ruler, checkerboard, colored triangles, beads, and many other attractive objects.

In Taipei, every classroom is equipped with a similar, but larger, set of such objects. In Beijing, where there is much less money available for purchasing such materials, teachers improvise with colored paper, wax fruit, plates, and other easily obtained objects. In all cases, these concrete objects are considered to be critically important tools for teaching mathematics, for it is through manipulating these objects that children can form important links between real-world problems and abstract mathematical notations.

American teachers are much less likely than Chinese or Japanese teachers to use concrete objects. At fifth grade, for example, Sendai teachers were nearly twice as likely to use concrete objects as the Chicago area teachers, and Taipei teachers were nearly five times as likely. There was also a subtle, but important, difference in the way Asian and American teachers used concrete objects. Japanese teachers, for example, use the items in the Math Set throughout the elementary school years and introduced small tiles in a high percentage of the lessons we observed in the first grade. American teachers seek variety and may use Popsicle sticks in one lesson, and in another, marbles, Cheerios, M&Ms, checkers, poker chips, or plastic animals. The American view is that objects should be varied in order to maintain children's interest. The Asian view is that using a variety of representational materials may confuse children, and thereby make it more difficult for them to use the objects for the representation and solution of mathematics problems. Having learned to add with tiles makes multiplication easier to understand when the same tiles are used.

Through the skillful use of concrete objects, teachers are able to teach elementary school children to understand and solve problems that are not introduced in American curricula until much later. An example occurred in a fourth-grade mathematics lesson we observed in Japan. The problem the teacher posed is a difficult one for fourth graders, and its solution is generally not taught in the United States until much later. This is the problem:

> There are a total of thirty-eight children in Akira's class. There are six more boys than there are girls. How many boys and how many girls are in the class?

This lesson began with a discussion of the problem and with the children proposing ways to solve it. After the discussion, the teacher handed each child two strips of paper, one six units longer than the other, and told the class that the strips would be used to help them think about the problem. One slip represented the number of girls in the class and the other represented the number of boys. By lining the strips next to each other, the children could see that the degree to which the longer one protruded beyond the shorter one represented 6 boys. The procedure for solving the problem then unfolded as the teacher, through skillful questioning, led the children to the solution: The number of girls was found by taking the total of both strips, subtracting 6 to make the strips of equal length, and then dividing by 2. The number of boys could be found, of course, by adding 6 to the number of girls. With this concrete visual representation of the problem and careful guidance from the teacher, even fourth graders were able to understand the problem and its solution.

Students Construct Multiple Solutions

A common Western stereotype is that the Asian teacher is an authoritarian purveyor of information, one who expects students to listen and memorize correct answers or correct procedures rather than to construct knowledge themselves. This may or may not be an accurate description of Asian high school teachers,[4] but, as we have seen in previous examples, it does not describe the dozens of elementary school teachers that we have observed.

Chinese and Japanese teachers rely on students to generate ideas and evaluate the correctness of the ideas. The possibility that they will be called upon to state their own solution as well as to evaluate what another student has proposed keeps Asian students alert, but this technique has two other important functions. First, it engages students in the lesson, increasing their motivation by making them feel they are participants in a group process. Second, it conveys a more realistic impression of how knowledge is acquired. Mathematics, for example, is a body of knowledge that has evolved gradually through a process of argument and proof. Learning to argue about mathematical ideas is fundamental to understanding mathematics. Chinese and Japanese children begin learning these skills in the first grade; many American elementary school students are never exposed to them.

We can illustrate the way Asian teachers use students' ideas with the following example. A fifth-grade teacher in Taiwan began her mathematics lesson by calling attention to a six-sided figure she had drawn on the blackboard. She asked the students how they might go about finding the area of the shaded region. "I don't want you to tell me what the actual area is, just tell me the approach you would use to solve the problem. Think of as many different ways as you can of ways you could determine the area that I have