

This exam is due on Monday November 27 at 10:45 in class

3. Two 4-sided dice are rolled independently. Let  $X$  be the maximum of the two outcomes, and  $Y$  the minimum of the two outcomes. Clearly  $1 \leq Y \leq X \leq 4$ .

- a. Draw a table for the joint probability mass function of  $X$  (rows) and  $Y$  (columns). Make sure to place the probability of each pair of  $x, y$  in its appropriate location.
- b. Display the marginal distribution of  $X$  and the marginal distribution of  $Y$ . (use 2 columns: one for values  $x$  and one for  $f(x)=P[X=x]$  and similarly for  $Y$ .)
- c. Are  $X$  and  $Y$  independent? Support your answer with appropriate calculations using your probability table.
- d. Compute the correlation between  $X$  and  $Y$
- e. Find the probability mass function, values and probabilities, for the conditional distribution of  $Y$  given that  $X=3$ . That is compute  $P[Y=y | X=3]$  for all relevant values of  $y$ .
- f. Compute  $E[Y | X=3]$  and  $\text{VAR}[Y | X=3]$ .