

MECHANICS OF MATERIALS

TORSION OF CIRCULAR SECTIONS – Verification of Torsion Equation

Background

Torsion is the form of loading that occurs when a twisting effort (Torque or Moment) is applied around an axis of a structural element. The element in question can have any shape of cross section, although circular sections (solid and hollow) most commonly occur. Generally the torque is applied around the longitudinal axis. The form of stress that results is termed a shear stress. If the *Shear Yield Point* is exceeded, some permanent plastic deformation will result. If the *Ultimate Shear Strength* of the section is exceeded, failure will result.

Important Quantities

Modulus of Rigidity, G
Polar Second Moment of Area J
Twist Angle θ

Theory

See class notes

Main Equations Used

1. Calculation of Polar Second Moment of Area (J):

$$J = (\pi(d^4))/32)$$

Note that this expression is valid only for a solid circular section.

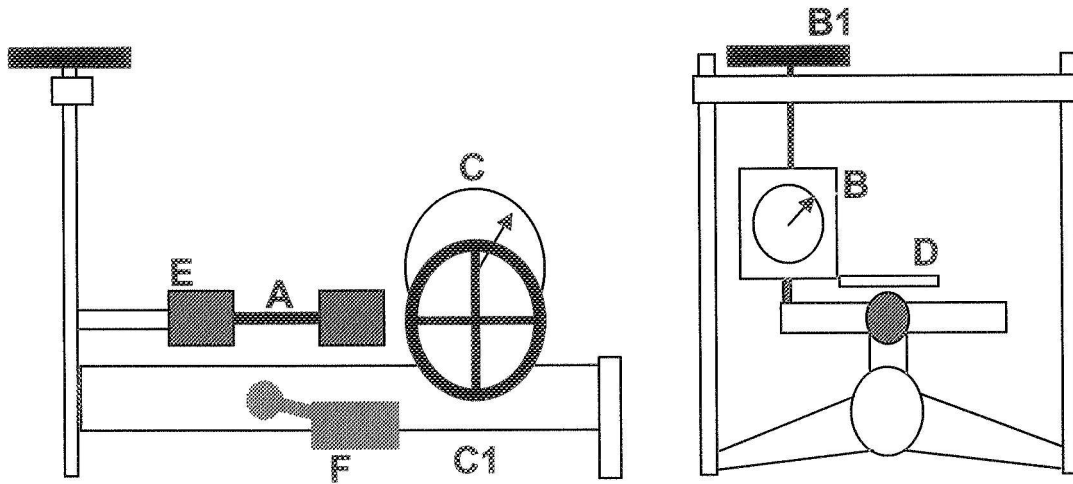
2. Torsion Equation (allows calculation of stress or twist angle)

$$T/J = \tau/r = (G.\theta)/L$$

Laboratory Experiment

Apparatus

Torsion Testing Machine,, Mild Steel specimen, Brass Specimen



- A Specimen
- B Torque Meter
- B1 Torque Wheel
- C Angle of Twist Meter (NB Degrees)
- C1 Angle Wheel
- D Spirit Level
- E Chuck
- F Slideway Clamp

Procedure

Note : The experiment is carried out using mild steel and brass specimens.

1. Using an accurate instrument, measure the specimen diameter at a number of selected locations. Compute the average value. Compare to the value provided in the spec. sheet. For the purposes of this experiment, the measured value is used.
2. Place the first specimen into the torsion testing machine. Leave the chucks loose and push the sliding chuck to the specimen end so that the ends of the specimen are secure. Now apply the slide-way clamp (F).
3. Open the release nut at the rear of the angle display so that this display can spin freely.
4. Tighten each chuck. Rotate wheels B1 and C1 so that the torque meter (B) reads zero and the spirit level (D) is balanced.
5. Set the angle display (C) to zero. Now tighten the release nut at the rear of the display.

6. Check both displays (B and C). Ideally both read zero. A non-zero value on either scale should be recorded as a zero error and subtracted from all readings from that scale.
7. Rotate wheel C1 so that the twist angle scale reads 0.4° . Note that the applied torque (registering as a reaction at the 'fixed' end) will now register on the torque meter. **This value is not the full torque as the end is not completely fixed. To get a correct value for torque, the wheel B1 must be rotated slightly so that the spirit level is brought back to the horizontal.** The scale will reflect this adjustment. Record the torque value (Nm)
8. Repeat step 7 in 0.4° increments up to a maximum of 6° . Note: the beam must be level before recording the torque..
9. Repeat the above for the second specimen.

Data Treatment

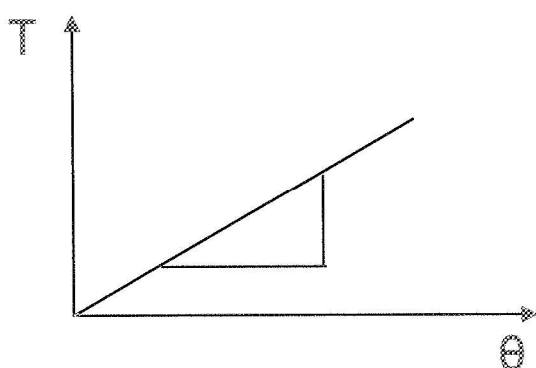
The aim of this experiment is to verify the Torsion Equation.

This will be achieved by inserting constants and measured data into the torsion equation but leaving the measured specimen diameter as an 'unknown'.

As the only unknown, the diameter of the specimen can then be calculated.

Comparison of the calculated value of diameter with its known (i.e. measured) value is then made. I.e. if the two values are in close agreement we can say that the torsion equation is correct.

For both specimens plot graphs of Torque T (y-axis) vs. Twist Angle θ (radians, x-axis).



$$\text{Slope} = T / \theta$$

The torsion equation is examined:

$$\underline{T/J} = \tau/r = \underline{(G.\theta)/L}$$

The equation is composed of three elements. This experiment only allows us to verify the two underlined elements.

$$T/J = (G.\theta)/L$$

$$(T . L) / (G . \theta) = J$$

$$\text{But } J = (\pi(d^4))/32$$

$$(T . L) / (G . \theta) = (\pi(d^4))/32$$

Rearranging L.H.S

$$(32 / \pi)(T / \theta) . (L / G) = d^4$$

$$\sqrt[4]{[(32 / \pi)(T / \theta) . (L / G)]} = d$$

Hence, using this equation, the specimen diameter can be calculated. Comparison can be made with measured values of diameter.

Note:

1. Average value for (T / θ) is simply the slope of the graph drawn earlier. This should be inserted in the equation above.
2. $G_{\text{STEEL}} \approx 80 \text{ GN/m}^2$
3. $G_{\text{BRASS}} \approx 38 \text{ GN/m}^2$
4. $\sqrt[4]{}$ is the fourth root of the value within the square brackets.

Error Analysis

Examine the equation used above to calculate the specimen diameter. The following inputs to the equation should be examined for possible error in the values used:

- (T / θ) - i.e. slope of the graph drawn
- L - specimen length
- G - assumed value of Modulus of Rigidity.

Select one quantity at a time from the equation.

Using your judgement, with supporting evidence where possible, apply a realistic error to the value of the quantity (i.e. simply add / subtract the error to / from the quantity as appropriate).

Recalculate the predicted specimen diameter using this new value that incorporates the error. Note all other values remain at their previous value.

Note: The validity of this approach is dependent on you estimating / judging a realistic value of measurement error for each input to the equation

Comment on the effect the error in measurement of this quantity has on the calculated value of diameter.

Repeat for the remaining inputs to the equation.

Discuss the relative importance of measurement accuracy for each input to the equation and relate this specifically to the equipment/process employed during the investigation.

Discuss, as an Engineer, how the accuracy of the experiment could be improved

Refer to Guidelines to Report Writing - available on Moodle.