

**This exam is due on Monday November 27 at 10:45 in class**

1. The duration of a summer shower in minutes is found to be a random variable  $X$  with a Cumulative Distribution Function

$$F(x) = .4 (1 - e^{-x/3}) + .6 [1 - (1 + x/5) e^{-x/5}] \quad \text{if } x > 0.$$

- a. Show that  $F$  is indeed a cumulative distribution function, and determine the probability density cumulative function  $f(x)$  of  $X$ . *Don't forget its **support**. (those values  $x$  for which  $f(x) > 0$ ). Check all assumptions!*
- b. The density you obtained is a 40% to 60% mixture of two well-known distributions. What are they? Include their parameters in your description.
- c.
- What is the conditional probability that a shower will last over 9 minutes, given that it has already lasted over 6 minutes? **Use  $F$  as given above.**
  - What is the probability that a shower lasts exactly 2 minutes?
  - What is the probability that a shower lasts between 4 and 7 minutes? Do the minimum amount of work!
  - Find the expected value of the duration  $X$  of a shower in minutes. **Hint:** Use the pdf which is a weighted sum of two densities whose means you know.