2.7 =
2. 7.9392 ÷ 0.96 =
3. 654.5 ÷ 35 =
4. 172.8 ÷ 2.4 =
5. 1386.0 ÷ 1.65 =
6. 25924.64 ÷ 31.6 =
7. 331.266 ÷ 80.6 =
8. 821.7 ÷ 83 =
9. 4401.25 ÷ 503 =
10. 2585.52 ÷ 26.6 =

Calculating Dimensions

Working drawings in construction projects are typically dimensioned in feet and inches. It is often necessary to add and subtract measurements in feet and inches to calculate overall distances and determine "missing" dimensions in dimension strings. Dimensions are made up of whole numbers and fractions. They are added and subtracted in the same way as adding and subtracting whole numbers and fractions.

When adding dimensions, inch values and feet values are added separately, beginning with the inch values first.

Example: Add 12′-3″ to 9′-6″.

```
  12′-3″
+ 9′-6″
  21′-9″
```

When adding inch values, convert inches to feet as needed. Every 12 inches is equal to 1 foot.

Example: Add 12′-3″ to 9′-10″.

```
  12′-3″
+ 9′-10″
 21′-13″ (Take 12″ from 13″ and convert to 1′.
          Add 1′ to 21′)
  22′-1″ (The leftover amount in inches is 1″)
```

Subtracting dimensions works in a similar manner. Calculate the value in inches first. When necessary, borrow from the foot value to subtract an inch value.

Example: Subtract 9′-1″ from 12′-3″.

```
  12′-3″
–  9′-1″
   3′-2″
```

Example: Subtract 9′-6″ from 12′-3″.

```
  12′-3″
–  9′-6″
   = ?
```

Borrow 12″ (1′) from the foot value and add to the inch value.

11′-15″
– 9′-6″
2′-9″

Fractions in dimensions are calculated in the same manner as other fractions. The main item to remember is that you must keep the denominators of fractions the same when adding and subtracting.

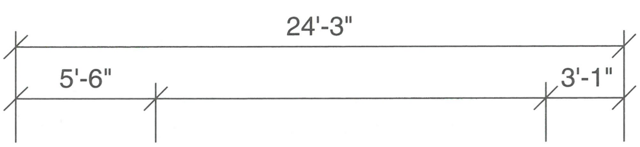
Example: Add 3′-4 1/4″ to 7′-9 1/2″.

Feet	Inches	Fraction
3	4	1/4
7	9	2/4
10	13	3/4

or 11′-1 3/4″

Addition and subtraction operations are commonly performed to determine "missing" dimensions in dimension strings. When adding, convert inches to feet as needed. When subtracting, borrow from the foot value to subtract an inch value as needed.

Example: Find the missing dimension in the string of dimensions below.



Add:

5′-6″ + 3′-1″ = 8′-7″

Subtract:

Overall dimension = 24′-3″

24′-3″ – 8′-7″ or 23′-15″ – 8′-7″ = 15′-8″

Converting Dimensions to Decimals

In some cases, it may be easier to work with decimal values when calculating dimensions. Dimensions in feet and inches can be converted into decimals by converting inches into decimal feet. This is done by dividing the inch value in the dimension by 12. The resulting decimal is added to the foot value.

Example: Convert 12′-3″ to decimal feet.

3 ÷ 12 = .25 = .25′

.25′ + 12′ = 12.25′

The inch values must first be converted to decimal feet before adding or subtracting decimal feet dimensions. Decimal feet dimensions are added or subtracted in the same manner as whole numbers and decimal fractions. The conversion table in **Figure 2-2** lists common inch values, decimal inch equivalent values, and decimal foot equivalent values. As when adding dimensions in whole inches and feet, values in decimal inches and feet are added separately.

Inch to Decimal Values				
Fractional Inch Value	**Decimal Inch Value**	**Decimal Foot Value**	**Inch Value**	**Decimal Foot Value**
1/16″	.0625″	.0052′	1″	.0833′
1/8″	.125″	.0104′	2″	.1667′
3/16″	.1875″	.0156′	3″	.2500′
1/4″	.25″	.0208′	4″	.3333′
5/16″	.3125″	.0260′	5″	.4167′
3/8″	.375″	.0313′	6″	.5000′
7/16″	.4375″	.0365′	7″	.5833′
1/2″	.5″	.0417′	8″	.6667′
9/16″	.5625″	.0469′	9″	.7500′
5/8″	.625″	.0521′	10″	.8333′
11/16″	.6875″	.0573′	11″	.9167′
3/4″	.75″	.0625′	12″	1.000′
13/16″	.8125″	.0677′		
7/8″	.875″	.0729′		
15/16″	.9375″	.0781′		

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Figure 2-2. Common inch values and equivalents in decimal inches and decimal feet. Decimal foot values are added separately from decimal inch values.

See the Reference Section in this textbook for a listing of common fractions, decimal fraction equivalents, and metric equivalents.

Area Measurement

Often, it is necessary to know the amount of paving, floor space, window opening, or wall space in a particular part of the project. This measurement is known as ***area***, and it is given in square units (square feet, square yards, or square meters, for example).

Square and Rectangular Areas

To compute the area of a rectangle or square, multiply the length of one side times the length of an adjacent side (length × width). The lengths must have the same units of measure. The resulting area calculation is in square units. For example, if you multiply two lengths in feet together, the area will be in square feet. If the lengths are given in inches, the area will be in square inches. If the lengths are given in feet and inches, the lengths must be converted to decimal feet and then multiplied so the answer will be in square feet.

Example: 12′-3″ × 10′-6″ would convert to 12.25′ × 10.5′.

Example: Determine the area of the room shown in **Figure 2-3A**.

$$\begin{array}{r} 12 \text{ ft} \\ \times\, 10 \text{ ft} \\ \hline 120 \text{ ft}^2 \end{array}$$

The area of a wall is computed in the same way, except that the area of all openings (doors and windows) must be subtracted from the total.

Example: Determine the area of the wall surface shown in **Figure 2-3B**.

Total Wall Area:

$$\begin{array}{r} 20 \text{ ft} \\ \times\ \ 8 \text{ ft} \\ \hline 160 \text{ ft}^2 \end{array}$$

Window Area:

$$\begin{array}{r} 5 \text{ ft} \\ \times\, 4 \text{ ft} \\ \hline 20 \text{ ft}^2 \end{array}$$

(Total Wall Area) – (Window Area)

$$\begin{array}{r} 160 \text{ ft}^2 \\ -\ \ 20 \text{ ft}^2 \\ \hline 140 \text{ ft}^2 \end{array} \text{ of wall surface}$$

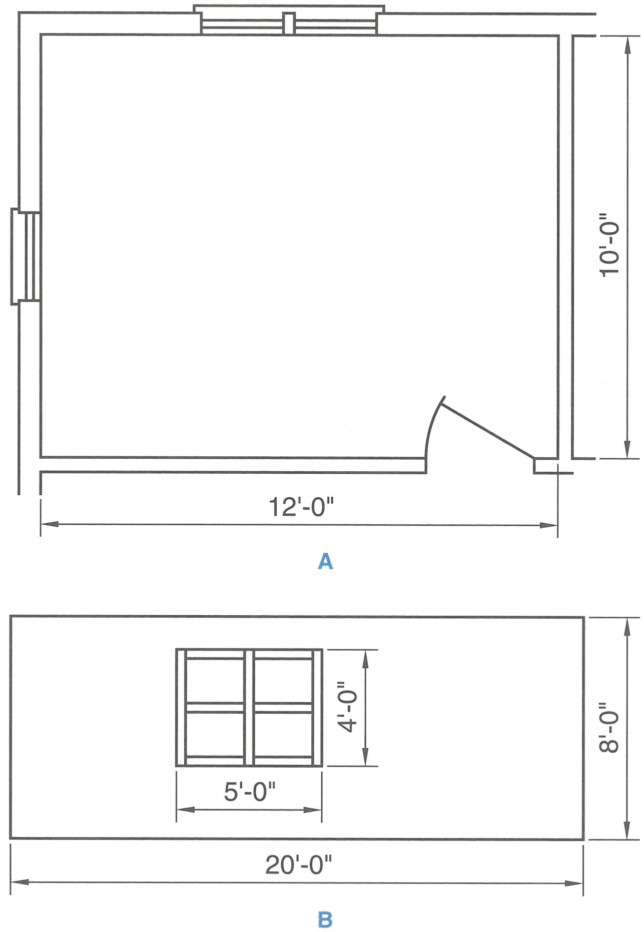

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Figure 2-3. Finding area of a rectangular area. A—Formula for finding area of a floor: A = L × W. B—Formula for finding area of a wall: A = L × W – area of openings.

Triangular Areas

A triangular area can be computed by multiplying the height times the base and then dividing by two. **Figure 2-4** illustrates the formula.

Example: Compute the area of the end of the gable roof shown in **Figure 2-4**.

$$5 \text{ ft} \times 24 \text{ ft} = 120 \text{ ft}^2$$

$$120 \text{ ft}^2 \div 2 = 60 \text{ ft}^2$$

Circles and Circular Areas

The characteristics of circles are shown in **Figure 2-5**. The ***circumference*** is the distance around the circle. The ***diameter*** is the length of a line running between two points on the circle and passing through the center. The ***radius*** is one-half the length of the diameter.

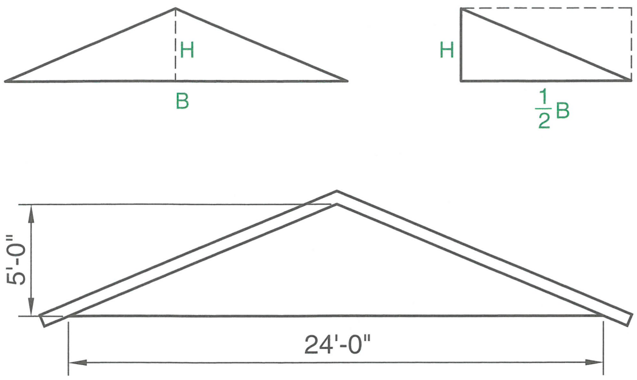

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Figure 2-4. Formula for finding area of a triangular area: A = (B × H) ÷ 2.

When determining the circumference, area, or volume of a circular object, the number π (pi) is used in the formula. ***Pi*** is the ratio of circumference to diameter, and is equal to 3.1416.

Circumference of a circle = $\pi \times d$

Example: Determine the circumference of a circle that has a 6′-6″ diameter.

$$3.1416 \times 6.5' = 20.42'$$

A portion of a circle is called an ***arc***. Lengths along an arc can be calculated by determining the circumference and multiplying the circumference by the degree percentage of the circle.

Arc length = 3.1416 × d; divide by 360/number of degrees in arc

Example: Determine the length of an arc that has a radius of 4′-3″ and has an arc of 90°.

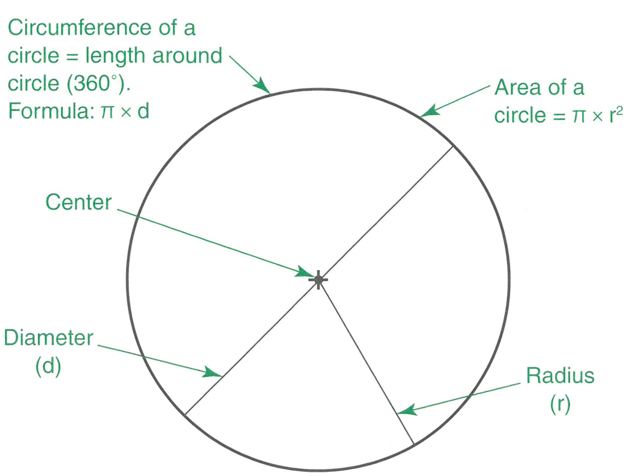

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Figure 2-5. To determine area and circumference of a circle (or portions of a circle), you must be familiar with the properties of the circle, such as radius and diameter.

First, determine the circumference of a complete circle:

$$3.1416 \times (4.25' \times 2) = 26.70'$$

Next, determine the portion of the arc:

$$360°/90° = 4 \text{ or } 25\%$$

Finally, divide the circumference by the portion of a circle:

$$26.7'/4 = 6.68' \text{ or } 6'\text{-}8''$$

The area of a circle can be found by multiplying π times the radius squared.

Area of a circle = $\pi \times r^2$

Example: Determine the area in square feet of the circular patio shown in **Figure 2-6**.

Patio diameter = 30′, radius = 15′

$$3.1416 \times 15^2 = \text{Area}$$

$$3.1416 \times 225 \text{ ft}^2 = \text{Area}$$

$$\begin{array}{r} 3.1416 \\ \times \quad 225 \text{ ft}^2 \\ \hline 706.86 \text{ ft}^2 \text{ patio} \end{array}$$

A portion of an area of a circle can be figured the same way by computing the area of the circle and then multiplying it by the portion of the complete circle.

Volume Measurement

Volume is a cubic measure. It is found by multiplying area by depth. The following example will compute the volume of ready-mix concrete required for a 4″ slab for the patio in **Figure 2-6**. When calculating volume, you must be certain that all of the numbers being multiplied together have the same units. Since the area

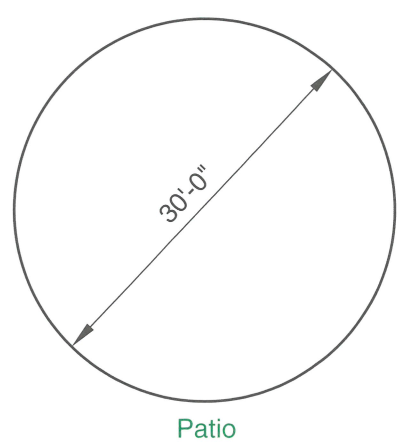

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Figure 2-6. Formula for finding area of a circle: $A = \pi \times r^2$.

of the patio is known to be 706.86 ft^2, the 4″ depth will be changed to 0.333′.

$$\begin{array}{r} 706.86\ \text{ft}^2\ \text{(area of patio)} \\ \times\ 0.333\ \text{ft} \\ \hline 235.38\ \text{ft}^3\ \text{in patio slab} \end{array}$$

Since concrete is sold by the cubic yard, it is necessary to change the cubic feet to cubic yards. There are 27 cubic feet in a cubic yard, so divide the number of cubic feet by 27:

$$\begin{array}{r} 8.71 = 8.71\ \text{cubic yards (cy)} \\ 27\overline{)235.38} \\ -216 \\ \hline 193 \\ -189 \\ \hline 48 \\ -27 \\ \hline 21 \end{array}$$

To allow for enough concrete for the patio slab, the computed volume is rounded up to the next largest cubic yard. This is done to account for waste and helps allow for enough material. In this example, the calculation is rounded up to 9 cy.

Area and volume calculations for metric measurements are made in the same manner using the appropriate units.

Example:

Patio diameter = 10 meters

Thickness = 10 centimeters

Determine area of patio:

$$\begin{array}{r} 3.1416 \\ \times\ 25\ \text{m}^2\ \text{(radius squared)} \\ \hline 78.54\ \text{m}^2\ \text{(area of patio)} \end{array}$$

Convert thickness to meters:

10 cm = 0.1 m

Multiply area by thickness:

$$\begin{array}{r} 78.54\ \text{m}^2 \\ \times\ 0.1\ \text{m} \\ \hline 7.854\ \text{m}^3\ \text{(volume of patio)} \end{array}$$

Note that fewer calculations are required with metric units.