1. Use structural induction to prove that the sum of the interior angles of an n-sided convex polygon is $((n-2)*180)$ degrees, where $n > 2$.

   Recall that an n-sided convex polygon can be specified by providing a sequence of vertices (points where sides connect) $p_1, p_2, ..., p_n$ such that for each i where $1 \leq i \leq n-1$ there is a side from p_i to p_{i+1}, and there is also a side from p_n to p_1, and where the interior angle between the two sides at each vertex is less than 180 degrees.

   (a) (5 points) State and Prove the Base Case: (You may use the fact that for $n = 3$ sides, a convex polygon is a triangle whose interior-angle sum is exactly 180 degrees.)

   (b) (5 points) State the Inductive Hypothesis:

   (c) (5 points) Show the Inductive step:

2. Use constructive induction to find/prove a formula for the sum of $k * 2^k$ for $k = 1$ to n, of this form: $an * 2^n + b * 2^n + c$, where $n > 0$ and a, b, and c are unknown constants.

   (a) (5 points) What do we learn from the Base Case?

   (b) (5 points) State the Inductive Hypothesis.

   (c) (5 points) Show the Inductive Step.

(d) (5 points) Derive the constants.

(e) (5 points) State the final result.

3. Use strong induction to prove that any amount of postage that is 12 cents or more can be achieved by using 4-cent and 5-cent stamps.
(Hint: use four base cases: n=12, 13, 14, 15)

   (a) (5 points) State and Prove the Base Case:

   (b) (5 points) State the Inductive Hypothesis:

   (c) (5 points) Show the Inductive step:

4. Use ordinary (weak) induction to prove that every tree T with height h has at least h edges, where h is in $\mathbb{N}$.

   (a) (5 points) State and Prove the Base Case:

   (b) (5 points) State the Inductive Hypothesis:

   (c) (5 points) Show the Inductive step:

5. For each of the following binary relations, mark whether it is **reflexive**, whether it is **symmetric**, and whether it is **transitive**:

   (a) (3 points) (x, y) in $\mathbb{R}$ iff x is a parent of y (x and y are people)

   (b) (3 points) (x, y) in $\mathbb{R}$ iff $x < y$ (x and y are integers)

   (c) (3 points) (x, y) in $\mathbb{R}$ iff $|x| = |y|$ (x and y are reals)

   (d) (3 points) (x, y) in $\mathbb{R}$ iff $|x - y| < 1$ (x and y are reals)

6. Given the equivalence relation (x, y) in $\mathbb{R}$ iff $(x - y)$ is even (where x and y are integers from 1 to 10). Write down the associated equivalence classes (partition) of 1,2,...,10. (Hint: there will be just two equivalence classes.)

7. For each of the following functions from the reals to the reals, mark whether it is **1-1** (injective), and whether it is **onto** (surjective):

   (a) (4 points) $f(x) = x^2$

   (b) (4 points) $f(x) = x^3$

   (c) (4 points) $f(x) = e^x$

   (d) (4 points) $f(x) = 1/(1 + e^{-x})$

8. Let f be a function from a domain D of 100 elements to a codomain C of 3 elements. Use the Pigeonhole Principle to show that for at least one element c in C, there must be at least 34 elements x in D such that $f(x) = c$.

9. Let S be a finite set with n elements. Use the binomial theorem to show that $|\mathcal{P}(S)| = 2^n$