Calculus- Derivatives part 3 :O Chapter Review Name Fuller

Section 1: Implicit differentiation.

1) $x + 2xy - y^2 = 2$

Find the value of $\frac{dy}{dx}$ at the point $(2, 4)$.

$1 + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x - 2y) = -2y - 1$

$\frac{dy}{dx} = \frac{-2y-1}{2x-2y} \rightarrow \frac{-2(4)-1}{2(2)-2(4)} = \frac{-9}{-4} = \boxed{\frac{9}{4}}$

2) $3y^2 + x^2 - xy = 1$

Find $\frac{dy}{dx}$.

$6y\frac{dy}{dx} + 2x - (y + x\frac{dy}{dx}) = 0$

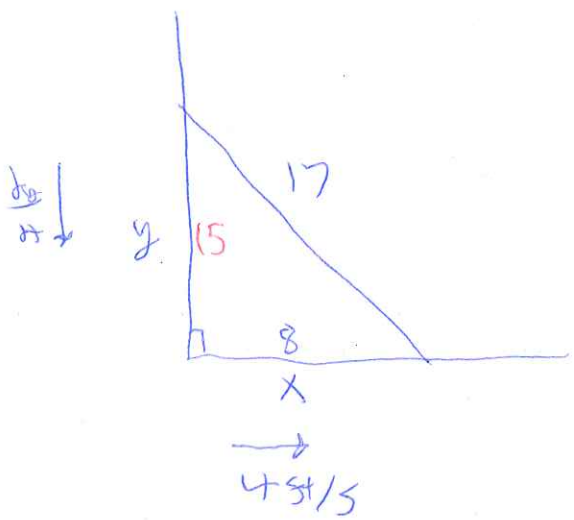
$6y\frac{dy}{dx} - x\frac{dy}{dx} + 2x - y = 0$

$\frac{dy}{dx}(6y - x) = y - 2x$

$\boxed{\frac{dy}{dx} = \frac{y-2x}{6y-x}}$

Section 2: Related rates.

3) There is a 17 foot ladder up against a building and the bottom starts to slide out at 4ft/s, how fast is the top sliding down when the ladder is 8 feet away from the base of the building?



$\frac{d}{dt}(x^2 + y^2 = 17^2)$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(8)(4) + 2(15)\frac{dy}{dt} = 0$

$64 + 30\frac{dy}{dt} = 0$

$\boxed{\frac{dy}{dt} = \frac{-64}{30} = -2.1 \text{ ft/s}}$

4) Air is being pumped into a balloon at a rate of 3.5 cubic feet/minute. Find the rate of change of the radius, when the radius is 5 feet.

$\frac{dV}{dT}$

$\frac{d}{dT}\left(V = \frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dT}$

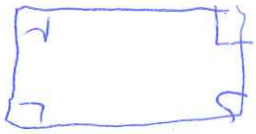
$3.5 = 4\pi (5)^2 \frac{dr}{dT}$

$\frac{.014}{5} = \frac{3.5}{4\pi (5)^2} = \frac{dr}{dt}$

very very slow

Section 3: Optimization.

5) A piece of 8.5 by 13 paper has its corners cut off with length x so that you can fold it up to form an open top box. What is the greatest volume this box can have?



$V = (13-2x)(8.5-2x)x$

$V = (110.5 - 43x + 4x^2)x$

$V = 110.5x - 43x^2 + 4x^3$

$V' = 110.5 - 86x + 12x^2$

$0 = 110.5 - 86x + 12x^2$

$x = 5.49$ or 1.68

$V = 83.24\ in^3$

6) Given the function $y = 8 - x^2$ find the point on the function which minimizes the distance between the function and the origin.

$y = 8 - x^2$

$d = \sqrt{(x-0)^2 + (y-0)^2}$

$d = \sqrt{x^2 + (8-x^2)^2}$

$d = \sqrt{x^2 + 64 - 16x^2 + x^4}$

$d = x^4 - 15x^2 + 64$

$d' = 4x^3 - 30x$

$0 = 2x(2x^2 - 15)$

$2x^2 = 15$

$x^2 = \frac{15}{2}$

$x = 0, \sqrt{\frac{15}{2}}, -\sqrt{\frac{15}{2}}$