* In each problem, determine the radius of convergence of the given power series.

3. $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

4. $\sum_{n=0}^{\infty} 2^n x^n$

* In each problem, Find the first nonzero Terms in the solution of the given initial-value problem

13. $y'' + xy' + 2y = 0 \qquad y(0) = 4 \quad y'(0) = -1$

14. $(1-x)y'' + xy' - y = 0 \qquad y(0) = -3 \quad y'(0) = 2$

* In each problem determine $\phi''(x_0)$, $\phi'''(x_0)$, and $\phi^{(4)}(x_0)$ for the given point x_0 if $y = \phi(x)$ is a solution of the given initial-value problem.

1. $y'' + xy' + y = 0 \qquad y(0) = 1, \quad y'(0) = 0$

3. $y'' + x^2 y' + (\sin x) y = 0 \qquad y(0) = a_0, \quad y'(0) = a_1$

* In each Problem

a- Show that the given differential equation has a regular singular point at $x = 0$

b- Determine the indicial equation, the recurrence relation, and the roots of the indicial equation

c- Find the series solution ($x > 0$) corresponding To the larger root.

d- If the roots are unequal and do not differ by an integer, find the series solution corresponding To the smaller root also.

4. $xy'' + y' - y = 0$

6. $xy'' + (1-x)y' - y = 0$

Answers

3. $\rho = \infty$

4. $\rho = 1/2$

Answers

13. $y = 4 - x - 4x^2 + \frac{1}{2}x^3 + \frac{4}{3}x^4 + \ldots$

14. $y = -3 + 2x - \frac{3}{2}x^2 - \frac{1}{2}x^3 - \frac{1}{8}x^4 + \ldots$

Answers.

1. $\phi''(0) = -1$ $\quad \phi'''(0) = 0$ $\quad \phi^4(0) = 3$

3. $\phi''(0) = 0$ $\quad \phi'''(0) = -a_0$ $\quad \phi^4(0) = -4a_1$

Answers

4. b. $r^2 = 0$; $a_n = \dfrac{a_{n-1}}{(n+r)^2}$; $r_1 = r_2 = 0$

c. $y_1(x) = 1 + \dfrac{x}{(1!)^2} + \dfrac{x^2}{(2!)^2} + \ldots + \dfrac{x^n}{(n!)^2} + \ldots$

6. b. $r^2 = 0$; $(n+r)a_n = a_{n-1}$; $r_1 = r_2 = 0$

c. $y_1(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots = e^x$