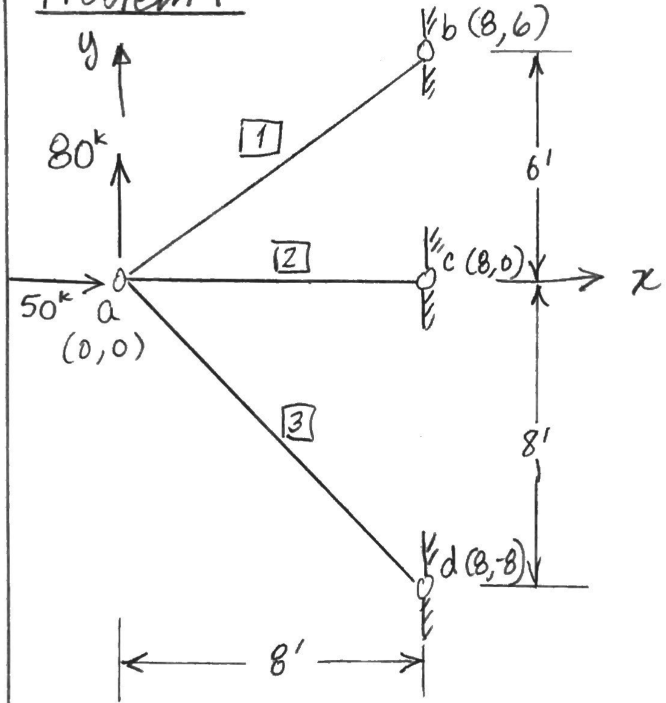
Problem 1



$E = 29{,}000\ ksi$

$A_{ab} = 1.2\ in^2$

$A_{ac} = 1.0\ in^2$

$A_{ad} = 3.6\ in^2$

– Member Details

Member No.	i-j	L (ft)	L (in)	A (in²)	AE/L (k/in²)	C	S	SC	C²	S²
1	a-b	10	120	1.2	290	0.8	0.6	0.48	0.64	0.36
2	a-c	8	96	1.0	302.0833	1	0	0	1	0
3	a-d	11.3137	135.7645	3.6	768.9786	0.7071	-0.7071	-0.5	0.5	0.5

$L_{ab} = \sqrt{(8-0)^2+(6-0)^2} = 10ft$; $\cos\phi = \frac{(8-0)}{10} = 0.8$; $\sin\phi = \frac{(6-0)}{10} = 0.6$

$L_{ad} = \sqrt{(8-0)^2+(-8-0)^2} = 11.3137ft$; $\cos\phi = \frac{8-0}{11.3137} = 0.7071$; $\sin\phi = \frac{(-8-0)}{11.3137} = -0.7071$

– Member stiffness matrix in global coord. [k]

$$[k] = \frac{AE}{L}\begin{bmatrix} C^2 & SC & -C^2 & -SC \\ & S^2 & -SC & -S^2 \\ \text{symm.} & & C^2 & SC \\ & & & S^2 \end{bmatrix}$$

$$[k]_1 = 290\begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ & 0.36 & -0.48 & -0.36 \\ & & 0.64 & 0.48 \\ & & & 0.36 \end{bmatrix} = \begin{array}{c c} \begin{array}{cccc} \Delta_{ax} & \Delta_{ay} & \Delta_{bx} & \Delta_{by} \end{array} & \\ \begin{bmatrix} 185.6 & 139.2 & -185.6 & -139.2 \\ 139.2 & 104.4 & -139.2 & -104.4 \\ & & 185.6 & 139.2 \\ & & & 104.4 \end{bmatrix} & \begin{array}{c} \Delta_{ax} \\ \Delta_{ay} \\ \Delta_{bx} \\ \Delta_{by} \end{array} \end{array}$$

$$[k]_2 = 302.0833\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{array}{c c} \begin{array}{cccc} \Delta_{ax} & \Delta_{ay} & \Delta_{cx} & \Delta_{cy} \end{array} & \\ \begin{bmatrix} 302.0833 & 0 & -302.0833 & 0 \\ 0 & 0 & 0 & 0 \\ -302.0833 & 0 & 302.0833 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} \Delta_{ax} \\ \Delta_{ay} \\ \Delta_{cx} \\ \Delta_{cy} \end{array} \end{array}$$

$$[k]_3 = 768.9786 \begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ & 0.5 & 0.5 & -0.5 \\ & & 0.5 & -0.5 \\ & & & 0.5 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} \Delta ax & \Delta ay & \Delta dx & \Delta dy \end{array} \\ \left[\begin{array}{cc|cc} 384.4893 & -384.4893 & -384.4893 & 384.4893 \\ -384.4893 & 384.4893 & 384.4893 & -384.4893 \\ & & 384.4893 & -384.4893 \\ & & & 384.4893 \end{array}\right] \end{array} \begin{array}{c} \Delta ax \\ \Delta ay \\ \Delta dx \\ \Delta dy \end{array}$$

- Assemble the structure stiffness matrix; assemble reduced $[K] = [K_{11}]$ since no prescribed displ.

$$[K_{11}] = \begin{array}{c} \begin{array}{cc} \Delta ax & \Delta ay \end{array} \\ \begin{bmatrix} (185.6 + 302.0833 + 384.4893) & (139.2 + 0 - 384.4893) \\ (139.2 + 0 - 384.4893) & (104.4 + 0 + 384.4893) \end{bmatrix} \end{array} \begin{array}{c} \Delta ax \\ \Delta ay \end{array}$$

$$= \begin{bmatrix} 872.1726 & -245.2893 \\ -245.2893 & 488.8893 \end{bmatrix}$$

- Assemble column matrix of nodal forces & write equilibrium eqs. $\{Q_f\} = [K_{11}]\{\Delta_f\}$ and solve:

$$\begin{Bmatrix} 50 \\ 80 \end{Bmatrix} = \begin{bmatrix} 872.1726 & -245.2893 \\ -245.2893 & 488.8893 \end{bmatrix} \begin{Bmatrix} \Delta ax \\ \Delta ay \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \Delta ax \\ \Delta ay \end{Bmatrix} = \begin{Bmatrix} 0.120328 \\ 0.224008 \end{Bmatrix} = \begin{Bmatrix} 0.120 \text{ in} \rightarrow \\ 0.224 \text{ in} \uparrow \end{Bmatrix}$$

- Member Forces:

Member 1: $$\{\delta\}_1 = \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{Bmatrix} \Delta_{ix} \\ \Delta_{iy} \\ \Delta_{jx} \\ \Delta_{jy} \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \delta_a \\ \delta_b \end{Bmatrix} = \begin{bmatrix} 0.8 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \end{bmatrix} \begin{Bmatrix} 0.120328 \\ 0.224008 \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.230667 \\ 0 \end{Bmatrix} \text{ in}$$

$$f_{ab} = \frac{AE}{L}(\delta_j - \delta_i) = 290(0 - 0.230667) = -66.89^{k} \text{ (C)}$$

Member 2: $$\{\delta\}_2 = \begin{Bmatrix} \delta_a \\ \delta_c \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} 0.120328 \\ 0.224008 \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.120328 \\ 0 \end{Bmatrix} \text{ in}$$

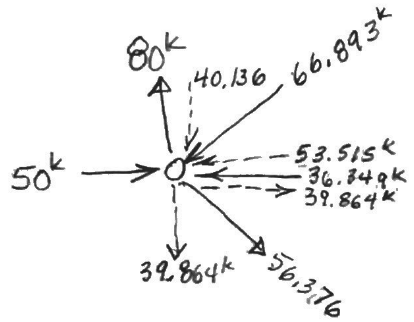
$$f_{a-c} = \frac{AE}{L}(\delta_j - \delta_i) = 302.0833(0-0.120328) = -36.35^k \ (C)$$

Member 3: $$\{\delta\}_3 = \begin{Bmatrix}\delta_a\\ \delta_d\end{Bmatrix} = \begin{bmatrix}0.7071 & -0.7071 & 0 & 0\\ 0 & 0 & 0.7071 & -0.7071\end{bmatrix}\begin{Bmatrix}0.120328\\0.224008\\0\\0\end{Bmatrix}$$

$$= \begin{Bmatrix}-0.0733128\\0\end{Bmatrix} \text{ in}$$

$$f_{a-d} = \frac{AE}{L}(\delta_j-\delta_i) = 768.9786[0-(-0.0733128)] = +56.38^k\ (T)$$

Equilibrium of node "a":



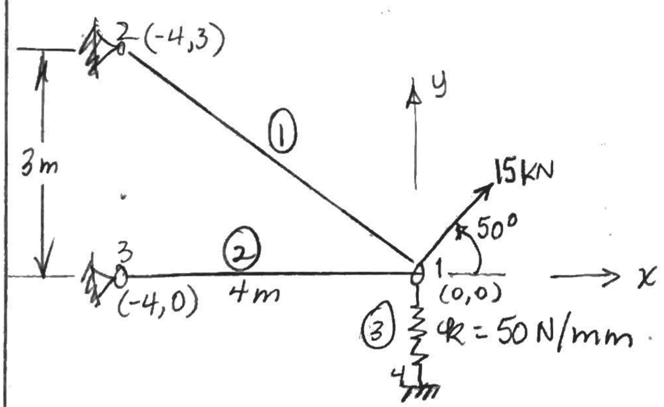
$\Sigma F_x = 0$:

$50 - 53.515 - 36.349 + 39.864 = 0$ ✓check

$\Sigma F_y = 0$:

$80 - 40.136 - 39.864 = 0$ ✓ check

Problem 2



$A = 300\,mm^2 = 300 \times 10^{-6}\,m^2$

$E = 80\,GPa = 80 \times 10^6\,kN/m^2$

$k = 50\,\frac{N}{mm} \times \frac{1000\,mm}{1\,m} \times \frac{1\,kN}{1000\,N}$

$= 50\,kN/m$

- Problem Details:
  - nodes/joints & members numbered
  - dof's: Δ_{1x} ; Δ_{1y}
  - no prescribed displ.

Member No.	i-j	L (m)	A (m²)	E (kN/m²)	AE/L (kN/m)	c	s	sc	c²	s²
1	1-2	5	0.0003	80x10⁶	4800	-0.8	0.6	-0.48	0.64	0.36
2	1-3	4	0.0003	80x10⁶	6000	-1	0	0	1	0
3	1-4	—	—	—	50	—	—	—	—	—

$L_1 = \sqrt{(-4-0)^2+(3-0)^2} = 5m$; $\cos\phi = \frac{(-4-0)}{5} = -0.8$; $\sin\phi = \frac{(3-0)}{5} = 0.6$

- Generate member stiffness in global coordinates [k]

$$[k]_1 = \frac{AE}{L}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ & s^2 & -sc & -s^2 \\ & & c^2 & sc \\ & & & s^2 \end{bmatrix} = 4800\begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ & 0.36 & 0.48 & -0.36 \\ & & 0.64 & -0.48 \\ & & & 0.36 \end{bmatrix}$$

$$= \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc} \Delta_{1x} & \Delta_{1y} & \Delta_{2x} & \Delta_{2y} \end{array}$$

$$= \begin{array}{c|c} \begin{bmatrix} 3072 & -2304 & -3072 & 2304 \\ -2304 & 1728 & 2304 & -1728 \\ & & 3072 & -2304 \\ & & & 1728 \end{bmatrix} & \begin{array}{c} \Delta_{1x} \\ \Delta_{1y} \\ \Delta_{2x} \\ \Delta_{2y} \end{array} \end{array}$$

$$[k]_2 = 6000\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{array}{cccc} \Delta_{1x} & \Delta_{1y} & \Delta_{3x} & \Delta_{3y} \end{array}\begin{bmatrix} 6000 & 0 & -6000 & 0 \\ 0 & 0 & 0 & 0 \\ -6000 & 0 & 6000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{array}{c} \Delta_{1x} \\ \Delta_{1y} \\ \Delta_{3x} \\ \Delta_{3y} \end{array}$$

$$[k]_3 = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} = \begin{array}{cc} \Delta_{1y} & \Delta_{4y} \end{array}\begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix}\begin{array}{c} \Delta_{1y} \\ \Delta_{4y} \end{array}$$

- Assemble the structure stiffness matrix [K]

$$[K_{11}] = \begin{array}{cc} \Delta_{1x} & \Delta_{1y} \end{array}\begin{bmatrix} (3072+6000) & (-2304+0) \\ (-2304+0) & (1728+0+50) \end{bmatrix}\begin{array}{c} \Delta_{1x} \\ \Delta_{1y} \end{array} = \begin{bmatrix} 9072 & -2304 \\ -2304 & 1778 \end{bmatrix}$$

- $\{Q_f\} = [K_{11}]\{\Delta_f\}$

$$\Rightarrow \begin{Bmatrix} 15\cos 50^\circ \\ 15\sin 50^\circ \end{Bmatrix} = \begin{Bmatrix} 9.6418 \\ 11.4907 \end{Bmatrix} = \begin{bmatrix} 9072 & -2304 \\ -2304 & 1778 \end{bmatrix}\begin{Bmatrix} \Delta_{1x} \\ \Delta_{1y} \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \Delta_{1x} \\ \Delta_{1y} \end{Bmatrix} = \begin{Bmatrix} 4.0306\times10^{-3}\ m \\ 11.6857\times10^{-3}\ m \end{Bmatrix} = \begin{Bmatrix} 4.0306mm \rightarrow \\ 11.6857mm \uparrow \end{Bmatrix}$$

- Force in member ①

Displ. in local coord., $\{\delta\} = \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{bmatrix} -0.8 & 0.6 & 0 & 0 \\ 0 & 0 & -0.8 & 0.6 \end{bmatrix}\begin{Bmatrix} 4.0306\times10^{-3} \\ 11.6857\times10^{-3} \\ 0 \\ 0 \end{Bmatrix}$

$$= \begin{Bmatrix} 0.0037869 \\ 0 \end{Bmatrix} m$$

$$F_{1-2} = \frac{AE}{L}(\delta_j - \delta_i) = 4800(0 - 0.0037869) = -18.177\,kN\ (C)$$

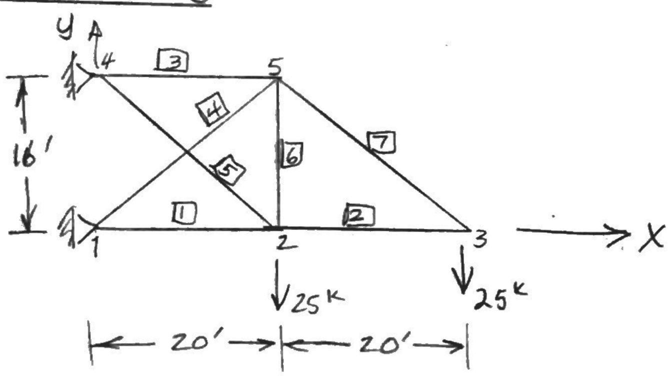
- Force in member ②

$$\begin{Bmatrix} \delta_1 \\ \delta_3 \end{Bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} 4.0306\times10^{-3} \\ 11.6857\times10^{-3} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -4.0306\times10^{-3} \\ 0 \end{Bmatrix} m$$

$$F_{1-3} = \frac{AE}{L}(\delta_j - \delta_i) = 6000\left(0 - (-4.0306\times10^{-3})\right) = +24.184\,kN\ (T)$$

- Force in spring $= F_s = k\Delta_{1y} = 50(11.6857\times10^{-3}) = 0.5843\,kN\ (T)$

Problem 3



$E = 29000\ ksi$

a) For member ⑤ : connection 2-4

$$L = \sqrt{(0-20)^2 + (16-0)^2} = 25.6125'$$

$$\cos\phi = \frac{(0-20)}{25.6125} = -0.78087 \qquad (c^2 = 0.60976)$$

$$(cs = -0.48780)$$

$$\sin\phi = \frac{(16-0)}{25.6125} = +0.62470 \qquad (s^2 = 0.390244)$$

$$\{\delta\} = \begin{Bmatrix} \delta_2 \\ \delta_4 \end{Bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{Bmatrix} \Delta_{2x} \\ \Delta_{2y} \\ \Delta_{4x} \\ \Delta_{4y} \end{Bmatrix}$$

$$= \begin{bmatrix} -0.78087 & 0.62470 & 0 & 0 \\ 0 & 0 & -0.78087 & 0.62470 \end{bmatrix} \begin{Bmatrix} -0.064655 \\ -0.19398 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix}\delta_2\\ \delta_4\end{Bmatrix} = \begin{Bmatrix}-0.070691\\ 0\end{Bmatrix} \text{ in}$$

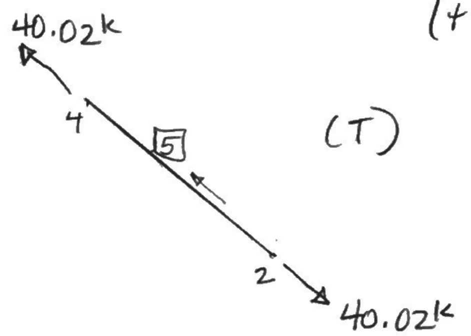
local coord

$$\{f\}_5 = \begin{Bmatrix}f_2\\ f_4\end{Bmatrix} = [k']\{\delta\} = \frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}\delta_2\\ \delta_4\end{Bmatrix}$$

$$= \frac{6(29000)}{25.6125 \times 12}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}-0.070691\\ 0\end{Bmatrix}$$

$$= 566.130\begin{Bmatrix}-0.070691\\ 0.070691\end{Bmatrix}$$

$$= \begin{Bmatrix}-40.02^k\\ +40.02^k\end{Bmatrix}$$



(T)

or $f_{2-4} = \frac{AE}{L}(\delta_j - \delta_i) = 566.130(0 - (-0.070691)) = +40.02^k$ (T)

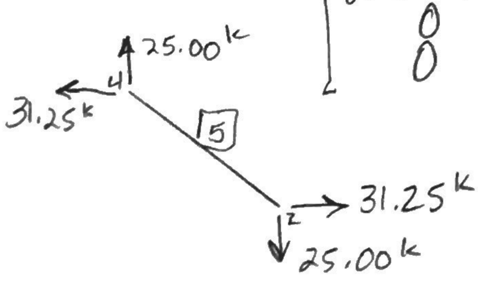
b) $\{F\} = [k]\{\Delta\}$ or $\{F\} = [T]^T\{f\}$

global coord.

$$\{F\}_5 = 566.130\begin{bmatrix}0.6098 & -0.4878 & -0.6098 & 0.4878\\ -0.4878 & 0.3902 & 0.4878 & -0.3902\\ -0.6098 & 0.4878 & 0.6098 & -0.4878\\ 0.4878 & -0.3902 & -0.4878 & 0.3902\end{bmatrix}\begin{Bmatrix}-0.064655\\ -0.19398\\ 0\\ 0\end{Bmatrix}$$

$$= 566.130\begin{Bmatrix}0.05520\\ -0.04416\\ -0.05520\\ 0.04416\end{Bmatrix} = \begin{Bmatrix}31.25^k\\ -25.00^k\\ -31.25^k\\ +25.00^k\end{Bmatrix}$$

or $$\{F\}_5 = \begin{bmatrix}-0.78087 & 0\\ 0.62470 & 0\\ 0 & -0.78087\\ 0 & 0.62470\end{bmatrix}_{4\times2}\begin{Bmatrix}-40.02\\ 40.02\end{Bmatrix}_{2\times1} = \begin{Bmatrix}31.25^k\\ -25.00^k\\ -31.25^k\\ +25.00^k\end{Bmatrix}$$

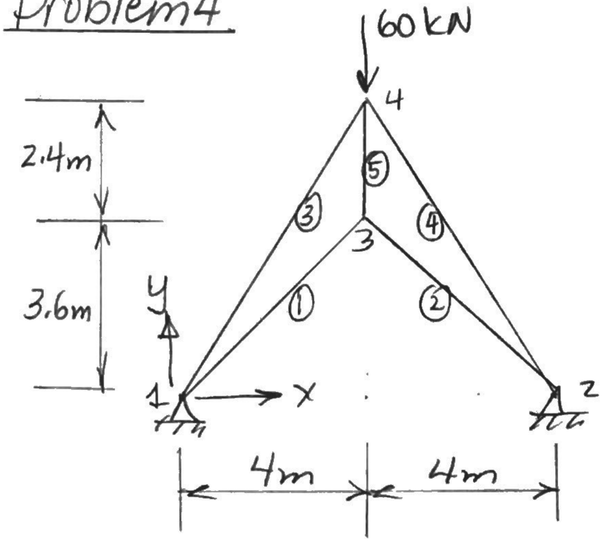


$\Sigma F_x = 31.25 - 31.25 = 0$ ✓

$\Sigma F_y = 25.00 - 25.00 = 0$ ✓

Member is in equilibrium

Problem 4



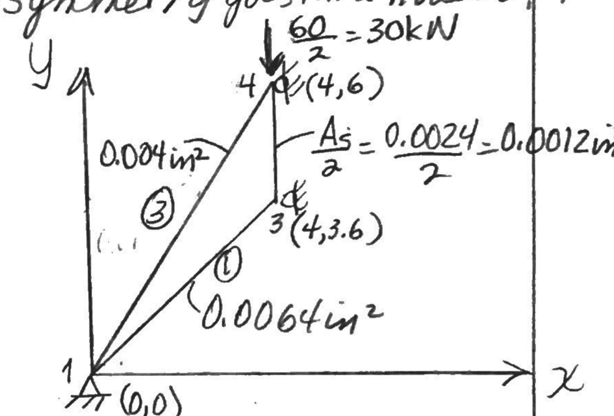
$E = 2\times10^{8}\ kN/m^2$

$A_1 = A_2 = 0.0064\ m^2$

$A_3 = A_4 = 0.004\ m^2$

$A_5 = 0.0024\ m^2$

- Problem details: - line of reflective symmetry goes thru nodes 3 & 4
  - reduced model:
    - half load
    - ½ A for member ⑤
    - $\Delta_{3x} = \Delta_{4x} = 0$
  - global x-axis thru node 1
  - dof's: Δ_{3y} & Δ_{4y}



- Member details

Bar No.	i-j	L (m)	A (m^2)	AE/L (kN/m)	C	S	SC	C^2	S^2
1	1-3	5.38145	0.0064	237854	0.7433	0.6690	0.4972	0.5525	0.4475
3	1-4	7.21110	0.004	110940	0.5547	0.8321	0.4615	0.3077	0.6923
½x5	3-4	2.4	0.0012	100000	0	1	0	0	1

$L_{1-3} = \sqrt{(4-0)^2 + (3.6-0)^2} = 5.38145\ m$; $\cos\phi = \dfrac{(4-0)}{5.38145} = 0.7433$

$\sin\phi = \dfrac{(3.6-0)}{5.38145} = 0.6690$

$L_{1-4} = \sqrt{(4-0)^2 + (6-0)^2} = 7.21110\ m$; $\cos\phi = \dfrac{(4-0)}{7.2111} = 0.5547$

$\sin\phi = \dfrac{(6-0)}{7.2111} = 0.8321$

- Member stiffness matrix [k] (global coord.) for each member:

$$[k]_{1\text{-}3} = 237854 \begin{bmatrix} 0.5525 & 0.4972 & -0.5525 & -0.4972 \\ & 0.4475 & -0.4972 & -0.4475 \\ & & 0.5525 & 0.4972 \\ & & & 0.4475 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} \Delta_{1x} & \Delta_{1y} & \Delta_{3x} & \Delta_{3y} \end{array} \\ \begin{bmatrix} 131414 & 118261 & -131414 & -118261 \\ & 106440 & -118261 & -106440 \\ & & 131414 & 118261 \\ & & & 106440 \end{bmatrix} \end{array} \begin{array}{c} \Delta_{1x} \\ \Delta_{1y} \\ \Delta_{3x} \\ \Delta_{3y} \end{array}$$

$$[k]_{1\text{-}4} = 110940 \begin{bmatrix} 0.3077 & 0.4615 & -0.3077 & -0.4615 \\ & 0.6923 & -0.4615 & -0.6923 \\ & & 0.3077 & 0.4615 \\ & & & 0.6923 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} \Delta_{1x} & \Delta_{1y} & \Delta_{4x} & \Delta_{4y} \end{array} \\ \begin{bmatrix} 34136 & 51199 & -34136 & -51199 \\ & 76803 & -51199 & -76803 \\ & & 34136 & 51199 \\ & & & 76803 \end{bmatrix} \end{array} \begin{array}{c} \Delta_{1x} \\ \Delta_{1y} \\ \Delta_{4x} \\ \Delta_{4y} \end{array}$$

$$[k]_{3\text{-}4} = 100000 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} \Delta_{3x} & \Delta_{3y} & \Delta_{4x} & \Delta_{4y} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 100000 & 0 & -100000 \\ 0 & 0 & 0 & 0 \\ 0 & -100000 & 0 & 100000 \end{bmatrix} \end{array} \begin{array}{c} \Delta_{3x} \\ \Delta_{3y} \\ \Delta_{4x} \\ \Delta_{4y} \end{array}$$

- Assemble the structure stiffness matrix [K]
  - assemble reduced [K] = [K₁₁] since no prescribed displ.

$$[K_{11}] = \begin{array}{c} \begin{array}{cc} \Delta_{3y} & \Delta_{4y} \end{array} \\ \begin{bmatrix} (100000+106440) & -100000 \\ -100000 & (100000+76803) \end{bmatrix} \end{array} \begin{array}{c} \Delta_{3y} \\ \Delta_{4y} \end{array}$$

$$= \begin{bmatrix} 206440 & -100000 \\ -100000 & 176803 \end{bmatrix} \text{ kN/m}$$

- Assemble column matrix of nodal forces & write equilibrium eqs. & solve:

$$\{Q_f\} = \begin{Bmatrix} Q_{3y} \\ Q_{4y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -30 \end{Bmatrix}$$

$$\{Q_f\} = [K_{11}]\{\Delta_f\}$$

$$\begin{Bmatrix} 0 \\ -30 \end{Bmatrix} = \begin{bmatrix} 206440 & -100000 \\ -100000 & 176803 \end{bmatrix} \begin{Bmatrix} \Delta_{3y} \\ \Delta_{4y} \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \Delta_{3y} \\ \Delta_{4y} \end{Bmatrix} = \begin{Bmatrix} -0.1132\times10^{-3}\text{ m} \downarrow \\ -0.2337\times10^{-3}\text{ m} \downarrow \end{Bmatrix} = \begin{Bmatrix} -0.1132\text{ mm} \downarrow \\ -0.2337\text{ mm} \downarrow \end{Bmatrix}$$

- Force in member (5):

$$\begin{Bmatrix} \delta_3 \\ \delta_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.1132\times10^{-3} \\ 0 \\ -0.2337\times10^{-3} \end{Bmatrix} = \begin{Bmatrix} -0.1132\times10^{-3} \\ -0.2337\times10^{-3} \end{Bmatrix}$$

$$F_{3-4} = \frac{AE}{L}(\delta_j - \delta_i) = 100000\left(-0.2337\times10^{-3} - (-0.1132\times10^{-3})\right)$$

$$= -12.05^k \text{ for } A/2$$

so actual force in member 5 $= 2 \times (-12.05^k)$

$$= -24.1^k \ (C)$$