This exam is due on Monday November 27 at 10:45 in class

5. A computerized test consists of two parts, A and B. A student must try successive questions until he gets one right. He can then proceed to part B of the test. Suppose that for each question in both parts P[success]= **p =1/3** and P[failure]=q=**1-p = 2/3**. Let N_1 = number of questions he requires to obtain the first success, or correct answer, in part A; Let N_2 = number of questions it takes him after finishing part A of the test, to get a question correctly (success) in part B. Thus if the sequence is **F F F S F F F F F F S** then $N_1=4$ and $N_2=7$. F=incorrect answer. S=correct answer.

i. Argue that N1 and N2 are independent random variables. Name their distributions. With what parameter? **Hint**: *Think of waiting for first success in Bernoulli trials.*

ii. Now write down the joint pdf $P[N_1=n_1, N_2=n_2]$ of N_1 and N_2 for $n_1 \geq 1$ and $n_2 \geq 1$. Pay attention to the support of the joint pdf.

iii. Write down the probability density function of $N = N_1+N_2$ = total number of questions attempted to pass part A and part B of the computerized test. Be sure to include the support of the distribution of N: $N \geq$???
Hint: think independent trials and waiting for second success.

iv. Find the conditional distribution of N_1 given N: $P[N_1 = n_1 \mid N=n]$ **for fixed** $n \geq 2$ and $1 \leq n_1 \leq (n-1)$ (because $n_2 \geq 1$). **Hint:** $P[N_1 = n_1 \mid N_1 + N_2 = n]$ **implies that N_2 must be $n-n_1$. Then use the fact that N1 and N2 are independent with the distribution you determined in part i.**

v. **Can you name the discrete distribution you found in part iv? Now** find the conditional mean $E[N_1 \mid N=n]$ for a fixed $n \geq 2$. No computation is required.