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Properties of time-series estimates of degree of leverage measures

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Abstract

Empirical studies suggest that time-series regression estimates of the degrees of operating and financial leverage have a tendency to produce measures less than one. According to ex ante theory, these measures should be greater than one for firms operating above the break-even point. There have also been suggestions that the biases in these estimates may be attributable to an underlying increase in unit sales. This work presents evidence that these counter-intuitive measures are produced by changes in the firm's operating parameters (unit price, variable cost, fixed cost and interest payments). It further suggests that attempts to control for the underlying change in unit sales substantially increase the volatility of predicted estimates.

Keywords: Operating leverage, financial leverage, capital structure
JEL classifications: G30/G32

1. Introduction

The use of time-series regression techniques to estimate the degree of leverage measures, degree of operating leverage and degree of financial leverage, has grown increasingly popular in empirical studies. However, considerable evidence exists that these methods have a tendency to produce signs and magnitudes of coefficients that are inconsistent with ex ante theory.

The purpose of this study is to show that these time-series regression estimates suffer from the same biases as point-to-point elasticity measures of degree of lever-

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age. These illogical results are attributable to fluctuations in the firm's operating parameters. The distortions are most prevalent for a firm with low volatility in demand, accompanied by relatively high volatility in an operating parameter, such as unit price, unit variable cost, periodic fixed costs, or the level of interest payments. Moreover, the tendency to produce counter-intuitive results is stronger, the higher above the break-even point a firm is operating.

If time-series estimates of the degree of leverage measures are to be employed in empirical research as linear or curvilinear proxies for leverage, the dangers are obvious. If the firm is operating far above the break-even point, these proxies have a tendency to signal the opposite. Also, this tendency is exacerbated if variability in demand is low. This result implies that the less risky the firm, the better the chance for the proxies to suggest that it is very risky. The aim of this work is merely to document the reasons for these biases, not to take on the far more ambitious task of suggesting a simple yet useful proxy for leverage.

The following section of the paper includes a brief review of recent literature on the degree of leverage measures. The third section demonstrates why the time-series regression methods for the estimation of the degree of leverage measures tend to produce illogical results. The subsequent section includes a presentation of a two-stage time-series regression technique for estimates of the degree of operating leverage when product demand is increasing. The core of this section demonstrates that this method also is susceptible to computational biases and tends to produce highly volatile results. The final section contains a summary of the results and conclusions.

2. Review of the literature

Finance and Managerial Accounting textbooks often include a discussion of the impact of leverage on firm profitability. Leverage usually is discussed in two distinct senses: operating leverage and financial leverage. These concepts obviously are very important and also reasonably simple to understand. Unfortunately, in practice, producing adequate proxies for leverage has proven very difficult. This is particularly true for operating leverage, since fixed costs are a very tenuous concept.[1]

One common textbook approach to proxying the leverage concepts is the use of the degree of leverage measures. For instance, the popular introductory text of Brigham (1995) contains an extensive introduction to the Degree of Operating Leverage and the Degree of Financial Leverage (hereafter DOL and DFL, respectively).

Brigham defines DOL as:

$$\text{DOL} = (p - v)\,Q/[(p - v)\,Q - F], \tag{1}$$

where p is the unit price, v the unit variable cost, F the periodic fixed cost, and Q

[1] Dugan and Shriver (1989) conducted a study of correlations between various common proxies for operating leverage.

the unit output or quantity demanded (it is assumed throughout that output and quantity demanded are interchangeable). It is implicitly assumed that p, v, and F are all constant, and that output is the only stochastic variable.[2]

When a firm incorporates a fixed cost into its production technology, we can estimate a break-even point (hereafter BE), where BE = F/(p − v). As long as the firm is operating above the BE, DOL > 1. If output were to fall below the BE, then DOL < 0. If the firm were operating very near the BE, the absolute values of the measures of DOL would tend to be very high. If the firm were operating far above the BE, the DOL would asymptotically approach one; and if the firm sold zero units, DOL = 0.

Usually it is difficult to classify externally reported costs into fixed and variable components, and therefore it is common to define DOL as an elasticity measure of the percentage change in earnings before interest and taxes (EBIT = (p − v) Q − F) for a given change in unit demand:

$$\text{DOL} = \%\Delta\text{EBIT}/\%\Delta\text{Q}. \tag{2}$$

Also, since many firms either do not report data on unit output or manufacture multiple products, an even more popular alternative is:

$$\text{DOL} = \%\Delta\text{EBIT}/\%\Delta\$\text{Sales}, \tag{3}$$

where \$Sales = pQ (note that if p were a constant, %ΔQ and %ΔpQ are interchangeable).

Two alternative definitions of DFL can be made along the same lines. The point estimate would be:

$$\text{DFL} = [(p - v)\,Q - F]/[(p - v)\,Q - F - I], \tag{4}$$

where I is the periodic interest payment, which normally is assumed to be constant. The elasticity measure would be:

$$\text{DFL} = \%\Delta\text{EBT}/\%\Delta\text{EBIT}, \tag{5}$$

where EBT is the earnings before taxes, (p − v) Q − F − I.

Here, we can introduce a "total break-even point" (hereafter BET), necessary to cover both operating and financial fixed costs, where the BET = (F + I)/(p − v). Again, if the firm's unit sales were above this level, DFL > 1. Points with demand just above the BET would produce very large measures of DFL, which would decline asymptotically toward one as unit output increased above the BET. Clearly,

[2] It is also implicitly assumed that prices are determined in a perfectly competitive market and that there is not a downward sloping demand curve for the firm's product.

if the firm were operating below the BET, DFL < 0. This case often is not discussed in texts, as it implies the firm is bankrupt.[3]

While these presentations of DOL and DFL are popular in introductory texts to familiarize students with the concepts, they rarely are employed in actual empirical testing. Another approach to estimating DOL and DFL that has gained popularity in the finance literature was suggested and used by Mandelker and Rhee (1984). In this methodology, DOL and DFL are estimated through the use of time-series regression. This approach basically is a play on equations (2), (3) and (5). DOL can be estimated by calculating the parameter b of the regression equation:

$$\ln(\text{EBIT}) = a + b\ln(Q) + \tilde{e}_1 \quad (6)$$

A similar time-series regression approach to calculate DOL was used in a contemporary paper by Ang and Peterson (1984). In the same vein, the parameter d in the equation:

$$\ln(\text{EBT}) = c + d\ln(\text{EBIT}) + \tilde{e}_2, \quad (7)$$

can be interpreted as DFL. In equations (6) and (7), e_1 and e_2 are error terms with the usual ordinary least-squares properties.

In a later study, O'Brien and Vanderheiden (1987) suggest that the time-series estimates of DOL proposed by Mandelker and Rhee suffer from a construct validity problem. If unit sales are systematically rising, the time-series estimates of DOL are proxying for growth rather than operating leverage. In particular, in cases where unit sales are growing at the same rate as EBIT on average, the Mandelker and Rhee approach tends to produce estimates of DOL clustered around one.

O'Brien and Vanderheiden proposed accounting for the growth component in sales and EBIT by estimating two time-series regression models in the first stage:

$$\ln(\text{EBIT}_t) = \ln(\text{EBIT}_0) + g_{\text{EBIT}} * t + \tilde{M}_t^{\text{EBIT}}, \text{ and} \quad (8)$$

$$\ln(Q_t) = \ln(Q_0) + g_Q * t + \tilde{M}_t^Q, \quad (9)$$

where EBIT_0 and Q_0 are the initial levels of EBIT and Q respectively, g_{EBIT} is the periodic growth rate in EBIT, g_Q is the periodic growth rate in unit sales, $\tilde{M}^{\text{EBIT}}$ and $\tilde{M}^Q$ are the residual terms, and t is a time counter. In the second stage, the following model is estimated:

$$M_t^{\text{EBIT}} = qM_t^Q + \tilde{e}_4, \quad (10)$$

where the regression slope coefficient q can be interpreted as DOL.[4]

[3] As output declines to zero units, DFL would move toward -F/((p-v)Q-F-I) instead of zero as was the case for DOL.

[4] This methodology can be extended to make estimates of DFL in a similar fashion. Equation (8) would be estimated, along with the model $\ln(\text{EBT}_T) = \ln(\text{EBT}_0) + g_{\text{EBT}} + M_t^{\text{EBT}}$, where EBT_0 is the initial level of the EBT, g_{EBT} is the periodic growth rate in EBT, and and M_t^{EBT} is the residual term. In the second stage, the model $M_t^{\text{EBT}} = rM_t^{\text{EBIT}} + e_5$ would be estimated, where the regression slope coefficient, r, can be interpreted as DFL.

In an empirical study, Dugan and Shriver (1992) calculated estimates of DOL employing both the Mandelker and Rhee and the O'Brien and Vanderheiden methodologies for 245 firms. One of their research objectives was to determine which of the two methodologies produced more measures of DOL less than one. The very nature of this test suggests that these estimates of DOL often violate the classical ex ante assumption in equation (1) that firms operating above the BE should have a DOL greater than one. In fact, for their sample, roughly 85% of the estimates of DOL calculated by the Mandelker and Rhee methodology and approximately 45% of those calculated by the O'Brien and Vanderheiden method take on values less than one. Operating below the BE is obviously not a plausible explanation for the majority of a 245 firm sample over fifteen years producing these low levels of DOL. Another interesting feature of their study is that while the O'Brien and Vanderheiden method produced a much higher proportion of DOL measures greater than one, the outcomes for DOL calculated by this method had a much higher standard deviation (by a factor of ten) than those calculated by the method of Mandelker and Rhee.[5] Dugan and Shriver present these empirical results without attempting to account for these troubling observations.

In a recent hypothetical study of DOL, Lord (1995) found that point-to-point elasticity measures of DOL, made with equations (2) and (3), tend to produce counter-intuitive estimates of the degree of leverage measures when the operating parameters (p, v, and F), which are assumed to be constant, actually fluctuate across the calculation period. He found that the tendency for a firm operating above both BE and BET to produce degree of leverage estimates less than one is most prevalent for small changes in demand associated with relatively large changes in the operating parameters.

For an example of this effect, rewrite equation (2) as:

$$DOL = (\Delta EBIT/\Delta Q) * (Q/EBIT). \tag{11}$$

Notice that if none of the operating parameters (p, v, and F) changed, the first term in this product, $\Delta EBIT/\Delta Q$, would always be equal to the operating margin (p – v). However, if unit price were allowed to fluctuate, then we could rewrite equation (11) as:

$$DOL = [(\Delta_Q EBIT + \Delta_P EBIT + \Delta_{PQ} EBIT)/\Delta Q] * (Q/EBIT).$$

Here $\Delta_Q EBIT$ is the change in EBIT attributable to the change in unit demand as if price had not changed; $\Delta_Q EBIT = p_1 (Q_2 - Q_1)$. This term would be the same as $\Delta EBIT/\Delta Q$ in equation (11) if price had not changed. The expression $\Delta_P EBIT$ is a change in EBIT attributable to the change in price as if output had not changed; $\Delta_P EBIT = (p_2 - p_1) Q_1$. Finally, $\Delta_{PQ} EBIT$ is a cross product term where $\Delta_{PQ} EBIT =$

[5] High cross-sectional variability in a proxy is not necessarily a limitation. However, when this high variability has a tendency to produce counter-intuitive observations, the implications are troubling.

$(p_2 - p_1)(Q_2 - Q_1)$. Assuming that $D_{PQ}EBIT$ is small, the vital term that determines deviations of the estimates of DOL from their expected values is $\Delta_P EBIT/\Delta Q$. Obviously, large changes in unit price coupled with small changes in output would tend to drive estimates of DOL further from their hypothetical values (as if price had not changed). Clearly, the same logic holds in the case of DFL in a world where interest payments fluctuate.

The earlier study of point-to-point estimates of the degree of leverage measures also found that the frequency and magnitude of these counter intuitive results tend to increase the higher above BE the firm is operating.[6] If the term $\Delta_P EBIT/\Delta Q$ is rewritten as $[(p_2 - p_1)\ Q_1]/(Q_2 - Q_1)$, clearly for a given change in $(p_2 - p_1)$ and $(Q_2 - Q_1)$, the magnitude of this term will be greater the higher the level of Q_1.

The primary purpose of the two following sections is, therefore, to demonstrate that these same tendencies hold for time-series estimates of DOL and DFL and can explain most of Dugan and Shriver's empirical results.

3. Mandelker and Rhee estimates of leverage

First, consider the time-series regression technique popularized by Mandelker and Rhee (1984) (hereafter referred to as MR) to estimate DOL and DFL. In each case, the slope coefficients from equations (6) and (7) are interpreted as the DOL_{MR} and DFL_{MR}, respectively.

This section shows that measures of DOL_{MR} and DFL_{MR} derived from the time-series regressions tend to take on illogical values ($DOL_{MR} < 1$ and $DFL_{MR} < 1$, for firms operating above the BE or BET) because of changes in the firm's operating parameters. To demonstrate the properties of MR's time-series estimates, a series of simulations will be presented using a random number generator to produce variable levels of unit demand and one of the operating parameters. Lord (1995) noted that changes in any of the operating parameters would cause distortions; therefore, the first case considered will be where Q and p are allowed to fluctuate.[7] The other parameters will be set to be constant with v = \$4.00, F = \$400.00, and I = \$200.00. The variables $\tilde{Q}$ and $\tilde{p}$ will be produced by a normal, random process about initial levels. Unit demand will be set to an initial level of 300 units, and unit price to \$8.00.

Random results then are produced allowing for two different volatility assumptions for each variable; first, for high volatility, and second, for low volatility. In the first case, unit output will vary normally about a mean of 300 units, with a standard deviation of 50 units ($\sigma_Q = 50$); in the second case, the standard deviation

[6] While the regions far beyond BET are of little interest to the manager, they are of critical importance to empirical researchers employing DOL measures as proxies for leverage, as most firms operate far above BET during normal operating periods.

[7] Lord (1995) noted that flucuations in any of the operating parameters (except interest payments for the case of DOL) would produce similar results. Unit price was simply chosen as a likely candidate to fluctuate outside the control of management.

will be only 5 units ($\sigma_Q = 5$). In the high volatility case, unit price will vary normally about a mean of \$8.00 with a standard deviation of \$1.00 (σ_p = \$1.00), and in the low volatility case with a standard deviation of \$0.10 (σ_p = \$0.10). This arrangement allows us to test four combinations:

(1) low volatility in both unit output and price;
(2) low volatility in output, but high volatility in price;
(3) high volatility in output, but low volatility in price; and
(4) high volatility in both output and unit price.

The initial assumptions make it likely that the firm will still be operating above the BE or BET and that $p \geq v$. At the initial conditions ($\bar{p}$ = \$8.00, v = \$4.00, F = \$400.00, I = \$200.00, and $\bar{Q} = 300$), equation (1) would suggest DOL = 1.50, and by equation (4) DFL = 1.33. Even if both variables were to simultaneously move more than one standard deviation from their means, the leverage measures would still remain greater than one. The actual point estimates of DOL and DFL for 49 random observations in the case of high volatility for each Q and p (the case most likely to produce extreme results) are shown in Table 1. All of the estimates are greater than one except for DFL associated with observation 48. By chance, at this point, both demand and unit price fell to very low levels (233.80 units at \$6.44 per unit). At this point, the firm would be bankrupt, unless the stockholders had accumulated some internal surplus funds (perhaps the firm was being managed by the bankruptcy court in period 49). Needless to say, for the other three combinations of 49 observations, none of the degree of leverage measures was less than one. The four sets of 49 random observations then were used to estimate a series of DOL_{MR} measures using a time-series regression of the form shown in equation (6).[8] A rolling time-series approach was employed, taking 20 observations at a time, then dropping the first observation, rolling ahead to the next 20 observations. This technique allowed the estimation of 30 time series measures of DOL_{MR} for each of the four combinations (based on the volatility of output and price). The results are summarized in Table 2. Note that in three of the four columns of Table 2, reasonable and stable estimates of DOL_{MR} appear, all near the level of DOL = 1.50. However, in the column based on small volatility in unit output and large variability in unit price, wildly fluctuating levels of DOL_{MR} are seen, including many observations less than one (most are in fact negative). This result is analogous to that noted by Lord (1995), where small changes in output coupled with relatively large changes in unit price have a tendency to produce point-to-point elasticity estimates of DOL < 1. Table 2 demonstrates that the time-series regression technique proposed and used by MR, based on equation (6), is subject to the same bias. When there are small changes

[8] A minor econometric concern is raised by the possibility that allowing normal random variability in p and Q might produce measures of EBIT which are not normally distributed. However, a Chi-square goodness-of-fit test, conducted at the 95% confidence level, gave no indication that the series of EBIT estimates should not be considered normal.

Table 1

Point estimates of DOL and DFL for randomly generated Q and p

(v = \$4.00, F = \$400.00, I = \$200.00, $\bar{p}$ = \$8.00, σ_p = \$1.00, $\overline{Q}$ = 300, σ_Q = 50)

OBS.	DOL	DFL	OBS.	DOL	DFL
1	1.387	1.240	26	1.465	1.303
2	1.361	1.221	27	1.756	1.607
3	1.499	1.333	28	1.428	1.272
4	1.553	1.382	29	1.499	1.333
5	1.772	1.629	30	1.395	1.246
6	1.839	1.722	31	1.851	1.741
7	1.659	1.491	32	1.359	1.219
8	2.004	2.007	33	2.288	2.809
9	1.374	1.230	34	1.425	1.270
10	1.816	1.689	35	1.389	1.242
11	1.598	1.427	36	1.315	1.187
12	1.266	1.153	37	2.257	2.690
13	1.409	1.257	38	1.793	1.657
14	1.672	1.506	39	1.712	1.552
15	1.564	1.393	40	1.415	1.262
16	1.333	1.200	41	1.363	1.222
17	1.705	1.545	42	1.543	1.373
18	2.382	3.235	43	1.655	1.487
19	1.461	1.300	44	1.394	1.245
20	2.245	2.649	45	1.721	1.563
21	1.525	1.356	46	1.302	1.178
22	1.528	1.359	47	1.547	1.377
23	2.243	2.641	48	3.355	-5.627
24	1.329	1.197	49	1.485	1.320
25	1.361	1.220			

in unit output and relatively large changes in price over time, there is a tendency to produce estimates of DOL < 1.

Lord (1995) also found that the tendency to produce estimates of DOL less than one is magnified, the further above BE the firm operates. To demonstrate that this same bias exists for the MR methodology, another set of four estimates of DOL_{MR} is calculated using an initial output level even further above the BE. Assume v = \$4.00, F = \$400, and I = \$200. Unit price, $\tilde{p}$, will, again, vary normally about a mean level of \$8.00. For these examples, the average level of $\overline{Q}$ will be set to 3000 units (at this level, the point estimate of DOL_2 = 1.034). The levels of variability will be left the same as in the previous example: with a high level of variability in price of σ_p = \$1.00, and a lower level of σ_p = \$0.10; and σ_Q = 50 for the high level of output variability, and σ_Q = 5 for the lower level.

Results for the four series of the estimates of DOL_{MR} at initial levels of unit output near 3000 units are shown in Table 3. The estimates of DOL_{MR} generated by low volatility in both Q and p are reasonably well-behaved, as only one result

Table 2

Levels of DOL_{MR} with changes in $\tilde{Q}$ and $\tilde{p}$

	(v and F constant and $\bar{Q}$ = 300 units)			
Observations Employed in Time-series	DOL_{MR} (σ_Q = 5.00 σ_p = \$0.10)	DOL_{MR} (σ_Q = 5.00 σ_p = \$1.00)	DOL_{MR} (σ_Q = 50.00 σ_p = \$0.10)	DOL_{MR} (σ_Q = 50.00 σs_p = \$1.00)
1-20	1.608	4.551	1.581	1.439
2-21	2.000	7.366	1.565	1.301
3-22	1.908	4.061	1.612	1.293
4-23	1.834	5.642	1.617	1.104
5-24	1.642	0.684	1.623	1.301
6-25	1.630	0.031	1.625	1.467
7-26	1.585	-0.733	1.611	1.454
8-27	1.746	0.295	1.622	1.513
9-28	1.840	0.301	1.644	1.422
10-29	1.839	0.602	1.637	1.291
11-30	1.720	-2.723	1.618	1.364
12-31	1.604	-2.474	1.588	1.445
13-32	1.583	-3.295	1.591	1.360
14-33	1.433	-3.499	1.602	1.478
15-34	1.572	-2.037	1.540	1.692
16-35	1.667	-0.608	1.545	1.798
17-36	1.735	-1.291	1.545	1.802
18-37	1.788	-1.942	1.547	1.940
19-38	1.775	-4.276	1.339	1.477
20-39	1.660	-4.330	1.586	1.275
21-40	1.660	-4.033	1.583	1.187
22-41	1.722	-3.047	1.597	1.329
23-42	1.738	-2.973	1.602	1.333
24-43	1.743	-3.019	1.600	1.473
25-44	1.754	-2.213	1.602	1.445
26-45	1.632	-2.741	1.606	1.428
27-46	1.628	-1.038	1.613	1.506
28-47	1.647	-0.772	1.607	1.463
29-48	1.657	0.232	1.613	1.772
30-49	1.775	-1.756	1.623	1.685

is less than one. The estimates are, however, much higher than the comparative point estimate of DOL = 1.034. In the series with high volatility in unit output and low volatility of unit price, over half of the estimates of DOL_{MR} are less than one (though none are negative). In the two series with high volatility in unit price, the prevalence of measures of DOL_{MR} less than one, and even less than zero, is obvious. As usual, the levels of DOL_{MR} are especially extreme, when the volatility of is $\tilde{Q}$ low and the volatility of $\tilde{p}$ is high. This evidence suggests that estimates of DOL_{MR} have the same tendency to produce measures of leverage less than one that was observed earlier in point-to-point elasticity estimates of DOL. This bias also is

Table 3

Levels of DOL_{MR} with changes in $\tilde{Q}$ and $\tilde{p}$

	(v and F constant and $\bar{Q}$ = 3000 units)			
Observations Employed in Time-series	DOL_{MR} (σ_Q = 5.00 σ_p = \$0.10)	DOL_{MR} (σ_Q = 5.00 σ_p = \$1.00)	DOL_{MR} (σ_Q = 50.00 σ_p = \$0.10)	DOL_{MR} (σ_Q = 50.00 σ_p = \$1.00)
1-20	1.739	18.891	1.263	0.036
2-21	2.900	35.260	1.196	-0.901
3-22	3.750	13.743	1.130	-1.144
4-23	2.512	22.768	1.175	-2.279
5-24	1.959	- 8.335	1.203	-0.940
6-25	1.875	-12.797	1.256	0.268
7-26	1.546	-17.915	1.171	0.133
8-27	2.675	-10.609	1.271	0.608
9-28	3.326	-11.087	1.415	-0.014
10-29	3.318	- 9.131	1.390	-0.837
11-30	2.509	-28.828	1.290	-0.403
12-31	2.849	-26.774	1.095	0.105
13-32	1.551	-32.248	1.108	-0.590
14-33	0.526	-33.050	0.928	-0.151
15-34	1.495	-23.042	0.678	0.959
16-35	2.151	-12.827	0.687	1.719
17-36	2.630	-17.545	0.685	1.742
18-37	3.003	-21.416	0.691	2.538
19-38	2.911	-35.330	0.716	-0.891
20-39	2.097	-34.678	0.801	-1.889
21-40	2.093	-32.383	0.750	-2.521
22-41	2.539	-24.899	0.801	-1.385
23-42	2.655	-26.058	0.806	-1.400
24-43	2.690	-26.274	0.813	-0.464
25-44	2.760	-20.125	0.820	-0.714
26-45	1.915	-24.060	0.837	-0.859
27-46	1.890	-12.009	0.876	-0.278
28-47	2.019	-10.111	0.795	-0.672
29-48	2.091	-3.426	0.855	1.060
30-49	2.891	-17.240	0.931	0.518

magnified, the higher above the BE the firm is operating. This result explains the prevalence of measures of DOL_{MR} less than one reported in the earlier empirical study by Dugan and Shriver (1992).

To demonstrate that the same biases exist for estimates of DFL, consider cases where unit output and the level of interest payments are allowed to vary simultaneously.[9] Assume that the parameters p = \$8.00, v = \$4.00 and F = \$400.00

[9] When the operating parameters other than annual interest payments (p, v, and F) are allowed to fluctuate, estimates of DFL_{MR} do not differ from ex ante theoretical estimates of DFL.

Table 4

Levels of DFL_{MR} with changes in $\tilde{Q}$ and $\tilde{I}$

	(p, v and F constant and $\bar{Q}$ = **3000 units**)			
Observations Employed in Time-series	DFL_{MR} (σ_Q = 5.00) (σ_I = \$2.00)	DFL_{MR} (σ_Q = 5.00) (σ_I = \$20.00)	DFL_{MR} (σ_Q = 50.00) (σ_I = \$2.00)	DFL_{MR} (σ_Q = 50.00) (σ_I = \$20.00)
1-20	1.069	0.828	1.019	1.096
2-21	1.054	0.330	1.020	1.066
3-22	1.076	0.453	1.029	0.958
4-23	1.071	0.502	1.029	0.953
5-24	1.070	0.298	1.028	0.971
6-25	1.076	0.262	1.029	0.985
7-26	1.078	0.299	1.028	0.996
8-27	1.085	0.341	1.028	0.992
9-28	1.071	0.429	1.025	1.000
10-29	1.072	0.407	1.024	1.006
11-30	1.018	-0.263	1.026	1.045
12-31	1.016	-0.043	1.026	1.085
13-32	1.003	-0.047	1.025	1.077
14-33	0.994	0.236	1.021	1.111
15-34	1.001	0.269	1.020	1.127
16-35	0.981	0.005	1.019	1.127
17-36	0.979	-0.090	1.019	1.120
18-37	0.980	-0.003	1.019	1.093
19-38	0.986	0.067	1.019	1.075
20-39	1.002	0.085	1.020	1.051
21-40	1.008	-0.033	1.018	1.063
22-41	1.043	0.158	1.017	1.098
23-42	1.030	0.282	1.016	1.106
24-43	1.033	0.269	1.016	1.109
25-44	1.028	0.457	1.016	1.099
26-45	0.999	0.374	1.017	1.098
27-46	1.020	0.545	1.018	1.097
28-47	1.018	0.537	1.017	1.105
29-48	1.036	0.389	1.020	1.084
30-49	1.029	0.592	1.020	1.094

are fixed. The level of interest payments, $\tilde{I}$, will vary normally about a mean of \$200.00, with a low volatility of σ_I = \$4.00 and a high volatility of σ_I = \$40.00. From previous experience, the interesting case is where $\tilde{Q}$ is far above BET. Therefore, allow unit output to vary normally about a mean level of 3000 units, with a high volatility level of σ_Q = 50 units and σ_Q = 5 for the low level of volatility. At these initial conditions, the point estimate of DFL = 1.018. The results for these four series estimated from equation (7) are presented in Table 4.

Note that the results for DFL_{MR} in Table 4 are very similar to those for DOL_{MR} in Table 2. The results in the third column, associated with high volatility in unit output and low volatility in interest payments, are reasonably well-behaved, and all estimates are close to the expected level of DFL = 1.018. In the two columns associated with low volatility for both variables or with high volatility in both cases, several estimates of DFL_{MR} less than one appear. In the second column, with low variability in unit output and high volatility in the level of interest payments, all the estimates of DFL_{MR} are less than one, and many are negative. Notice that the estimates of DFL_{MR} in Table 4 are much closer to their theoretical levels than measures of DOL_{MR} shown in Table 3.[10]

In this section, it has been demonstrated that time-series regression estimates of DOL_{MR} and DFL_{MR} suffer from the same biases as the point-to-point estimates of DOL and DFL. The frequent occurrence of measures of the degree of leverage less than one (despite the fact that the firm is operating above the BE or BET) is shown to be attributable to fluctuations in the firm's operating parameters. As was the case above, these problems appear to be more serious when small changes in unit output are accompanied by large changes in the operating parameters. The biases also become more serious the higher above the BE or BET the firm is operating.

4. O'Brien and Vanderheiden estimates of leverage

O'Brien and Vanderheiden (1987) (hereafter referred to as OV) noted a construct validity problem in the time-series regression method of MR for estimating DOL_{MR}. They found that in a situation where unit sales are growing at the same rate as EBIT on average, the time-series method tends to produce estimates of DOL_{MR} very near one. In their study, they demonstrated that adjusting the time-series methodology to account for the growth rate in sales and EBIT corrected this problem, and tended to produce values of DOL very near the point estimate. To make this adjustment, they employed a two-stage regression technique embodied in equations (8), (9) and (10) above to estimate DOL_{OV}.

Like most previous work on the degree of leverage measures, OV's result is based on the assumption that the operating parameters of the firm are constant. Therefore, the purpose of this section is to show that the two-stage method of OV in calculating DOL_{OV} suffers from the same biases as the other methods to calculate DOL, when the firm's operating parameters are stochastic rather than constant.

Again, results will be estimated for a hypothetical firm for which all parameters are observable. The initial levels will be set to $\bar{p}$ = \$8.00, v = \$4.00, F_0 = \$400.00, I_0 = \$200.00, and $\bar{Q}$ = 300 units. At these levels DOL = 1.50. It will then be assumed that unit output increases by an average of 1% per period for 49 periods, and that it will vary stochastically about this average by a normal process with a standard deviation of 5 units (the low volatility scenario). Similar to the OV study, assume

[10] Under most conditions, changes in DFL caused by changes in I should be less extreme than changes in DOL caused by changes in p because $dDOL/dp = -FQ/[((p-v)Q-F)^2] \gg dDFL/dI = [(p-v)Q-F)]/[((p-v)Q-F-I)^2]$.

that the fixed costs, F and I, increase at the same 1% as the level of demand. As was the case above, estimates of DOL_{OV} will be made employing a rolling 20 year time-series based on equations (8), (9) and (10).[11]

Results for DOL_{MR} and DOL_{OV} are shown in Table 5. The first two columns replicate the results of the OV study. Here the operating parameters p and v are held constant throughout, and F and I increase at 1% per period. Note estimates of DOL_{MR}, in the first column, are very near one, while those of DOL_{OV} in the second column are very close to the theoretical level of DOL = 1.50. This result is identical to that presented by OV.

In the next case, one of the operating parameters is allowed to fluctuate randomly. In this case, unit price will vary normally about its mean level of $8.00 with a standard deviation of $1.00. (Earlier tests have suggested that these cases with reasonably small variability in unit output and large variability in the operating parameter tend to produce the most dramatic results.)

The results for the MR time-series estimates appear in the third column of Table 5. Here the usual volatility in the estimates of DOL_{MR} is seen, with many results less than one, and also several negative observations (despite the fact that the firm is operating above the BE). Notice in the case where $\tilde{p}$ is variable that estimates of DOL_{OV}, shown in the fourth column, are not well-behaved, as was the case in the second column. In fact, the results seem to fluctuate much more wildly than those for DOL_{MR}. This result is consistent with the results of Dugan and Shriver (1992) where the standard deviation of a series of estimates for DOL_{OV} was much higher than those for DOL_{MR} (by a factor of ten). This situation would suggest that while the methodology of OV corrects for the bias associated with growth, it is even more susceptible to the distortions caused by fluctuating operating parameters than the method proposed by MR. As was the case in the other methodologies to calculate DOL, this bias is magnified, the higher above the BE the firm is operating.

This section has demonstrated that if the firm's operating parameters fluctuate, DOL_{OV} suffers from the same shortcomings as all other proxies of the degree of leverage measures.[12] In fact, they seem to produce much more volatile series than the MR measures (which OV had noted tend to cluster very tightly near one). As is usually the case, this volatility increases the higher above the break-even point the firm is operating.

5. Summary and conclusions

In recent years, time-series regression models, as suggested by Mandelker and Rhee (1984), have become increasingly popular to estimate the degree of leverage

[11] Again, a Chi-square goodness-of-fit test confirmed that it is reasonable to assume that the estimates of EBIT throughout were normally distributed.

[12] Clearly estimates of DFL that controlled for growth, as outlined in Note #4 above, would demonstrate similar behavior when interest payments increased in a variable manner.

Table 5

Levels of DOL_{MR} and DOL_{OV} with growing demand

Observations Employed in Time-series	DOL_{MR} (σ_Q = 5.00 σ_p = \$0.00)	DOL_{OV} (σ_Q = 5.00 σ_p = \$0.00)	DOL_{MR} (σ_Q = 5.00 σ_p = \$1.00)	DOL_{OV} (σ_Q = 5.00 σ_p = \$1.00)
1-20	1.067	1.500	1.152	5.765
2-21	1.048	1.497	1.186	9.605
3-22	1.025	1.498	1.217	4.967
4-23	1.023	1.497	0.290	6.804
5-24	1.017	1.499	1.170	1.192
6-25	1.011	1.499	0.949	0.271
7-26	1.006	1.499	0.701	-0.882
8-27	1.015	1.500	0.324	0.512
9-28	1.020	1.500	0.065	0.571
10-29	1.020	1.500	0.936	0.943
11-30	0.972	1.500	-0.232	-6.956
12-31	0.965	1.500	-0.226	-7.153
13-32	0.960	1.500	1.278	-4.982
14-33	0.953	1.500	1.895	-3.913
15-34	0.973	1.500	3.190	0.401
16-35	0.994	1.501	3.230	1.128
17-36	1.000	1.502	4.452	0.772
18-37	1.013	1.502	3.662	-2.502
19-38	1.025	1.501	1.363	-6.298
20-39	1.037	1.502	1.907	-8.422
21-40	1.033	1.502	1.428	-6.804
22-41	1.034	1.503	1.145	-5.148
23-42	1.037	1.502	1.141	-5.205
24-43	1.035	1.502	-0.427	-3.508
25-44	1.015	1.501	0.191	-3.416
26-45	1.023	1.501	0.060	-3.789
27-46	1.031	1.501	1.072	-2.221
28-47	1.028	1.500	0.625	-1.459
29-48	1.033	1.500	-0.523	1.363
30-49	1.028	1.501	0.262	-2.720

measures. However, Dugan and Shriver (1992) found that estimates of degree of operating leverage made with Mandelker and Rhee's methodology have a strong tendency to produce estimates with values less than one. According to ex ante theory, such results are highly unlikely. Recently, Lord (1995) has shown that point-to-point elasticity estimates of the degree of leverage measures have a strong tendency to take on such counter-intuitive values when the firm's operating parameters are allowed to fluctuate. He found this tendency was especially strong when small changes in demand were coupled with relatively large changes in one of the operating parameters. Interestingly, the tendency to produce these distortions grows more dramatic the higher above the break-even point the firm is operating. The first goal of this study is to show that this same phenomenon could explain the prevalence of illogical values for DOL and DFL produced by the time-series regression methodology of Mandelker and Rhee.

O'Brien and Vanderheiden (1987) suggested an alternative two-stage time-series regression methodology to estimate DOL, which controlled for growth-related biases in the Mandelker and Rhee technique. This study shows that the two-stage method of O'Brien and Vanderheiden controls for this particular problem. However, it also is shown that the O'Brien and Vanderheiden methodology is subject to the usual distortions when the firm's operating parameters are allowed to fluctuate. In fact, considerable evidence exists that their estimates are much more volatile than for any other calculational technique.

The results of this study explain the puzzling prevalence of degree of leverage measures less than one calculated in actual empirical studies. They also should make clear the potential hazards involved in employing time-series regression estimates of degree of leverage measures as simple linear or curvilinear proxies for the underlying leverage constructs.

The implications of these results for future research are obvious. First, along the lines of O'Brien and Vanderheiden, there may be value in fine-tuning the existing equations to correct these biases. However, at a deeper level, there is a need to consider anew whether or not the degree of leverage measures actually represent what they are intended to proxy.

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