

drawn in yellow chalk." She allowed the students several minutes to work in small groups and then called upon a child from each group to describe the group's solution. After each proposal, many of which were quite complex, the teacher asked members of the other groups whether the procedure described could yield a correct answer. After several different procedures had been suggested, the teacher moved on to a second problem with a different embedded figure and repeated the process. Neither teacher nor students actually carried out a solution to the problem until all of the alternative solutions had been discussed. The lesson ended with the teacher affirming the importance of coming up with multiple solutions. "After all," she said, "we face many problems every day in the real world. We have to remember that there is not only one way we can solve each problem."

American teachers are less likely to give students opportunities to respond at such length. Although a great deal of interaction appears to occur in American classrooms—with teachers and students posing questions and giving answers—American teachers generally pose questions that are answerable with a yes or no or with a short phrase. They seek a correct answer and continue calling on students until one produces it. "Since we can't subtract 8 from 6," says an American teacher, "we have to . . . what?" Hands go up, the teacher calls on a girl who says "Borrow." "Correct," the teacher replies. This kind of interchange does not establish the student as a valid source of information, for the final arbiter of the correctness of the student's opinions is still the teacher. The situation is very different in Asian classrooms, where children are likely to be asked to explain their answers and other children are then called upon to evaluate their correctness.

Clear evidence of these differing beliefs about the roles of students and teachers appears in the observations of how teachers evaluate students' responses. The most frequent form of evaluation used by American teachers was praise, a technique that was rarely used in either Taiwan or Japan. In Japan, evaluation most frequently took the form of a discussion of children's errors.

Praise serves to cut off discussion and to highlight the teacher's role as the authority. It also encourages children to be satisfied with their performance rather than informing them about where they need improvement. Discussing errors, on the other hand, encourages argument and justification and involves students in the exciting quest of assessing the strengths and weaknesses of the various alternative solutions that have been proposed.

Why are American teachers often reluctant to encourage students to participate at greater length during mathematics lessons? One possibility is that they feel insecure about the depth of their own mathemat-

ical training. Placing more emphasis on students' explanations necessarily requires teachers to relinquish some control over the direction the lesson will take. This can be a frightening prospect to a teacher who is unprepared to evaluate the validity of novel ideas that students inevitably propose.

USING ERRORS EFFECTIVELY

We have been struck by the different reactions of Asian and American teachers to children's errors. For Americans, errors tend to be interpreted as an indication of failure in learning the lesson. For Chinese and Japanese, they are an index of what still needs to be learned. These divergent interpretations result in very different reactions to the display of errors—embarrassment on the part of the American children, calm acceptance by Asian children. They also result in differences in the manner in which teachers utilize errors as effective means of instruction.

We visited a fifth-grade classroom in Japan the first day the teacher introduced the problem of adding fractions with unequal denominators. The problem was a simple one: adding one-third and one-half. The children were told to solve the problem and that the class would then review the different solutions.

After everyone appeared to have completed the task, the teacher called on one of the students to give his answer and to explain his solution. "The answer is two-fifths," he stated. Pointing first to the numerators and then to the denominators, he explained: "One plus one is two; three plus two is five. The answer is two-fifths." Without comment, the teacher asked another boy for his solution. "Two point one plus three point one, when changed into a fraction adds up to two-fifths." The children in the classroom looked puzzled. The teacher, unperturbed, asked a third student for her solution. "The answer is five-sixths." The student went on to explain how she had found the common denominator, changed the fractions so that each had this denominator, and then added them.

The teacher returned to the first solution. "How many of you think this solution is correct?" Most agreed that it was not. She used the opportunity to direct the children's attention to reasons why the solution was incorrect. "Which is larger, two-fifths or one-half?" The class agreed that it was one-half. "It is strange, isn't it, that you could add a number to one-half and get a number that is smaller than one-half." She went on to explain how the procedure the child used would result in the odd situation where, when one-half was added to one-half, the answer yielded is one-half. In a similarly careful, interactive manner, she discussed how the second boy had confused fractions with decimals to come up with his surprising