the former case the computation is direct, but the latter sometimes requires NTU be extracted as the root of a transcendental equation (except for $C = 0$). The MathCad solve block structure is used for these cases. HX.MCD is divided into three segments: (1) $C = 0$, (2) given NTU and C, find ξ and (3) given ξ and C, find NTU. For the case of $C = 0$, all cases (except for multiple shell passes) reduce to $\xi = 1 - \exp(-\mathrm{NTU})$ or $\mathrm{NTU} = -\ln(1 - \xi)$. For $C = 1$ some of the relations in Table 2–1 are indeterminate and l'Hopital's rule must be used. Figure 2–9 reproduces the MathCad worksheet for the HX.MCD. The three divisions are evident; the input quantities (C, NTU, and ξ in various places) are simply convenient quantities to illustrate the capabilities of HX. Consider the following example.

EXAMPLE 2-1

Use MathCad software element HX to find the NTU required for a single-pass counterflow heat exchanger with $C = 0.5$ and $\xi = 0.7$.

Solution. Software element HX will generate ξ-NTU relations for all the types of heat-exchanger configurations examined in this text. For this problem only single-pass counterflow results are needed. The values of C and ξ are entered as 0.5 and 0.7, respectively. For the single-pass counterflow configuration, the output value of NTU is 1.546. The single-pass counterflow device information is abstracted from the complete worksheet and is reproduced as Fig. 2–10.

The NTU method can be used for heat exchangers with phase change (condensers or evaporators) by noting that the capacity for the fluid undergoing the phase change is essentially unbounded, since the temperature remains constant. Thus, for condensers or evaporators $C = 0$, and all the expressions in Table 2–1 reduce to a much simpler form.

EXAMPLE 2-2

A heat exchanger with one shell pass and eight tube passes raises 100,000 lbm/hr of water from 180° to 300°F. The tube-side fluid is air $(C_p = 0.24$ Btu/lbm °F$)$, which enters at 650°F and exits at 350°F. If $U = 5$ Btu/hr-ft^2-°F, find the surface area required.

Worksheet for computing heat exchanger ξ-NTU relations.

NOTE: This worksheet is divided into three segments:
(1) computations for C = 0,
(2) given NTU and C, find ξ,
(3) given ξ and C, find NTU.

Computations for C = 0:
For C = 0, all ξ-NTU relations (except for multiple shell passes) reduce to $\xi = 1 - \exp(-NTU)$.

Given NTU, find ξ:

Enter the value of NTU

$NTU := 0.5$

$\xi := 1 - \exp(-NTU)$ $\qquad \xi = 0.393$

Given ξ, find NTU:

Enter the value of ξ

$\xi := 0.5$

$NTU := -\ln(1-\xi)$ $\qquad NTU = 0.693$

Given NTU and C, find the effectiveness values:
Enter the value of NTU and C

$NTU := 2.0$

$C := 0.5$

Parallel flow, single pass (PFSP):

$$\xi_{PFSP} := \frac{1 - \exp(-NTU\cdot(1+C))}{1+C} \qquad \xi_{PFSP} = 0.633$$

Counterflow, single pass (CFSP):

$$\xi_{CFSP} := \begin{cases} \dfrac{NTU}{1+NTU} & \text{if } C=1 \\ \dfrac{1-\exp(-NTU\cdot(1-C))}{1-\exp(-NTU\cdot(1-C))\cdot C} & \text{otherwise} \end{cases} \qquad \xi_{CFSP} = 0.775$$

Shell and tube, one shell pass, multiple of two tube passes (ST1):

$$\xi_{ST1} := 2\cdot\left[1 + C + \frac{1+\exp\left[-NTU\cdot\left(1+C^2\right)^{0.5}\right]}{1-\exp\left[-NTU\cdot\left(1+C^2\right)^{0.5}\right]}\cdot\left(1+C^2\right)^{0.5}\right]^{-1} \qquad \xi_{ST1} = 0.693$$

Shell and tube, n shell passes, multiple of 2n tube passes (STn):

$n := 2 \qquad NTU_1 := \dfrac{NTU}{n}$

FIGURE 2–9 MathCad worksheet for software element HX.

Solution. This is a design problem, since a geometrical attribute, A, is to be calculated. The counterflow temperature-area diagram for the situation is given in Fig. 2–11. We have

$$LMTD = \frac{350 - 170}{\ln 350/170} = 249.3°F$$

$$\xi_1 := 2\cdot\left[1 + C + \frac{1+\exp\left[-NTU_1\cdot\left(1+C^2\right)^{0.5}\right]}{1-\exp\left[-NTU_1\cdot\left(1+C^2\right)^{0.5}\right]}\cdot\left(1+C^2\right)^{0.5}\right]^{-1}$$

$$\xi_{STn} := \begin{cases} \dfrac{n\cdot\xi_1}{\xi_1\cdot(n-1)+1} & \text{if } C=1 \\ \left[\left(\dfrac{1-\xi_1\cdot C}{1-\xi_1}\right)^n - 1\right]\cdot\left[\left(\dfrac{1-\xi_1\cdot C}{1-\xi_1}\right)^n - C\right]^{-1} & \text{otherwise} \end{cases} \qquad \xi_{STn} = 0.752$$

Cross flow, both streams unmixed (CFBU):

$$\xi_{CFBU} := 1 - \exp\left[C^{-1}\cdot NTU^{0.22}\cdot\left(\exp\left(-C\cdot NTU^{0.78}\right) - 1\right)\right] \qquad \xi_{CFBU} = 0.739$$

Cross flow, both stream mixed (CFBM):

$$\xi_{CFBM} := NTU\cdot\left(\frac{NTU}{1-\exp(-NTU)} + \frac{NTU\cdot C}{1-\exp(-NTU\cdot C)} - 1\right)^{-1} \qquad \xi_{CFBM} = 0.691$$

Cross flow, stream C_{min} unmixed (CFMIN):

$$\xi_{CFMIN} := C^{-1}\cdot(1-\exp(-C\cdot(1-\exp(-NTU)))) \qquad \xi_{CFMIN} = 0.702$$

Cross flow, stream C_{max} unmixed (CFMAX):

$$\xi_{CFMAX} := 1 - \exp\left[-C^{-1}\cdot(1-\exp(-NTU\cdot C))\right] \qquad \xi_{CFMAX} = 0.718$$

Given ξ and C, find the NTU values:
Enter the value of ξ and C

$\xi := 0.5$

$C := 0.5$

Parallel flow, single pass (PFSP):

$$NTU_{PFSP} := -\frac{\ln(1-\xi\cdot(1+C))}{1+C} \qquad NTU_{PFSP} = 0.924$$

Counterflow, single pass (CFSP):

$$NTU_{CFSP} := \begin{cases} \dfrac{\xi}{1-\xi} & \text{if } C=1 \\ \left(\dfrac{1}{C-1}\cdot\ln\left(\dfrac{\xi-1}{\xi\cdot C-1}\right)\right) & \text{otherwise} \end{cases} \qquad NTU_{CFSP} = 0.811$$

FIGURE 2–9 *(continued)*

The total energy transfer is

$$q = \dot{m}C_p\Delta T$$

$$= 100{,}000\,\frac{\text{lbm}}{\text{hr}}\;1\,\frac{\text{Btu}}{\text{lbm}\cdot°\text{F}}\,(300 - 180)°\text{F}$$

$$= 12 \times 10^6\ \text{Btu/hr}$$

Shell and tube, one shell pass, multiple of two tube passes (ST1):

$$E := \frac{\frac{2}{\xi} - (1 + C)}{(1 + C^2)^{0.5}} \qquad NTU_{ST1} := -(1 + C^2)^{-0.5} \cdot \ln\left(\frac{E - 1}{E + 1}\right) \qquad NTU_{ST1} = 0.861$$

Shell and tube, n shell passes, multiple of 2n tube passes (STn):

$$n := 2 \qquad F := \left(\frac{\xi C - 1}{\xi - 1}\right)^{\frac{1}{n}} \qquad \xi_1 := \begin{cases} \dfrac{\xi}{\xi - n\cdot(\xi - 1)} & \text{if } C=1 \\ \dfrac{F - 1}{F - C} & \text{otherwise} \end{cases}$$

$$E_1 := \frac{2\xi_1^{-1} - (1 + C)}{(1 + C^2)^{0.5}} \qquad NTU_1 := -(1 + C^2)^{-0.5} \cdot \ln\left(\frac{E_1 - 1}{E_1 + 1}\right)$$

$$NTU_{STn} := n \cdot NTU_1 \qquad NTU_{STn} = 0.822$$

Cross flow, both streams unmixed (CFBU):

Initial guess of NTU: $NTU_{CFBU} := 1$

Given

$$\xi = 1 - \exp\left[C^{-1} \cdot NTU_{CFBU}^{0.22} \cdot \left(\exp\left(-C\, NTU_{CFBU}^{0.78}\right) - 1\right)\right]$$

$$NTU_{CFBU} := \mathrm{Find}(NTU_{CFBU}) \qquad NTU_{CFBU} = 0.858$$

Cross flow, both stream mixed (CFBM):

Initial guess of NTU: $NTU_{CFBM} := 1.0$

Given

$$\xi = NTU_{CFBM}\left(\frac{NTU_{CFBM}}{1 - \exp(-NTU_{CFBM})} + \frac{NTU_{CFBM}\, C}{1 - \exp(-NTU_{CFBM}\, C)} - 1\right)^{-1}$$

$$NTU_{CFBM} := \mathrm{Find}(NTU_{CFBM}) \qquad NTU_{CFBM} = 0.861$$

Cross flow, stream C_{min} unmixed (CFMIN):

$$NTU_{CFMIN} := -\ln\left(1 + C^{-1} \cdot \ln(1 - \xi C)\right) \qquad NTU_{CFMIN} = 0.857$$

Cross flow, stream C_{max} unmixed (CFMAX):

$$NTU_{CFMAX} := -C^{-1} \cdot \ln(C \cdot \ln(1 - \xi) + 1) \qquad NTU_{CFMAX} = 0.851$$

FIGURE 2–9 *(concluded)*

and

$$P = \frac{T_{h_{out}} - T_{h_{in}}}{T_{c_{in}} - T_{h_{in}}} = \frac{350 - 650}{180 - 650} = 0.638$$

$$Z = \frac{T_{c_{in}} - T_{c_{out}}}{T_{h_{out}} - T_{h_{in}}} = \frac{180 - 300}{350 - 650} = 0.4$$

Given ξ and C, find the NTU values:
Enter the value of ξ and C

$\xi := 0.7$

$C := 0.5$

Counterflow, single pass (CFSP):

$$NTU_{CFSP} := \begin{cases} \dfrac{\xi}{1 - \xi} & \text{if } C=1 \\ \left(\dfrac{1}{C - 1} \cdot \ln\left(\dfrac{\xi - 1}{\xi C - 1}\right)\right) & \text{otherwise} \end{cases} \qquad NTU_{CFSP} = 1.546$$

FIGURE 2–10 HX solution for Example 2-1.

FIGURE 2–11 Temperature-area diagram for Example 2-2.

from whence F is found from Fig. 2–7(a) to be

$$F = 0.88$$

But

$$12 \times 10^6 \frac{\text{Btu}}{\text{hr}} = 5 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F}} A(249.3\ {}^\circ\text{F})(0.88)$$

from which

$$A = 10{,}940 \text{ ft}^2$$

ALTERNATIVE METHOD

The NTU approach can also be used. Assuming negligible energy lost to the surroundings, we have

$$\Delta T_h(\dot{m}C_p)_h = \Delta T_c(\dot{m}C_p)_c$$

$$\dot{m}_{air}\, 0.24 \frac{\text{Btu}}{\text{lbm} \cdot {}^\circ\text{F}} (650 - 350)^\circ\text{F} = 100{,}000 \frac{\text{lbm}}{\text{hr}}\, 1 \frac{\text{Btu}}{\text{lbm} \cdot {}^\circ\text{F}} (300 - 180)^\circ\text{F}$$

$$\dot{m}_{air} = 166{,}667 \frac{\text{lbm}}{\text{hr}}$$

The capacities are:

$$(\dot{m}C_p)_h = 166{,}667\,\frac{\text{lbm}}{\text{hr}}\,0.24\,\frac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{F}} = 40{,}000\,\frac{\text{Btu}}{\text{hr}\cdot{}^\circ\text{F}}$$

$$(\dot{m}C_p)_c = 100{,}000\,\frac{\text{lbm}}{\text{hr}}\,1\,\frac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{F}} = 100{,}000\,\frac{\text{Btu}}{\text{hr}\cdot{}^\circ\text{F}}$$

Thus, $(\dot{m}C_p)_h$ is minimum. Then

$$C = \frac{C_{min}}{C_{max}} = 0.40$$

and

$$\xi = \frac{q}{q_{max}} = \frac{12 \times 10^6}{C_{min}(650 - 180)} = 0.6383$$

Using Fig. 2–8(c) or HX, we obtain

$$\text{NTU} = \frac{UA}{C_{min}} = 1.35$$

$$A = 1.35\,\frac{C_{min}}{U} = 10{,}800\ \text{ft}^2$$

The difference in the two areas computed is caused by errors in reading the various figures.

EXAMPLE 2-3

What would the water exit temperature be if the hot-air flow rate were reduced by 30 percent in the heat exchanger in Example 2-2, assuming that all other quantities remained unchanged?

Solution. This is an analysis problem, since the heat-exchanger area is known and an exit temperature is to be computed. If the LMTD method is used, the solution is *iterative*, since P and Z are functions of the unknown exit temperature of the water. The NTU method, however, can be used to *directly* compute the water exit temperature.

The new air flow rate is 0.7 $\dot{m}_{air}$ and is 116,667 lbm/h. Then

$$(\dot{m}C_p)_h = 116{,}667\,\frac{\text{lbm}}{\text{hr}}\,0.24\,\frac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{F}} = 28{,}000\,\frac{\text{Btu}}{\text{hr}\cdot{}^\circ\text{F}}$$

and

$$C = \frac{28{,}000}{100{,}000} = 0.28$$

Here we know the NTU, since

$$\text{NTU} = \frac{UA}{C_{min}} = 5\,\frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}}\,10{,}940\ \text{ft}^2\,\frac{\text{h}\cdot{}^\circ\text{F}}{28{,}000\ \text{Btu}} = 1.95$$

from which Fig. 2–8(c) or HX yields

$$\xi = 0.76$$

and

$$q = \xi C_{min}(T_{h_{in}} - T_{c_{in}})$$

$$= (0.76)28{,}000\,\frac{\text{Btu}}{\text{hr}\cdot{}^\circ\text{F}}(650 - 180)^\circ\text{F}$$

$$= 10 \times 10^6\,\frac{\text{Btu}}{\text{hr}}$$

Writing an energy balance on the cold fluid, we obtain

$$q = (\dot{m}C_p)_c\,\Delta T_c = (\dot{m}C_p)_c(T_{c_{out}} - T_{c_{in}})$$

$$T_{c_{out}} = T_{c_{in}} + \frac{q}{C_c}$$

$$= 180\ {}^\circ\text{F} + 100\ {}^\circ\text{F} = 280\ {}^\circ\text{F}$$

Thus, the exit temperature of the water is 280°F. The gas exit temperature for this case is 293°F. Reducing the mass flow rate of air results in a lower exit temperature for the air but less total energy transfer.

Special case: heat transfer from a pipe immersed in an ambient environment. A recurring situation for many energy systems is a fluid-conveying pipe or duct exposed to an ambient environment. This scenario, illustrated in Fig. 2–12, occurs in situations such as chilled or hot-water heating, ventilating, and air-conditioning (HVAC) systems and in many industrial processes where hot or cold compounds must be piped from one location to another. For the arrangement illustrated in the figure, the driving potential for the heat transfer process is often taken to be the difference between T_∞ and the average of T_{in} and T_{out}; that is,

$$\overline{\Delta T} = T_\infty - \tfrac{1}{2}(T_{in} + T_{out}) \tag{2-36}$$

so that Eq. (2-1) holds:

$$q = UA\,\overline{\Delta T}$$

However, as pointed out by Wolf (13) and as discussed by Suryanarayana (14), when UA/C_{min} is greater than 2.0, the second law of thermodynamics would be violated since $(T_{in} - T_\infty)$ and $(T_{out} - T_\infty)$ would have different signs. The reason for this anomaly is that the situation illustrated in Fig. 2–12 has a temperature-area diagram similar to that

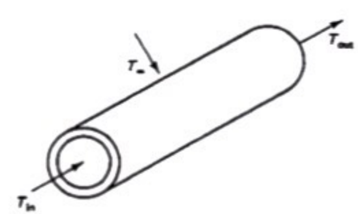

FIGURE 2–12 Pipe in an ambient environment.