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1. (15 points) Sketch graphs of $y = \sin x$ and $y = \cos x$ on the interval $[0, 2\pi]$. Explain why it makes sense that the derivative of $\sin x$ should be $\cos x$ from the graphs of both functions. Be sure to compare what is happening at several places/intervals along the curve.

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2. (20 points) Show how we determined the derivative of $y = \sin^{-1} x$ (or $\arcsin x$). Be sure to briefly explain what technique we used to find the derivative, and how we adjusted the end result to be strictly in terms of x without any trigonometric functions.

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3. (20 points) How did we determine that the exponential function was appropriate to use for problems involving growth and decay? Be sure to explain the meaning of the differential equation we set up and how we found that $y = e^x$ satisfies that differential equation.

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4. (15 points) Pick ONE of the following to explain:

- (a) State the derivative rule for rational exponents and provide an example. Give a proof of the rule and briefly explain it. In other words, how did we determine this rule was true?
- (b) State the derivative rule for irrational exponents and provide an example. Give a proof of the rule and briefly explain it. In other words, how did we determine this rule was true?

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5. (35 points total) Find the following pieces of information for the function $f(x) = \frac{-2x^2}{x^2 - 1}$:

(a) (2 points) The domain of $f(x)$.

(b) (3 points) Horizontal and vertical asymptotes (if any). (Write asymptotes as equations.)

(c) (2 points) Compute $f'(x)$.

(d) (3 points) Find all critical points of $f(x)$.

(e) (4 points) Determine intervals where $f(x)$ is increasing or decreasing.

(f) (3 points) Use the first derivative test to classify critical points as local maximum, local minimum, or neither.

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5. Continued.

(g) (3 points) Compute $f''(x)$.

(h) (2 points) Find all inflection points of $f(x)$.

(i) (3 points) Determine intervals where $f(x)$ is concave up and concave down.

(j) (3 points) Use the second derivative test to confirm critical points as local maximum, local minimum, or neither.

(k) (7 points) Sketch a graph of the function.

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6. (15 points) Solve the following related rate problem using implicit differentiation: Water is being pumped into a cylindrical fish tank that is 6 feet tall with a radius of 2 feet. If the water level is rising at 2 inches per minute, how fast is the water being pumped into the tank?