9:20am 10-25-17

Chapter 10

Section 5: Optimization II

Extreme Value Theorem

If f is continuous on a closed interval $[a,b]$ then f has an absolute max value and absolute min value on $[a,b]$.

Finding absolute extrema of a continuous function f on $[a,b]$:

1. Find critical numbers of f on (a,b) (where $f'(x)=0$ or $f'(x)$ is undefined.)

2. Compute $f(c)$ for each critical number c and also compute $f(a)$ + $f(b)$.

3. The absolute max value is the biggest y-value + absolute min value is the smallest y-value.

Solving all optimization problems:

1. Assign a letter to each variable in the problem. If possible, draw and label a picture.

2. Find an expression for the thing we want to optimize.

3. Use physical conditions (extra info) to write our expression in terms of one variable.

Note any physical restrictions (like sides of a rectangle must be ≥ 0, ect) on the function to make an interval.

4. Optimize the function over the interval like we did in Section 10.4.

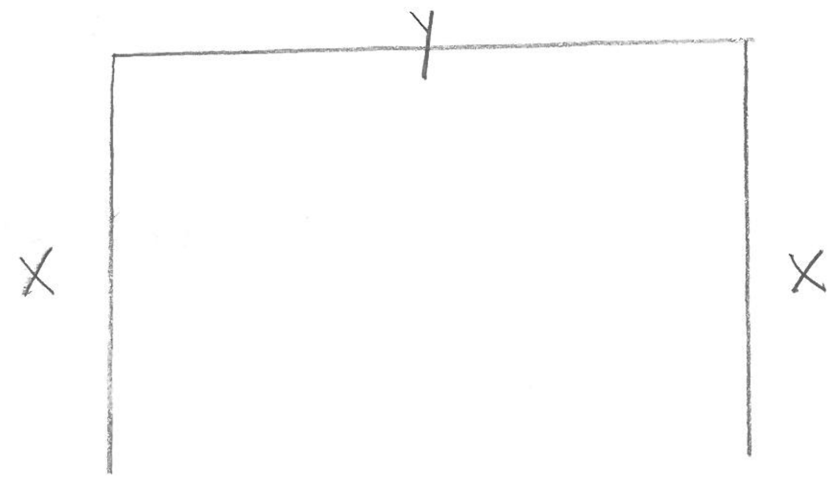
* Units!

We want to build a rectangular parking lot on campus. We want to fence three of the four sides of the rectangle and we have 500 feet of fence. What is the largest area the parking lot can have?

Looking for: Maximum area

Answer: Area in ft^2

1.



Sides

Let x and y be the sides of the parking lot in feet.

Thing we want to optimize

Let A be the area of the parking lot in $feet^2$.

2. $A = lw = xy$

3. 500 feet of fence

$\leftarrow$ Add up the sides

$$x + y + x = 500$$

$$2x + y = 500$$

$$\boxed{y = 500 - 2x}$$

$$A = xy = x(500 - 2x)$$

$$A(x) = -2x^2 + 500x$$

Physical restriction: Sides can't be negative!

$x \geq 0$ $\qquad$ $y \geq 0 \Rightarrow 500 - 2x \geq 0$

$$\frac{-2x}{-2} \geq \frac{-500}{-2}$$

$\leftarrow$ Flipped!

$$x \leq 250$$

Interval: $[0, 250]$

4. Maximize $A(x) = -2x^2 + 500x$ on $[0, 250]$

$A'(x) = -4x + 500 = 0$

$-4x = -500$

$x = 125$

$\leftarrow$ original function

$A(0) = 0$

$A(125) = 31{,}250$

$A(250) = 0$

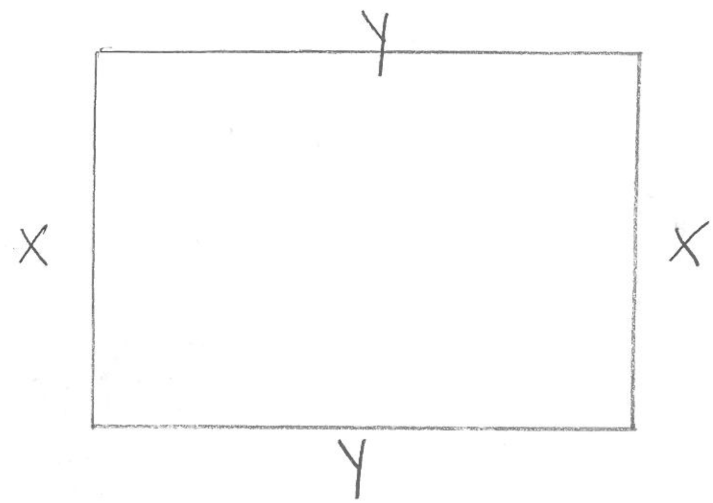
The maximum area is $31{,}250\ ft^2$.

We wish to enclose a rectangular pool that has an area of $300\ \text{ft}^2$ with a fence. Find the dimensions of the pool that uses the smallest amount of enclosure. Round to one decimal place.

Looking for: Minimum perimeter

Answer: Dimensions in feet

1.



Sides

Let x and y be the sides of the pool in feet.

Thing we want to optimize

Let P be the perimeter of the pool in feet.

* We do not need a variable for area! It is not unknown!

Add up sides

2. $P = 2x + 2y$

3. $\dfrac{300}{x} = \dfrac{xy}{x}$ $\quad y = \dfrac{300}{x}$

$$P = 2x + \frac{2}{1}\left(\frac{300}{x}\right)$$

$$P(x) = 2x + \frac{600}{x}$$

$$= 2x + 600x^{-1}$$

Physical restrictions:

$x \geq 0$

$y \geq 0 \Rightarrow \frac{300}{\cancel{x}} \geq 0 \cdot x$

$300 \geq 0$

Gives us no info on x

Interval: $[0, \infty)$

4. Minimize $P(x) = 2x + 600x^{-1}$ on $[0, \infty)$.

$$P'(x) = 2 - 600x^{-2}$$

$$= \frac{2 \cdot x^2}{1 \cdot x^2} - \frac{600}{x^2}$$

$$= \frac{2x^2 - 600}{x^2}$$

$2x^2 - 600 = 0$

$2(x^2 - 300) = 0$

$x^2 = 300$

$x = \pm\sqrt{300}$

$x = \sqrt{300}$ is in $[0, \infty)$.

$x^2 = 0$

$x = 0$

$P(0) =$ undefined

$P(\sqrt{300}) =$

$2\sqrt{300} + \frac{600}{\sqrt{300}}$

≈ 69.3 Feet

anything else

$P(1) = 2 + 600$

$= 602 > 69.3$

Dimensions: $x + y$

$$x = \sqrt{300} \text{ feet} \approx 17.3 \text{ feet}$$

$$y = \frac{300}{x} = \frac{300}{\sqrt{300}}$$

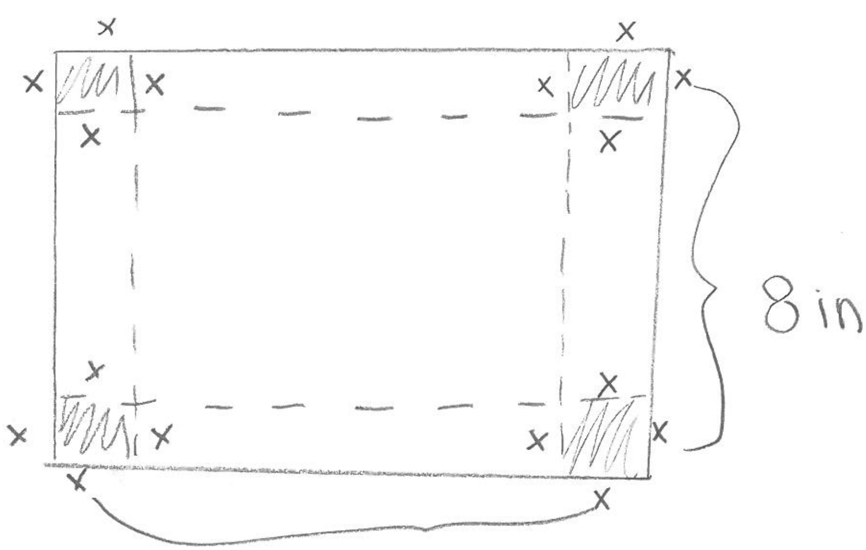
$$\approx 11.3 \text{ feet}$$

Make sure you answer with dimensions, not perimeter!

A box can be constructed by cutting identical squares from each corner of a piece of cardboard. then bending up the flaps. If the piece of cardboard is 8 in by 12 in what is the maximum volume of the box? Round to one decimal place.

Looking for: Maximum volume

Answer: Volume in in^3



1. Let x be the sides of the squares we cut out from each corner.

   Let V be the volume in in^3.

2. $V = lwh = (12-2x)(8-2x)x$

   $V(x) = 4x^3 - 40x^2 + 96x$

Physical restrictions:

$12-2x \geq 0$ $8-2x \geq 0$ $x \geq 0$

$\frac{-2x}{-2} \geq \frac{-12}{-2}$ $\frac{-2x}{-2} \geq \frac{-8}{-2}$

$x \leq 6$ $x \leq 4$

Interval: $[0, 4]$

Pick the tighter restriction

4. Maximize $V(x) = 4x^3 - 40x^2 + 96x$ over $[0, 4]$.

We will continue on Friday!