1. (13 Points) For the matrix

$$A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1/2 & -1/2 \\ 1 & -1/2 & 1/2 \\ 1 & 3/2 & 5/2 \\ 1 & 3/2 & 5/2 \end{bmatrix}$$

(a) (3 Points) Find an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2\}$ for the column space of A.

(b) (2 Points) Using the above, find an orthonormal basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for the column space of A.

(c) (1 Point) From the above, create a matrix $Q = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{bmatrix}$. Is this an orthogonal matrix? Why or why not?

(d) (2 Points) Using A and your matrix Q, create a matrix R such that $A = QR$.

(e) (5 Points) Using your matrices R and Q, find the least-squares solution $\hat{\mathbf{x}}$ to the equation $A\mathbf{x} = \mathbf{b}$ using

$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

Is this solution unique? Why or why not?

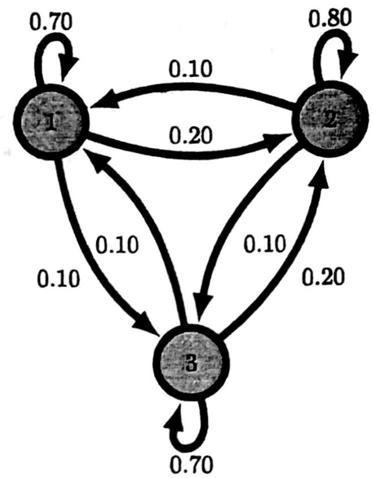
2. (6 Points) For the matrix

$$A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$$

(a) (4 Points) Find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ by first constructing the normal equations and then solving for $\mathbf{x}$.

(b) (2 Points) Compute the least-squares error associated with the least-squares solution you found above.

3. (6 Points) For the system below



(a) (3 Points) Find the matrix P associated with this system.

(b) (3 Points) Find the Steady-state vector $\mathbf{q}$ for matrix P using Reduced Row Echelon Form ($RREF$).