

Real Options

Assignment problems are in "Real Options Exercises" by Timothy Luehrman.). Complete Excel spreadsheet and any other material (e.g., decision tree diagrams, Word or pdf files, etc.)

- **Problem 2, all parts.**
 - You will need to use the NPVq and σ t framework laid out in the Luehrman reading "Investment Opportunities as Real Options: Getting Started on the Numbers.
 - **Part A:** Use the Excel File attached to this assignment for all your calculations. Complete the worksheet "NPV Only" by calculating NPV for Rounds 2 and 3 (Round 1 is already completed for you see Exercises: Question 1a , etc.). Vary the discount rates in the spreadsheet and use the results to support your answer to part A.
 - **Part B: Copy** your completed "NPV Only" worksheet and rename the copy "Option Value R2." Modify this worksheet to reflect the value of rounds 1 and 2 when round 2 is valued as a real option rather than using NPV. Ignore Round 3 (i.e., do not include it in the overall project value).
 - **Part C: Copy** your completed "Option Value R2" worksheet and rename the copy "Option Value R3." Modify this worksheet to reflect the value of the entire project when both round 2 and round 3 are valued as real options.
- **Problem 3, all parts.** This problem can be answered using either the NPVq and σ t framework or the decision tree method explained during class. If you choose to use decision trees, hand drawn and scanned diagrams or computer drawn diagrams are acceptable.

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-\$100

NOW

Exercises: Question 1a

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Exercises: Question 1a

Exercises: Question 1a

$NPV = -100 + 50\% \cdot \frac{160}{1.1} + 50\% \cdot \frac{60}{1.1}$

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Exercises: Question 1a

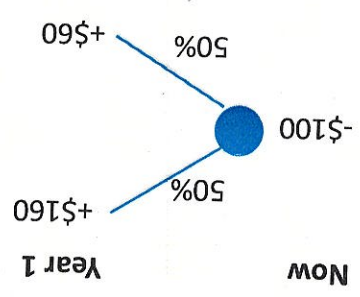
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$$NPV = -100 + 50\% * (160 / (1.1)) + 50\% * (60 / (1.1))$$

$$NPV = -100 + 50\% * (145.5 + 54.5)$$

$$NPV = -100 + 50\% * (200)$$

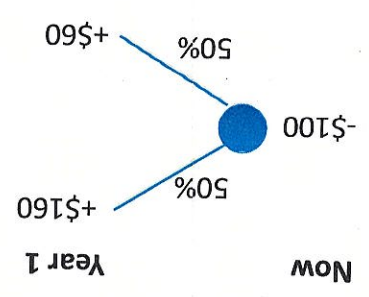
$$NPV = 0$$



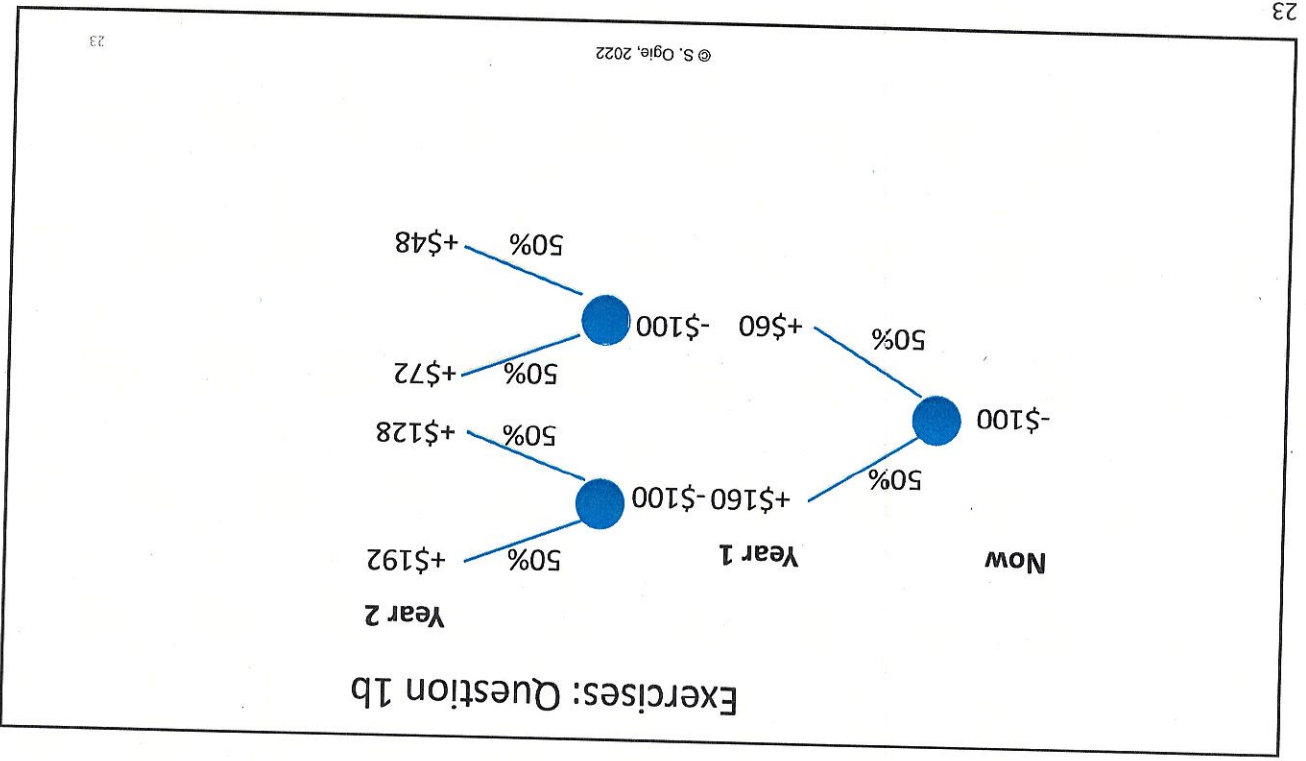
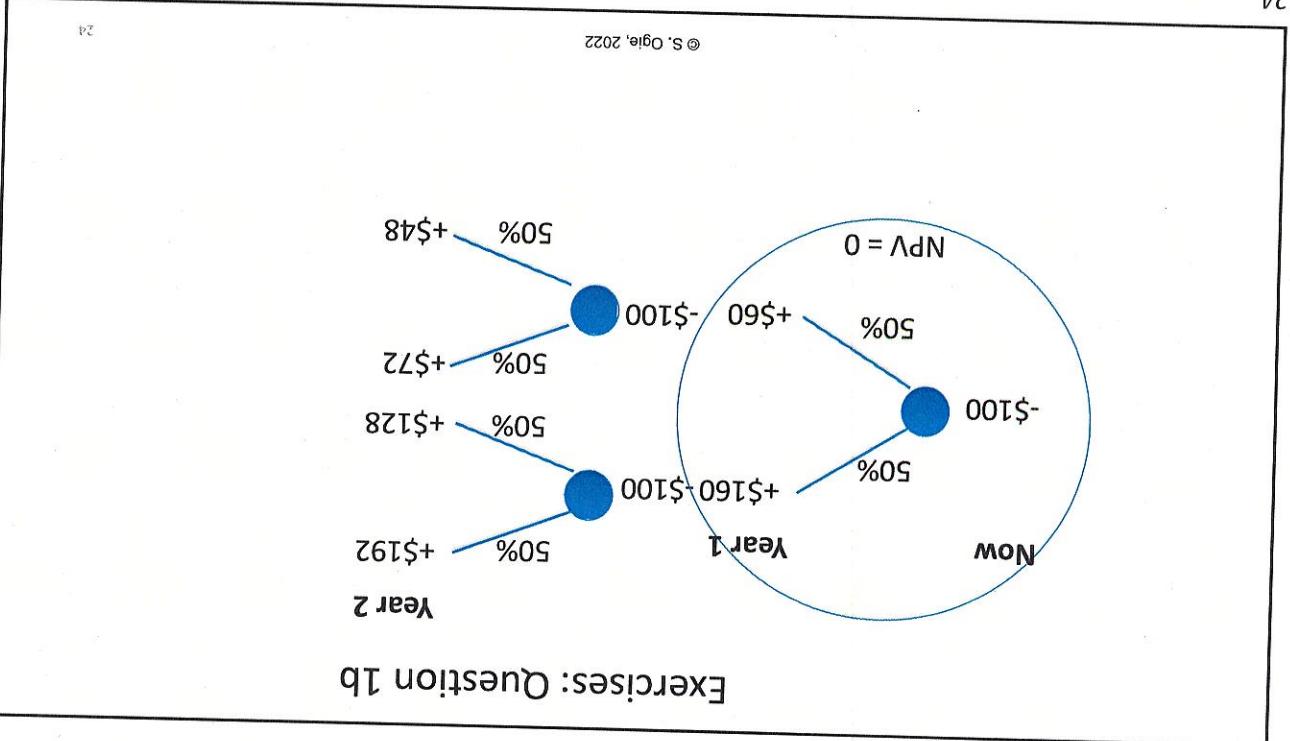
Exercises: Question 1a

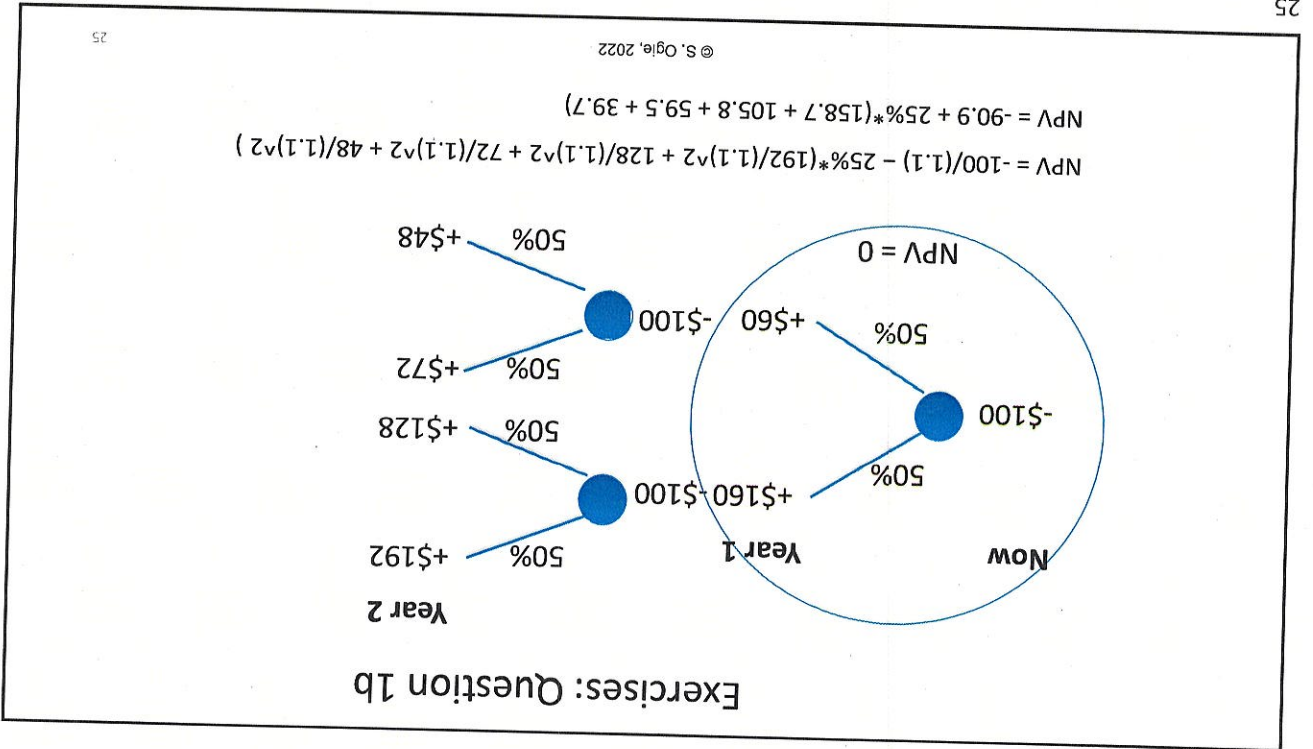
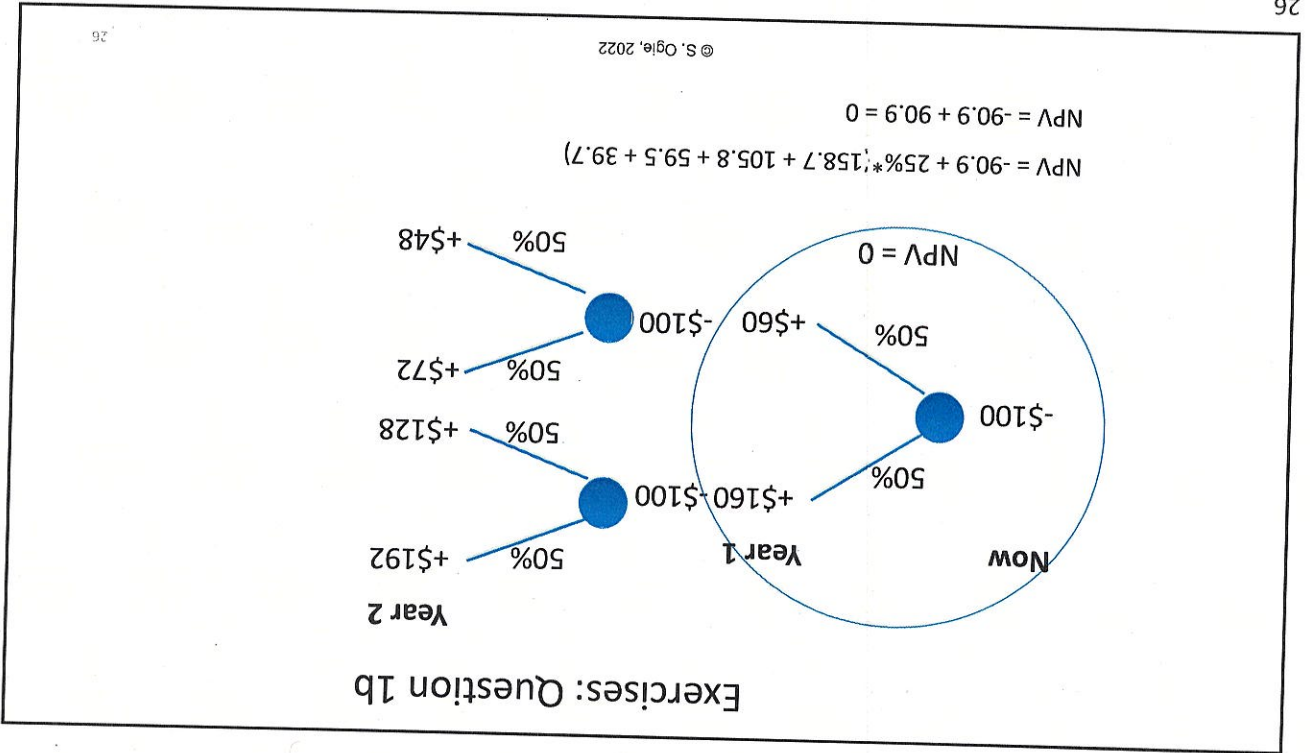
$$NPV = -100 + 50\% * (160 / (1.1)) + 50\% * (60 / (1.1))$$

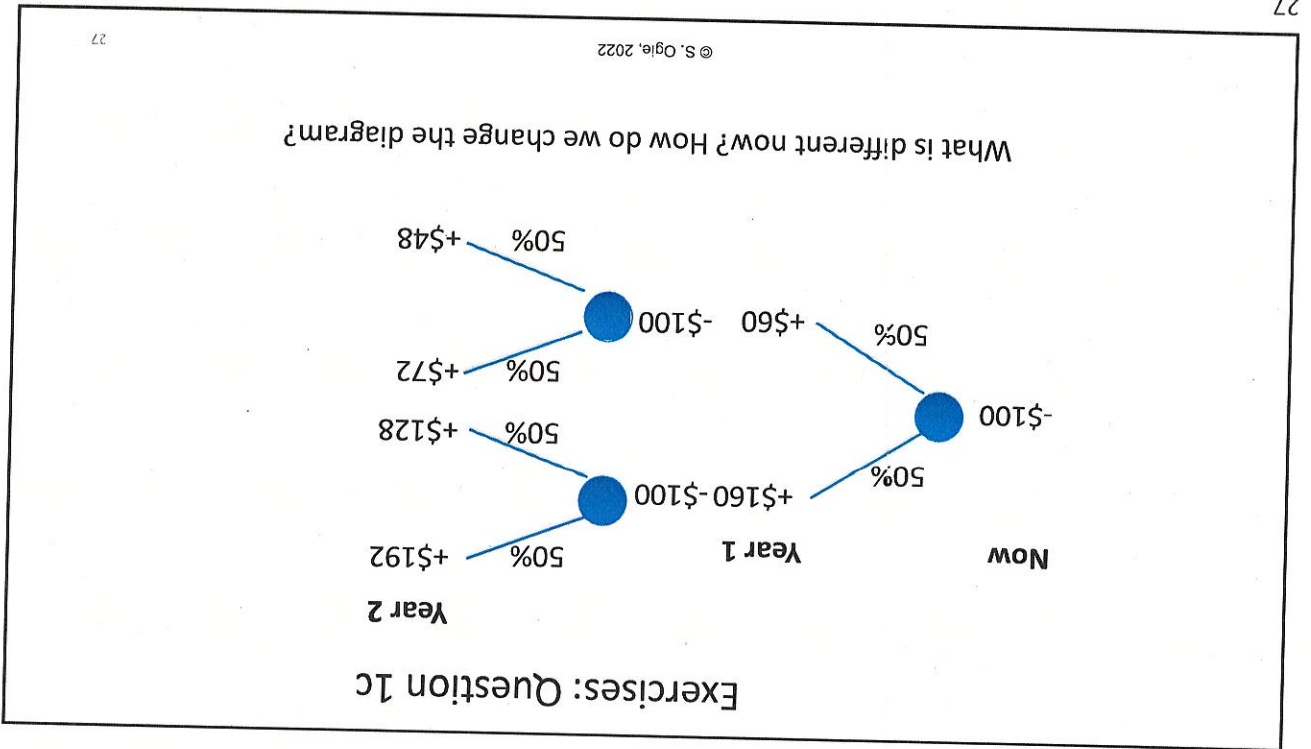
$$NPV = -100 + 50\% * (145.5 + 54.5)$$



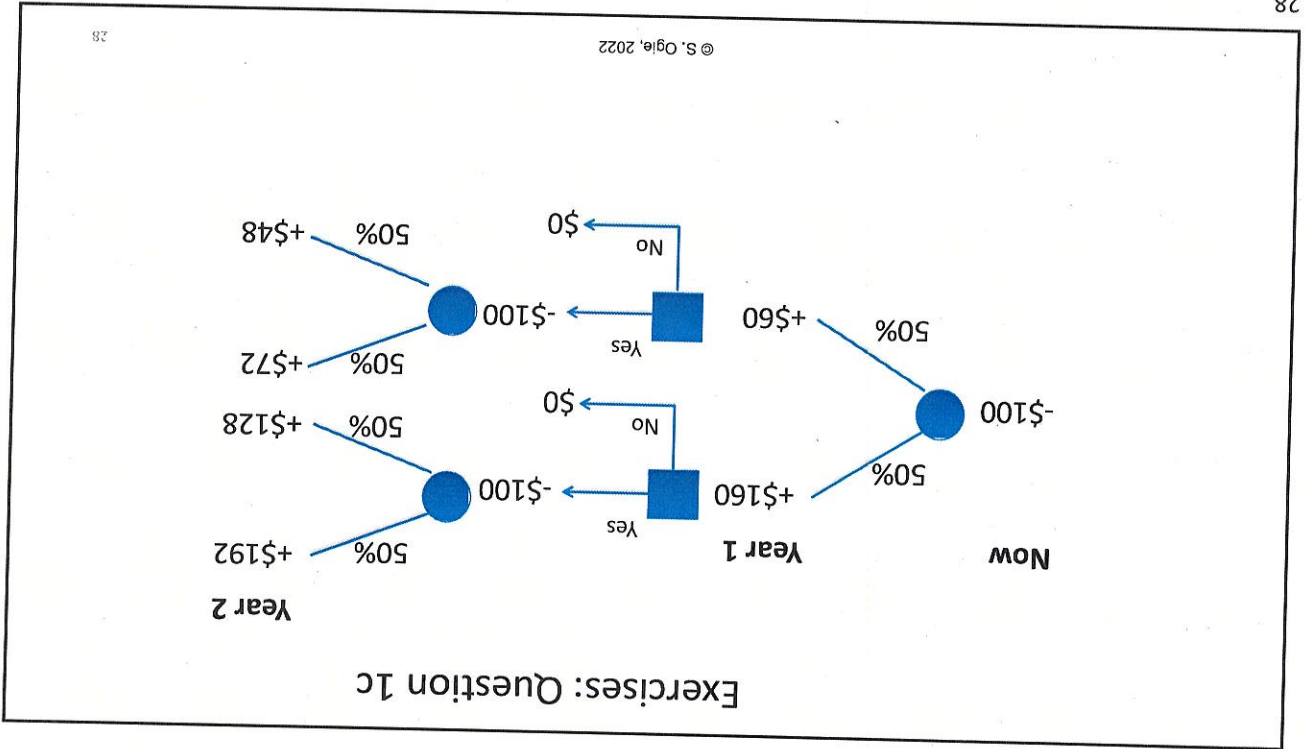
Exercises: Question 1a

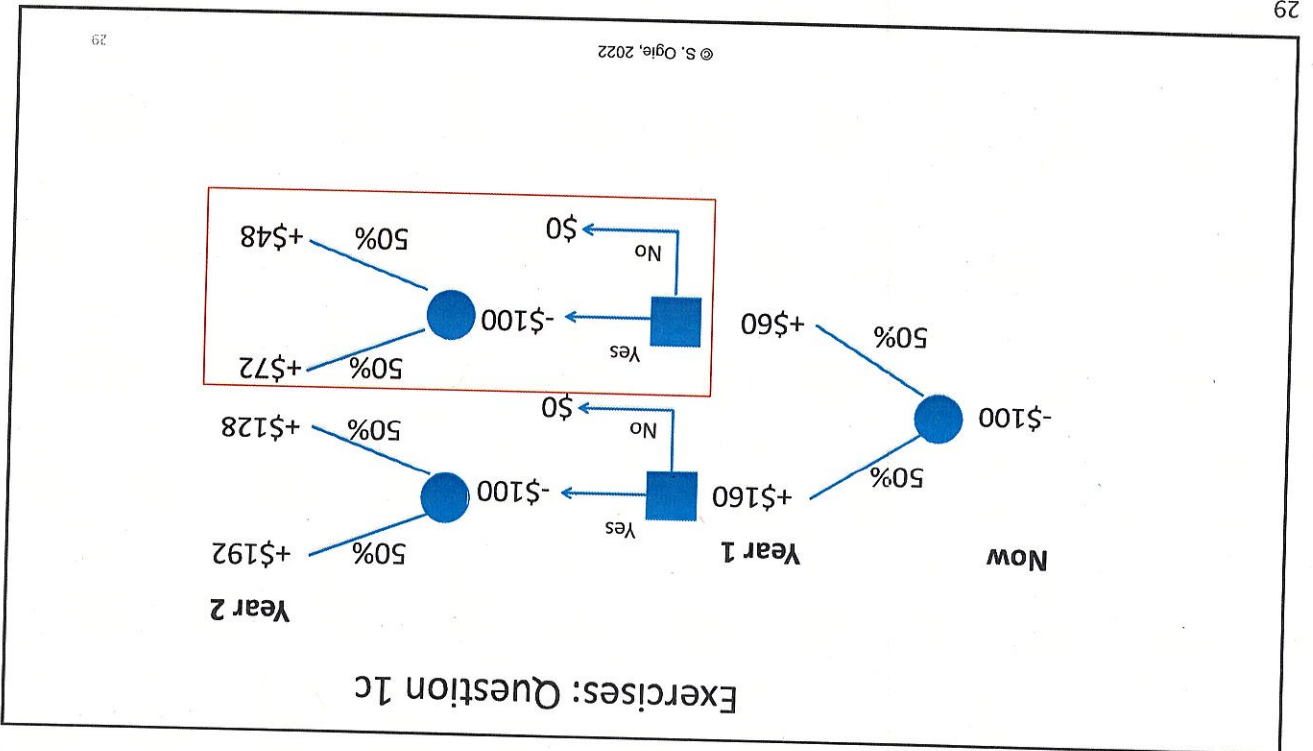
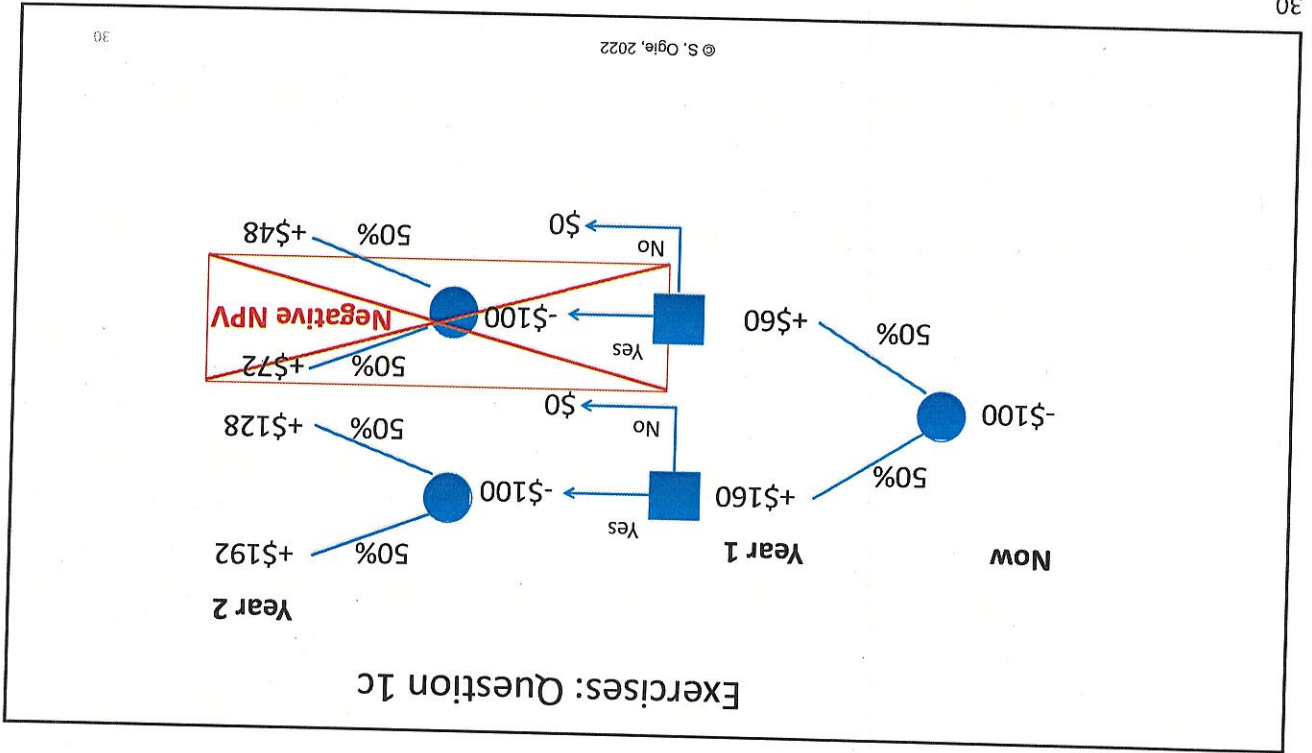


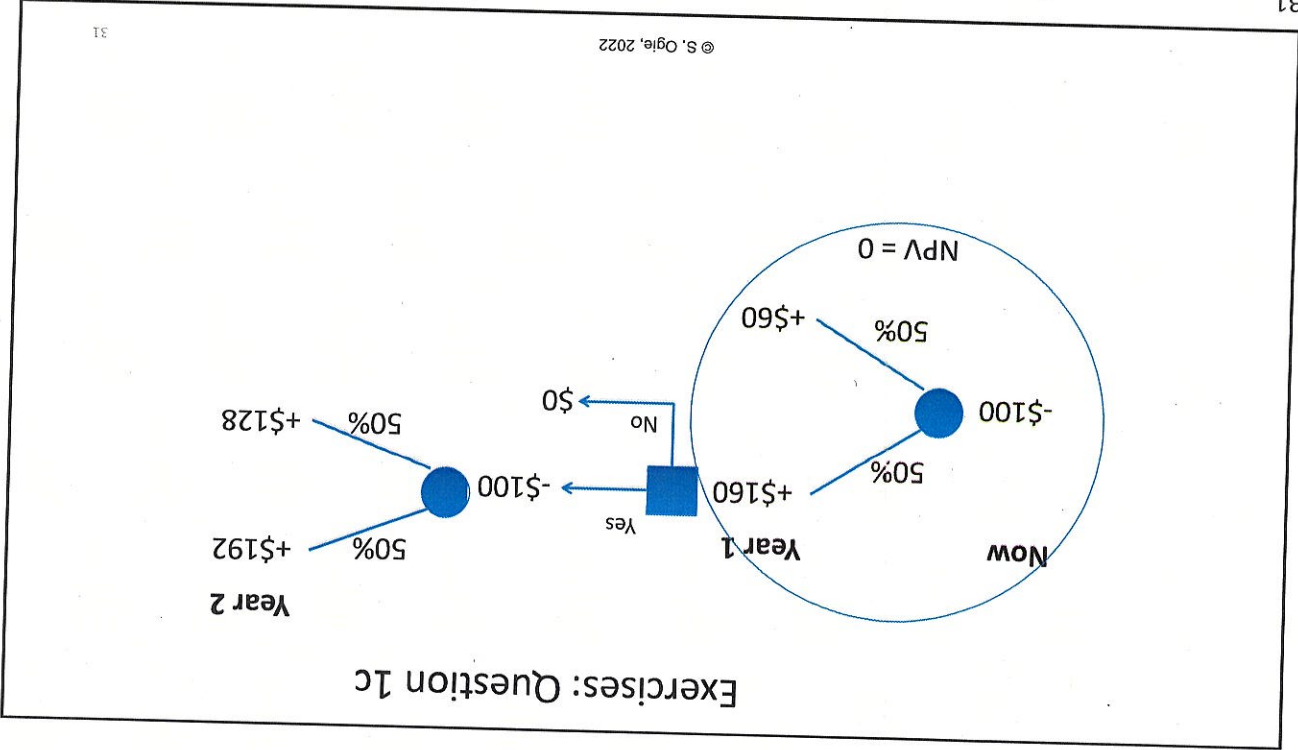
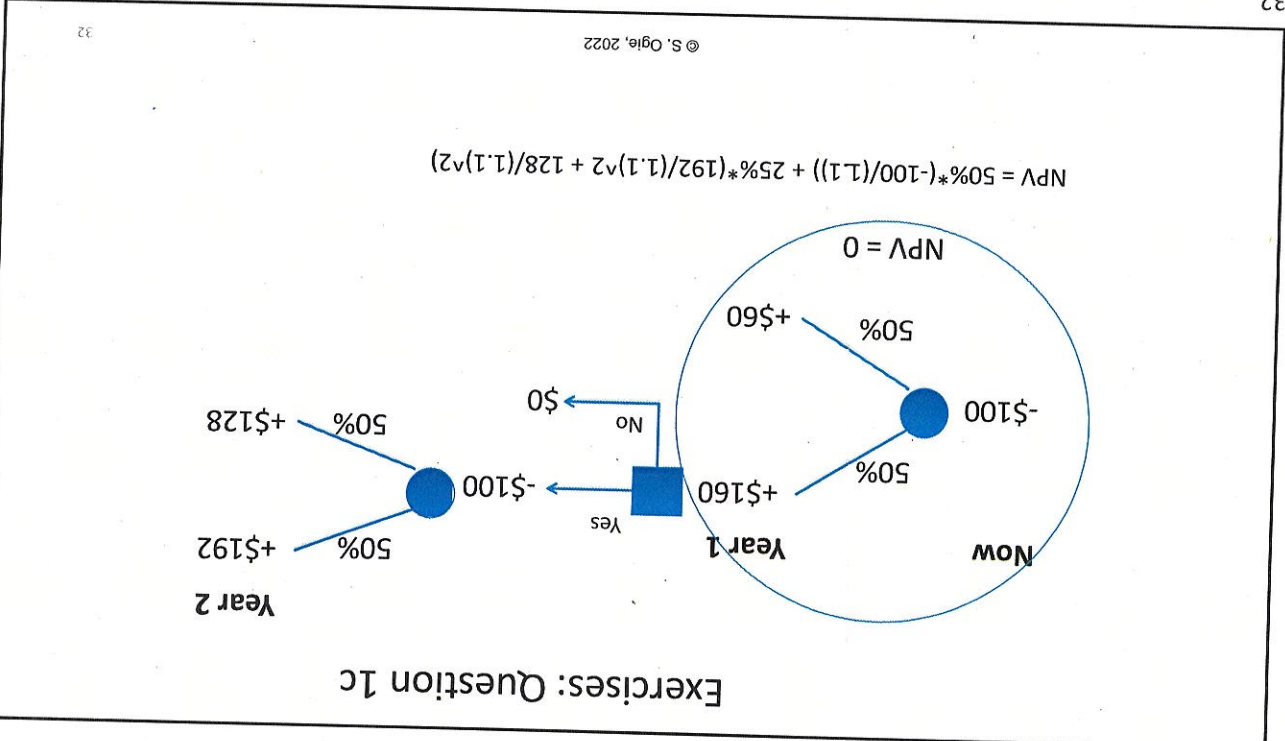


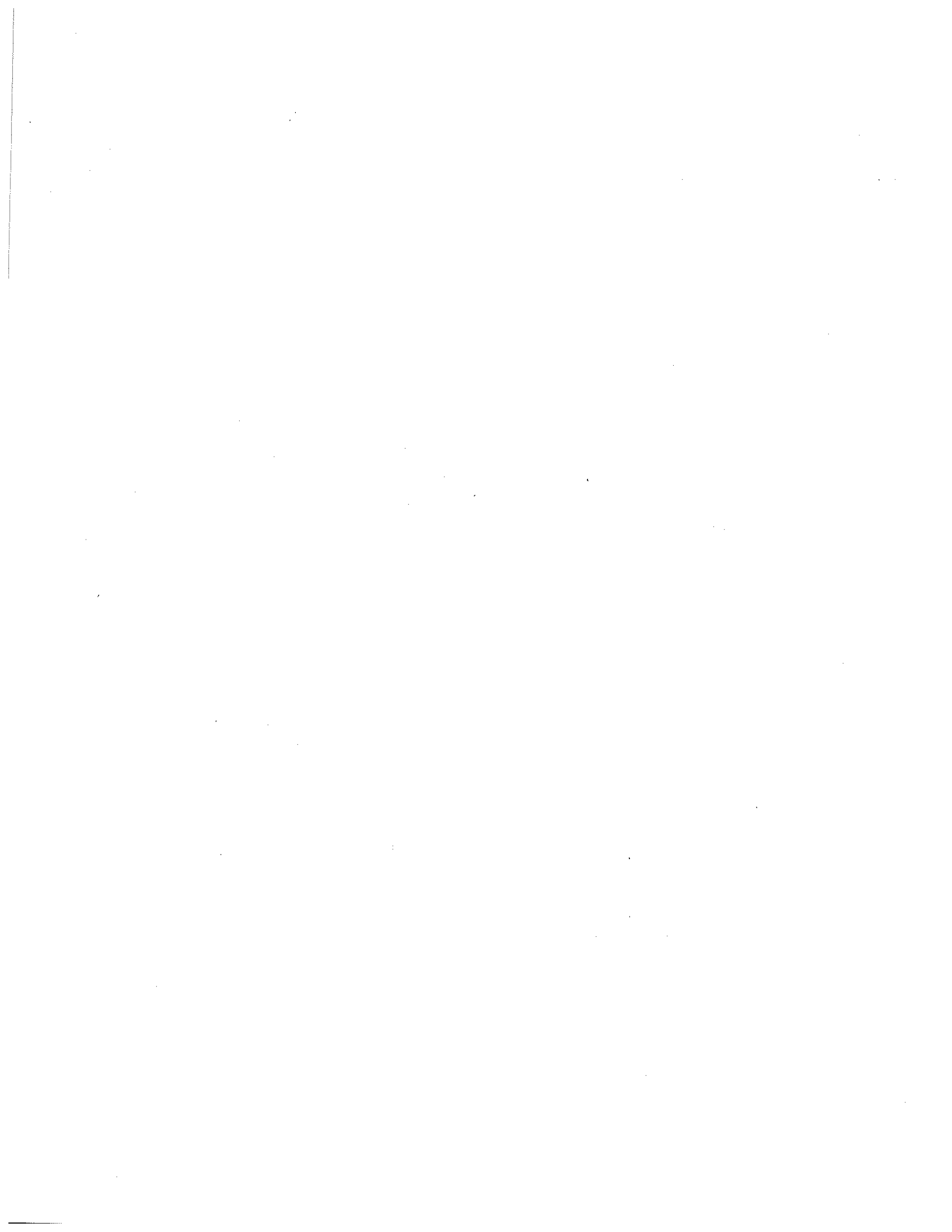


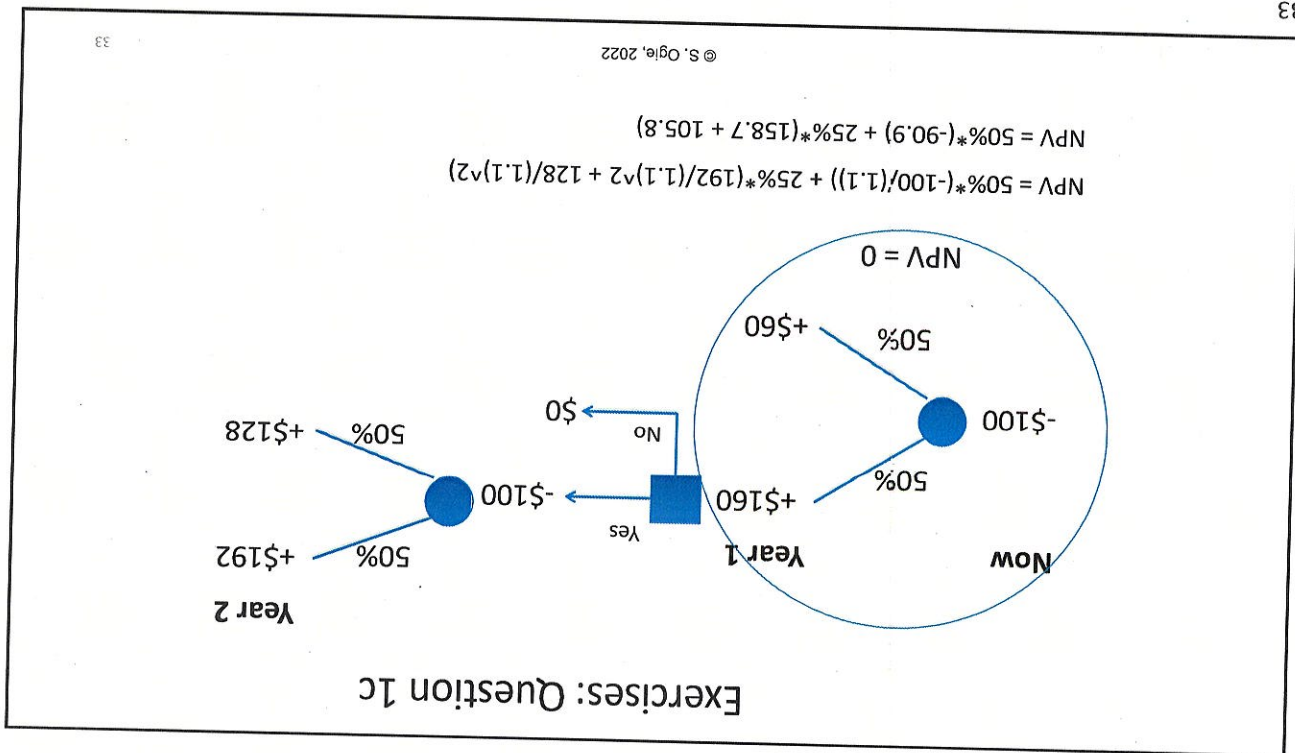
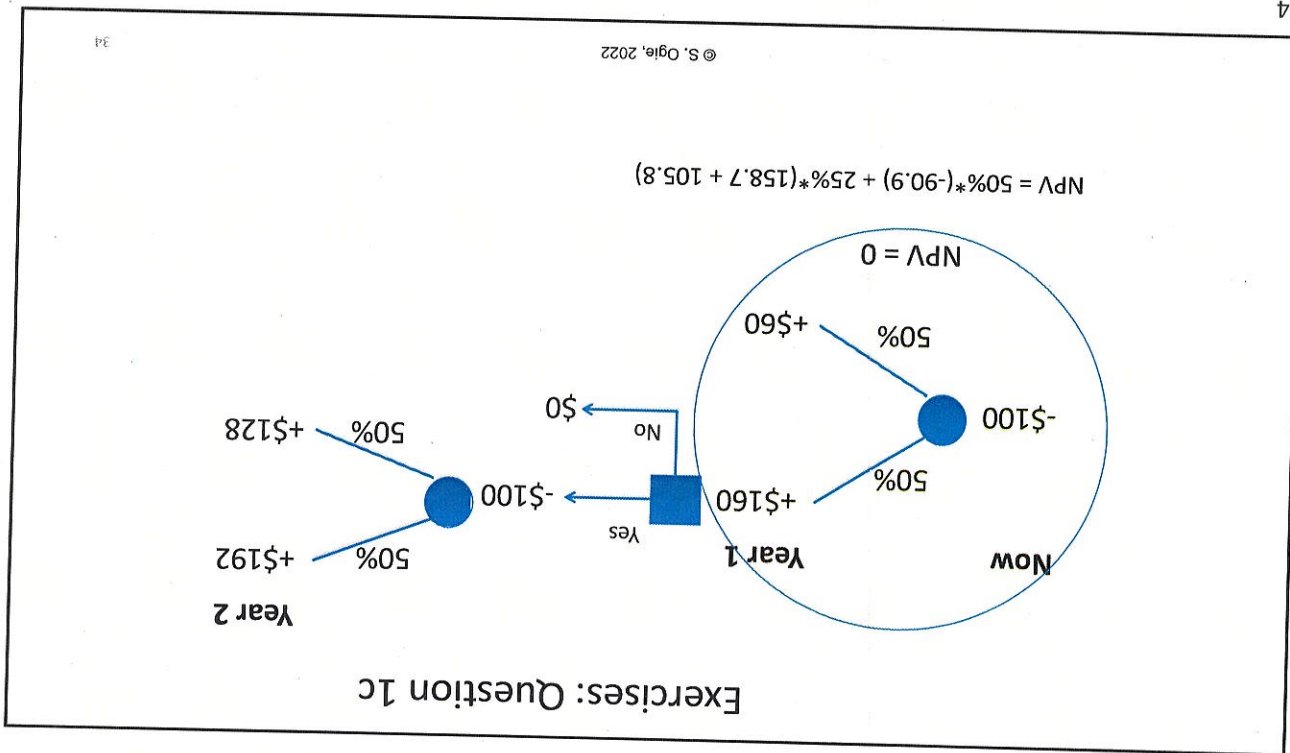
What is different now? How do we change the diagram?

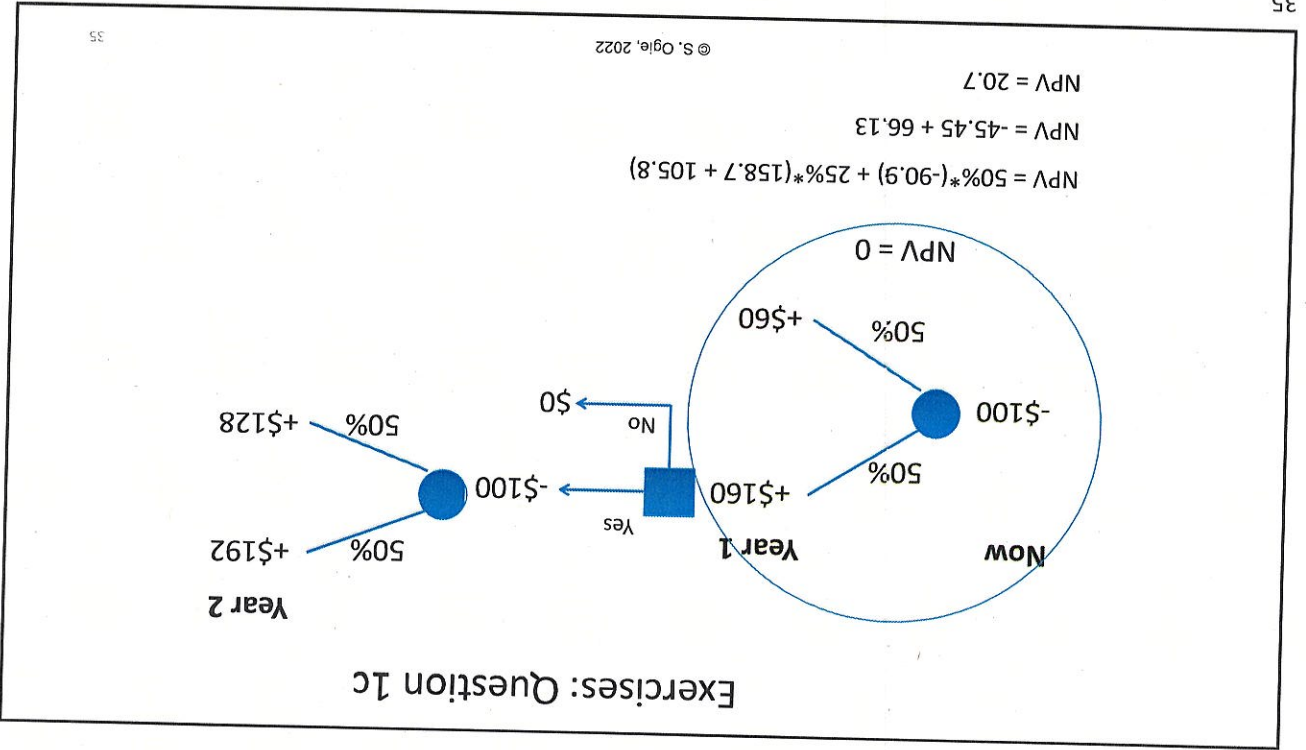












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TIMOTHY A. LUEHRMAN

Real Options Exercises

Don't answer questions in this case

Problem 1

Use simple decision trees and a 10% per year discount rate to evaluate Project Sable, which has two phases. You may invest in the first, in both or in neither. You may not invest in the second phase without investing in the first. Phase 1 requires an investment of \$100. One year later the project delivers either \$160 or \$60 with equal probability. At that time, after the phase 1 payout has been received, you may invest an additional \$100 for phase 2. One year later, phase 2 pays out either 20% more cash than phase 1 actually delivered or (equally likely) 20% less. Assume no taxes.

a. How much would Project Sable be worth if it offered only the phase 1 opportunity?

b. How much would phase 2 be worth if you had to choose today, once and for all, whether or not to invest in it?

c. How much is Project Sable worth if you have access to both phases and can wait to decide whether to invest in phase 2?

d. How would you allocate the value of Project Sable between assets-in-place and real options?

e. Assuming that investment in Project Sable will require external funding, what dollar amounts of debt and equity do you recommend (assume capital markets are informed and efficient, and that the risk-free rate of interest is 5%)?

Problem 2

You are a member of the investment committee for a large manufacturing company that two years ago licensed a new technology from a small start-up firm. Based largely on that technology, engineers in one of your company's divisions developed a new product that is now ready for launch. The division president recommends proceeding with the project and has requested authorization and funding to begin the launch. Your committee has to decide whether to endorse the recommendation and authorize the necessary funding. If your company does not proceed with the launch within the

Professor Timothy A. Luehrman prepared the original version of this exercise, "Corporate Financial Management: Options Exercises," HBS No. 293-095. This version was also prepared by him as the basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative situation.

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year, the licensor has the right to suspend your license and attempt to market the technology to another manufacturer.

The product launch will require a significant investment: \$145 million during the first year for manufacturing capacity, working capital, and marketing and distribution. Net after-tax cash flows are expected to be \$10, \$12, and \$15 million in the following three years of operation, at which point the capacity will be fully utilized. The manufacturing assets are expected to have a very long useful life and necessary maintenance expenditures are included in the after-tax cash flow projections. Your company uses a hurdle rate of 10% for investments in its core business. However, the investment committee considers this product launch to be riskier than the core business and has stipulated that a 15% hurdle rate will apply. The risk-free rate is currently 5%.

The headquarters analyst assigned to examine the project prepared a report showing that, at 15%, the NPV of the proposed investment is negative. When the division president was shown a preliminary copy of the report, she took strong exception to the analysis. In a memo to the CFO, who also chairs the investment committee, she outlined her objections.

First, she argued, the discount rate is inappropriate. The committee's insistence on 15% is myopic and penalizes the project unjustifiably. In her view the project should be treated the same, at least, as other projects, which suggests a hurdle rate of 10%. In fact, given the new product's strategic importance to the division and to the corporation as a whole, an even lower discount rate – she suggested 8% – would be justified.

Second, she argued, the analyst's cash flows are wrong. The new product targets a very large market and is a key part of the division's growth plans. After three years, once the original capacity is fully utilized, she can add twice again as much capacity for less than twice the additional cash expenditure. Specifically, an incremental investment of \$250 million will generate additional cash flows of \$20, \$24, and \$30 million in the subsequent three years, and this capacity, too, would have a very long life. Three years after this second investment, the division can invest another \$450 million and receive incremental net cash flows of \$40, \$48, and \$60 million in the subsequent three years. Thus, the memo noted, the new product's annual net cash flow is expected to grow from \$0 to \$100+ million within ten years, implying a compound annual growth rate of more than 30%. By comparison, the analyst used a growing perpetuity to estimate the terminal value of the project in year 3, and applied a growth rate of only 3%. Accordingly, the analyst is either wrong to adopt the three-year horizon, and/or wrong to apply such a low growth rate.

The division president's memo concluded by observing that although all the analytical errors should be corrected for the record, even if only one was fixed, the project's NPV would be positive. Not only is it a good investment, it is actually a *very good* investment.

In a follow-up memo the analyst defended his original figures. It was perfectly legitimate, he argued, to separate the sequence of investments because each could be analyzed on its own merits at the appropriate time. In fact, only the initial investment of \$145 million was now before the committee. Nevertheless, he claimed to be indifferent about whether he evaluated them separately (see Table A) or added them all together (Table B). Discounting at 15%, none of the three investments had a positive NPV and so neither did the whole program. At 10% the project currently being reviewed looked better, naturally, and the committee was free to accord it the special treatment requested. But the analyst respectfully observed that a new product launch was almost certainly riskier than the established core business. Even if 15% was too high (which he doubted) surely 10% was too low. At any hurdle rate which reflected the higher risk of a product introduction, the project was unimpressive.

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Table A

Round 1										
Year	0	1	2	3	4	5	6	7	8	9
After-tax cash flow	10.0	10.0	12.0	15.0	15.5	15.9	16.4	16.9	17.4	17.9
Investment	-145.0									
Net cash flow	-145.0	10.0	12.0	15.0	15.5	15.9	16.4	16.9	17.4	17.9
Round 2										
Year	0	1	2	3	4	5	6	7	8	9
After-tax cash flow	0	0	0	20.0	24.0	30.0	30.9	31.8	32.8	32.8
Investment	-250.0									
Net cash flow	-250.0	0	0	20.0	24.0	30.0	30.9	31.8	32.8	32.8
Round 3										
Year	0	1	2	3	4	5	6	7	8	9
After-tax cash flow	0	0	0	0	0	0	40.0	48.0	60.0	60.0
Investment	-450.0									
Net cash flow	-450.0	0	0	0	0	0	40.0	48.0	60.0	60.0

Table B

Rounds 1-3

Year	0	1	2	3	4	5	6	7	8	9
After-tax cash flow	0.0	10.0	12.0	15.0	35.5	39.9	46.4	87.8	97.2	110.7
Investment	-145.0									
Net cash flow	-145.0	10.0	12.0	15.0	35.5	39.9	46.4	87.8	97.2	110.7

- Evaluate the proposed launch using a standard discounted cash flow approach. How should the committee address the issue of which discount rate to use? Which phases should be included? What growth rate should be used?
- Evaluate the program as a portfolio of options (you should begin by ignoring, at first, the opportunity to invest \$450 in year 6 and focus only on the first two rounds of investment). You may use the Black-Scholes-Merton model and a calculator or the pricing table in the Appendix. Assume that, though you are unsure of the standard deviation of returns on the assets under consideration, you are confident that it is at least 40% per year and no more than 60%. Based on this approach, what course of action do you recommend to the committee?
- How should the year 6 opportunity be incorporated into an options-based analysis of the whole program?

Done + Answer questions

Problem 3

on this page

A product based on a new technology has two major potential markets. The dominant uncertainty associated with it has to do with the technology rather than the markets. Accordingly, the product will succeed in both or fail in both, with equal probability. The markets are otherwise independent and may be entered sequentially or simultaneously either now, one year from now, or two years from now. Market A requires an initial investment of \$100 regardless of when it is entered. If the product is successful, market A will have a present value of \$160 one year after entry. If the product fails, market A will be worth \$80 one year after entry. Market B requires an initial investment of \$55 regardless of when it is entered. One year after entry, B will have a present value of \$140 or \$25 for success and failure, respectively. For simplicity, perform all discounting in this problem at 5%.

- a. What is the NPV for each market, assuming each is entered immediately?
- b. Examine the possible combinations of time and place for introducing the new product. Can any possibilities be eliminated as suboptimal without further calculations? Why or why not? Which entry strategy is optimal?
- c. State a general capital budgeting rule for selecting the optimal strategy in this and similar problems.
- d. Suppose there are three potential markets, A, B, and C, where A and B are as above and C requires an investment of \$40 regardless of when entered and will be worth either \$55 or \$40 one year later. Test the decision rule you formulated in part (c.) above to check that it produces the optimal decision for this revised problem.

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Appendix

Black-Scholes-Merton Value of a European Call Option, Expressed as a Percentage of Underlying Asset Value

$$NPVq = (\text{Underlying asset value}) / PV(\text{Exercise price})$$

	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.75	2.00	2.50																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
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0.45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	1.0	1.3	1.7	2.2	2.8	3.5	4.2	5.1	6.0	7.0	8.0	9.1	10.2	11.4	12.6	13.8	15.0	16.2	17.4	20.4	23.3	26.0	28.6	31.1	33.3	42.9	50.0	60.0	0.45																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
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0.55	0.5	1.0	1.7	2.6	3.8	5.1	6.6	8.3	10.0	11.9	13.8	14.6	15.4	16.1	16.9	17.7	18.5	19.3	20.1	20.9	21.7	22.4	23.2	24.0	24.8	25.5	26.3	27.0	27.8	28.5	29.2	29.9	30.6	31.3	32.0	32.7	33.3	34.0	34.6	35.3	35.9	37.4	38.9	40.3	41.7	43.0	44.3	45.6	47.0	48.3	49.6	50.9	52.2	53.5	54.8	56.1	57.4	58.7	60.0	61.3	62.6	63.9	65.2	66.5	67.8	69.1	70.4	71.7	73.0	74.3	75.6	76.9	78.2	79.5	80.8	82.1	83.4	84.7	86.0	87.3	88.6	89.9	91.2	92.5	93.8	95.1	96.4	97.7	99.0	100.3	101.6	102.9	104.2	105.5	106.8	108.1	109.4	110.7	112.0	113.3	114.6	115.9	117.2	118.5	119.8	121.1	122.4	123.7	125.0	126.3	127.6	128.9	130.2	131.5	132.8	134.1	135.4	136.7	138.0	139.3	140.6	141.9	143.2	144.5	145.8	147.1	148.4	149.7	151.0	152.3	153.6	154.9	156.2	157.5	158.8	160.1	161.4	162.7	164.0	165.3	166.6	167.9	169.2	170.5	171.8	173.1	174.4	175.7	177.0	178.3	179.6	180.9	182.2	183.5	184.8	186.1	187.4	188.7	190.0	191.3	192.6	193.9	195.2	196.5	197.8	199.1	200.4	201.7	203.0	204.3	205.6	206.9	208.2	209.5	210.8	212.1	213.4	214.7	216.0	217.3	218.6	219.9	221.2	222.5	223.8	225.1	226.4	227.7	229.0	230.3	231.6	232.9	234.2	235.5	236.8	238.1	239.4	240.7	242.0	243.3	244.6	245.9	247.2	248.5	249.8	251.1	252.4	253.7	255.0	256.3	257.6	258.9	260.2	261.5	262.8	264.1	265.4	266.7	268.0	269.3	270.6	271.9	273.2	274.5	275.8	277.1	278.4	279.7	281.0	282.3	283.6	284.9	286.2	287.5	288.8	290.1	291.4	292.7	294.0	295.3	296.6	297.9	299.2	300.5	301.8	303.1	304.4	305.7	307.0	308.3	309.6	310.9	312.2	313.5	314.8	316.1	317.4	318.7	320.0	321.3	322.6	323.9	325.2	326.5	327.8	329.1	330.4	331.7	333.0	334.3	335.6	336.9	338.2	339.5	340.8	342.1	343.4	344.7	346.0	347.3	348.6	349.9	351.2	352.5	353.8	355.1	356.4	357.7	359.0	360.3	361.6	362.9	364.2	365.5	366.8	368.1	369.4	370.7	372.0	373.3	374.6	375.9	377.2	378.5	379.8	381.1	382.4	383.7	385.0	386.3	387.6	388.9	390.2	391.5	392.8	394.1	395.4	396.7	398.0	399.3	400.6	401.9	403.2	404.5	405.8	407.1	408.4	409.7	411.0	412.3	413.6	414.9	416.2	417.5	418.8	420.1	421.4	422.7	424.0	425.3	426.6	427.9	429.2	430.5	431.8	433.1	434.4	435.7	437.0	438.3	439.6	440.9	442.2	443.5	444.8	446.1	447.4	448.7	450.0	451.3	452.6	453.9	455.2	456.5	457.8	459.1	460.4	461.7	463.0	464.3	465.6	466.9	468.2	469.5	470.8	472.1	473.4	474.7	476.0	477.3	478.6	479.9	481.2	482.5	483.8	485.1	486.4	487.7	489.0	490.3	491.6	492.9	494.2	495.5	496.8	498.1	499.4	500.7	502.0	503.3	504.6	505.9	507.2	508.5	509.8	511.1	512.4	513.7	515.0	516.3	517.6	518.9	520.2	521.5	522.8	524.1	525.4	526.7	528.0	529.3	530.6	531.9	533.2	534.5	535.8	537.1	538.4	539.7	541.0	542.3	543.6	544.9	546.2	547.5	548.8	550.1	551.4	552.7	554.0	555.3	556.6	557.9	559.2	560.5	561.8	563.1	564.4	565.7	567.0	568.3	569.6	570.9	572.2	573.5	574.8	576.1	577.4	578.7	580.0	581.3	582.6	583.9	585.2	586.5	587.8	589.1	590.4	591.7	593.0	594.3	595.6	596.9	598.2	599.5	600.8	602.1	603.4	604.7	606.0	607.3	608.6	609.9	611.2	612.5	613.8	615.1	616.4	617.7	619.0	620.3	621.6	622.9	624.2	625.5	626.8	628.1	629.4	630.7	632.0	633.3	634.6	635.9	637.2	638.5	639.8	641.1	642.4	643.7	645.0	646.3	647.6	648.9	650.2	651.5	652.8	654.1	655.4	656.7	658.0	659.3	660.6	661.9	663.2	664.5	665.8	667.1	668.4	669.7	671.0	672.3	673.6	674.9	676.2	677.5	678.8	680.1	681.4	682.7	684.0	685.3	686.6	687.9	689.2	690.5	691.8	693.1	694.4	695.7	697.0	698.3	699.6	700.9	702.2	703.5	704.8	706.1	707.4	708.7	710.0	711.3	712.6	713.9	715.2	716.5	717.8	719.1	720.4	721.7	723.0	724.3	725.6	726.9	728.2	729.5	730.8	732.1	733.4	734.7	736.0	737.3	738.6	739.9	741.2	742.5	743.8	745.1	746.4	747.7	749.0	750.3	751.6	752.9	754.2	755.5	756.8	758.1	759.4	760.7	762.0	763.3	764.6	765.9	767.2	768.5	769.8	771.1	772.4	773.7	775.0	776.3	777.6	778.9	780.2	781.5	782.8	784.1	785.4	786.7	788.0	789.3	790.6	791.9	793.2	794.5	795.8	797.1	798.4	799.7	801.0	802.3	803.6	804.9	806.2	807.5	808.8	810.1	811.4	812.7	814.0	815.3	816.6	817.9	819.2	820.5	821.8	823.1	824.4	825.7	827.0	828.3	829.6	830.9	832.2	833.5	834.8	836.1	837.4	838.7	840.0	841.3	842.6	843.9	845.2	846.5	847.8	849.1	850.4	851.7	853.0	854.3	855.6	856.9	858.2	859.5	860.8	862.1	863.4	864.7	866.0	867.3	868.6	869.9	871.2	872.5	873.8	875.1	876.4	877.7	879.0	880.3	881.6	882.9	884.2	885.5	886.8	888.1	889.4	890.7	892.0	893.3	894.6	895.9	897.2	898.5	899.8	901.1	902.4	903.7	905.0	906.3	907.6	908.9	910.2	911.5	912.8	914.1	915.4	916.7	918.0	919.3	920.6	921.9	923.2	924.5	925.8	927.1	928.4	929.7	931.0	932.3	933.6	934.9	936.2	937.5	938.8	940.1	941.4	942.7	944.0	945.3	946.6	947.9	949.2	950.5	951.8	953.1	954.4	955.7	957.0	958.3	959.6	960.9	962.2	963.5	964.8	966.1	967.4	968.7	970.0	971.3	972.6	973.9	975.2	976.5	977.8	979.1	980.4	981.7	983.0	984.3	985.6	986.9	988.2	989.5	990.8	992.1	993.4	994.7	996.0	997.3	998.6	999.9	1001.2	1002.5	1003.8	1005.1	1006.4	1007.7	1009.0	1010.3	1011.6	1012.9	1014.2	1015.5	1016.8	1018.1	1

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by Timothy A. Luehrman



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*Here's a way to apply option pricing
to strategic decisions without
hiring an army of Ph.D.'s.*

Investment Opportunities as Real Options: Getting Started on the Numbers

by Timothy A. Luehrman

THE ANALOGY between financial options and corporate investments that create future opportunities is both intuitively appealing and increasingly well accepted. Executives readily see why investing today in R&D, or in a new marketing program, or even in certain capital expenditures (a phased plant expansion, say) can generate the possibility of new products or new markets tomorrow. But for many nonfinance managers, the journey from insight to action, from the puts and calls of financial options to actual investment decisions, is difficult and deeply frustrating.

Experts do a good job of explaining what option pricing captures that conventional discounted-cash-flow (DCF) and net-present-value (NPV) analyses do not. Moreover, simple option pricing for exchange-traded

puts and calls is fairly straightforward, and many books present the basics lucidly. But at that point, most executives get stuck. Their interest piqued, they want to know: How can I use option pricing on *my* project? and How can I use this with real numbers rather than with sterilized examples? Unfortunately, how-to advice is scarce on this subject and mostly aimed at specialists, preferably with Ph.D.'s. As a result, corporate analyses that generate real numbers have been rare, expensive, and hard to understand.

The framework presented here bridges the gap between the practicalities of real-world capital projects

and the higher mathematics associated with formal option-pricing theory. It produces quantitative output, can be used repeatedly on many projects, and is compatible with the ubiquitous DCF spreadsheets that are at the heart of most corporate capital-budgeting systems. What this framework cannot supply is absolute precision: when a very precise number is required, managers will still have to call on technical experts with specialized financial tools. But for many projects in many companies, a "good enough" number is not only good enough but considerably better than the number a plain DCF analysis would generate. In such cases, forgoing some precision in exchange for simplicity, versatility, and explicability is a worthwhile trade.

We'll begin by examining a generic investment opportunity—a capital budgeting project—to see what makes it similar to a call option. Then we'll compare DCF with the option-pricing approach to evaluating the project. Instead of looking only at the differences between the two approaches, we will also look for points of commonality. Recognizing the differences adds extra insight to the analysis, but exploiting the commonalities is the key to making the framework understandable and compatible with familiar techniques. In fact, most of the data the framework uses come from the DCF spreadsheets that managers routinely prepare to evaluate investment proposals. And for option values, the framework uses the Black-Scholes option-pricing table instead of complex equations. Finally, once we've built the framework, we'll apply it to a typical capital-investment decision.

Mapping a Project Onto an Option

A corporate investment opportunity is like a call option because the corporation has the right, but not the obligation, to acquire something—let us say, the operating assets of a

Timothy A. Luehrman is a professor of finance at Thunderbird, the American Graduate School of International Management, in Glendale, Arizona. He is the author of "What's It Worth: A General Manager's Guide to Valuation," HBR May-June 1997.

This common ground between NPV and option value has great practical significance. It means that corporate spreadsheets set up to compute conventional NPV are highly relevant for option pricing. Any spreadsheet that computes NPV already contains the information necessary to compute S and X , which are two of the five option-pricing variables. Accordingly, executives who want to begin using option pricing need not discard their current DCF-based systems.

When do NPV and option pricing diverge? When the investment decision may be deferred. The possibility of deferral gives rise to two additional sources of value. First, we would always rather pay later than sooner, all else being equal, because we can earn the time value of money on the deferred expenditure. Second, while we're waiting, the world can change. Specifically, the value of the operating assets we intend to acquire may change. If their value goes up, we haven't missed out; we still can acquire them simply by making the investment (exercising our option). If their value goes down, we might decide not to acquire them. That also is fine (very good, in fact) because, by waiting, we avoid making what would have turned out to be a poor investment. We have preserved the ability to participate in good outcomes and insulated ourselves from some bad ones.

For both of these reasons, being able to defer the investment decision is valuable. Traditional NPV misses the extra value associated with deferral because it assumes the decision cannot be put off. In contrast, option pricing *presumes* the ability to defer and provides a way to quantify the value of deferring. So to value the investment, we need to develop two new metrics that capture these extra sources of value.

Quantifying Extra Value: NPVq. The first source of value is the interest you can earn on the required capital expenditure by investing later rather than sooner. A good way to capture that value is to suppose you put just enough money in the bank now so that when it's time to invest, that money plus the interest it has

earned is sufficient to fund the required expenditure. How much money is that? It is the discounted present value of the capital expenditure. In option notation, it's the present value of the exercise price, or $PV(X)$. To compute $PV(X)$, we discount X for the requisite number of periods (t) at the risk-free rate of return (r_f):

$$PV(X) = X \div (1 + r_f)^t$$

The extra value is the interest rate (r_f) times X , compounded over however many time periods (t) are involved. Alternatively, it is the difference between X and $PV(X)$.

We know that conventional NPV is missing that extra value, so let's put it in. We have seen that NPV can be expressed in option notation as:

$$NPV = S - X$$

Let's rewrite it using $PV(X)$ instead of X . Thus:

$$\text{"modified" NPV} = S - PV(X)$$

Note that our modified NPV will be greater than or equal to regular NPV because it explicitly includes interest to be earned while we wait. It picks up one of the sources of value we are interested in.

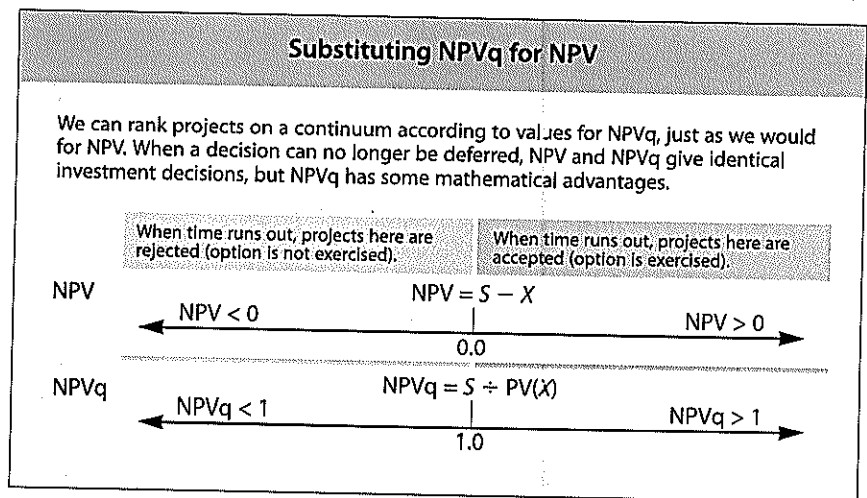
Modified NPV, then, is the difference between S (value) and $PV(X)$ (cost adjusted for the time value of money). Modified NPV can be positive, negative, or zero. However, it will make our calculations a lot easier if we express the relationship between cost and value in such a way that the number can never be negative or zero.

So instead of expressing modified NPV as the *difference* between S and $PV(X)$, let's create a new metric: S divided by $PV(X)$. By converting the difference to a ratio, all we are doing, essentially, is converting negative values to decimals between zero and one. We'll call this new metric NPVq, where "q" reminds us that we are expressing the relationship between cost and value as a quotient:

$$NPVq = S \div PV(X)$$

Modified NPV and NPVq are not equivalent; that is, they don't yield the same numeric answer. For example, if $S = 5$ and $PV(X) = 7$, $NPV = -2$ but $NPVq = 0.714$. But the difference in the figures is unimportant because we haven't lost any information about the project by substituting one metric for another. When modified NPV is positive, NPVq will be greater than one; when NPV is negative, NPVq will be less than one. Anytime modified NPV is zero, NPVq will be one. There is a perfect correspondence between them, as the diagram "Substituting NPVq for NPV" shows.

Quantifying Extra Value: Cumulative Volatility. Now let's move on to the second source of additional value, namely that while we're waiting, asset value may change and affect our investment decision for the better. That possibility is very important, but naturally it is more difficult to quantify because we are not actually sure that asset values will change or, if they do, what the future values will be. Fortunately, rather than measuring added value directly,



we can measure *uncertainty* instead and let an option-pricing model quantify the value associated with a given amount of uncertainty. Once again, we'll go through two steps. First, we'll identify a sensible way to measure uncertainty. Then we'll express the metric in a mathematical form that will be easier for us to use but will not cause us to lose any practical content.

The only way to measure uncertainty is by assessing probabilities. Imagine that the project's future value is to be drawn from an urn containing all possible future values, weighted according to their likelihood of occurring. That is, if a value of \$100 were twice as likely as \$75 or \$125, there would be twice as many \$100 balls in the urn as \$75 balls or \$125 balls.

How can we quantify this uncertainty? Perhaps the most obvious measure is simply the range of all possible values: the difference between the lowest and the highest possibilities. But we can do better than that by taking into account the relative likelihood of values between those extremes. If, for example, very high and very low values are less likely than "medium" or "average" values, our measure of uncertainty should reflect that. The most common probability-weighted measure of dispersion is *variance*, often denoted as *sigma squared* (σ^2). Variance is a summary measure of the likelihood of drawing a value far away from the average value in the urn. The higher the variance, the more likely it is that the values drawn will be either much higher or much lower than average. In other words, we might say that high-variance assets are riskier than low-variance assets.

Variance is an appealing measure of uncertainty, but it's incomplete. We have to worry about a time dimension as well: how much things can change while we wait depends on how long we can afford to wait. For business projects, things can change a lot more if we wait two years than if we wait only two months. So in option valuation, we speak in terms of variance *per period*. Then our measure of the total

amount of uncertainty is variance per period times the number of periods, or $\sigma^2 t$.

This sometimes is called *cumulative variance*. An option expiring in two years has twice the cumulative variance as an otherwise identical option expiring in one year, given the same variance per period. Alternatively, it may help to think of cumulative variance as the amount of variance in the urn times the number of draws you are allowed, which again is $\sigma^2 t$.

Cumulative variance is a good way to measure the uncertainty associated with business investments. Now we'll make two modifications, again for mathematical convenience, that won't affect the ability of the variable to tell us what we want to know about uncertainty. First, instead of using the variance of project values, we'll use the variance of project returns. In other words, rather than working with the actual dollar value of the project, we'll work with the percentage gained (or lost) per year. There is no loss of content because a project's return is completely determined by the project's value:

$$\text{return} = \frac{\text{future value} - \text{present value}}{\text{present value}}$$

The probability distribution of possible values is usually quite asymmetric; value can increase greatly but cannot drop below zero. Returns, in contrast, can be positive or negative, sometimes symmetrically positive or negative, which makes their probability distribution easier to work with.

Second, it helps to express uncertainty in terms of *standard deviation* rather than variance. Standard deviation is simply the square root of variance and is denoted by σ . It tells us just as much about uncertainty as variance does, but it has the advantage of being denominated in the same units as the thing being measured. In our business example, future asset values are denominated in units of currency—say, dollars—and returns are denominated in percentage points. Standard deviation, then, is likewise denominated in

dollars or percentage points, whereas variance is denominated in *squared* dollars or *squared* percentage points, which are not intuitive. Since we are going to work with returns instead of values, our units will be percentage points instead of dollars.

To make these refinements to our measure of total uncertainty, we do the following:

First, stipulate that σ^2 denotes the variance of returns per unit of time on our project.

Second, multiply variance per period by the number of periods (t) to get cumulative variance ($\sigma^2 t$).

Third, take the square root of cumulative variance to change units, expressing the metric as standard deviation rather than variance. Let's call this last quantity *cumulative volatility* ($\sigma\sqrt{t}$) to distinguish it from cumulative variance.

Valuing the Option. Together, our two new call-option metrics, NPVq and $\sigma\sqrt{t}$, contain all the information needed to value our project as a European call option using the Black-Scholes model. They capture the extra sources of value associated with opportunities. And they are composed of the five fundamental option-pricing variables onto which we mapped our business opportunity. NPVq is actually a combination of four of the five variables: S , X , r_f , and t . Cumulative volatility combines the fifth, σ , with t . (See the diagram "Linking Our Metrics to the Black-Scholes Model.") By combining variables in this way, we get to work with two metrics instead of five. Not only is that easier for most of us to grasp, it also allows us to plot two-dimensional pictures, which can be helpful substitutes for equations in managers' discussions and presentations. Finally, each of the metrics has a natural business interpretation, which makes option-based analysis less opaque to non-finance executives.

The graph "Locating the Option Value in Two-Dimensional Space" shows how to use NPVq and $\sigma\sqrt{t}$ to obtain a value for the option. NPVq is on the horizontal axis, increasing from left to right. As NPVq rises, so does the value of the call option.

What causes higher values of NPVq? Higher project values (S) or lower capital expenditures (X). Note further that NPVq also is higher whenever the present value of X is lower. Higher interest rates (r_f) or longer time to expiration (t) both lead to lower present values of X . Any of these changes (lower X or higher S , r_f , or t) increases the value of a European call.

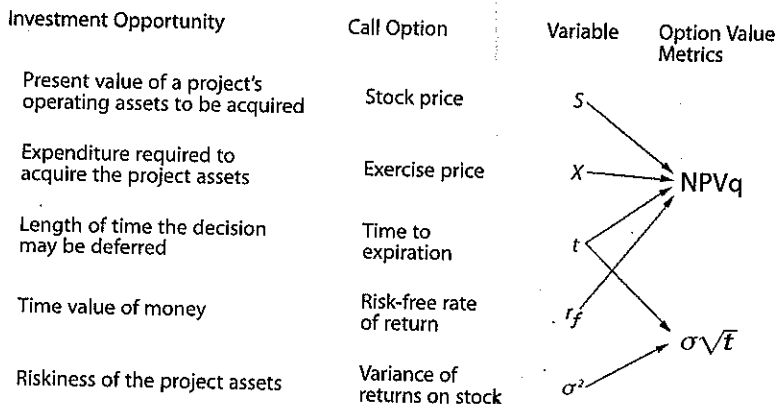
Cumulative volatility is on the vertical axis of the graph, increasing

from top to bottom. As $\sigma\sqrt{t}$ increases, so does call value. What causes higher values of $\sigma\sqrt{t}$? Greater uncertainty about a project's future value and the ability to defer a decision longer. Either of these changes (higher σ or t) likewise increases the value of a European call.

Plotting projects in this two-dimensional space creates a visual representation of their relative option values. No matter where you start in the graph, call value increases

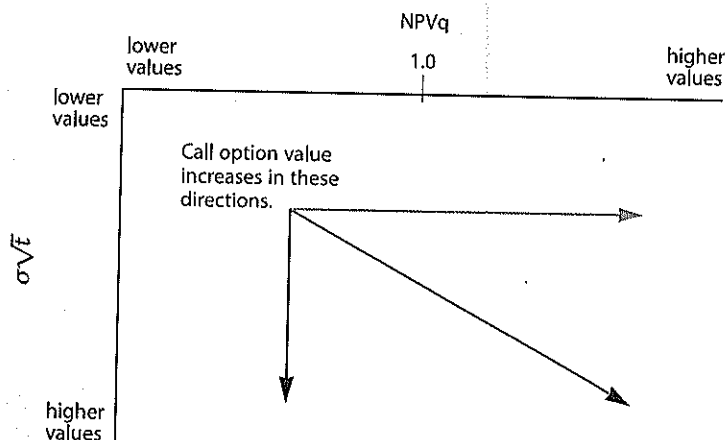
Linking Our Metrics to the Black-Scholes Model

Our two new metrics together contain all five variables in the Black-Scholes model. Combining five variables into two lets us locate opportunities in two-dimensional space.



Locating the Option Value in Two-Dimensional Space

We can locate investment opportunities in this two-dimensional space.



when you move down, to the right, or in both directions at once. Projects in the lower-right corner of the graph are high on both NPVq and $\sigma\sqrt{t}$ metrics and their option value is high compared with projects in the upper-left corner.

Locating various projects in the space reveals their value relative to one another. How do we get absolute values? That is, how can we get a number? Having gotten this far, we find that getting a number is easy. Because NPVq and $\sigma\sqrt{t}$ contain all five Black-Scholes variables, we can fill in a table with Black-Scholes call values that correspond to every pair of NPVq and $\sigma\sqrt{t}$ coordinates. I call this "pricing the space," and the table does it for us.

The exhibit "Using the Black-Scholes Option-Pricing Model to 'Price the Space'" shows part of the filled-in Black-Scholes table. Each

number expresses the value of a specific call option as a percentage of the underlying project's (or asset's) value. For example, for a project whose NPVq equals 1.0 and $\sigma\sqrt{t}$ equals 0.5, the value given in the table is 19.7%. Any European call option for which NPVq is 1.0 and $\sigma\sqrt{t}$ is 0.5 will have a value equal to 0.197 times S . If the assets associated with a particular project have a value (S) of \$100, then the project viewed as a call option has a value of \$19.70. If S were \$10, the call option value would be \$1.97, and so forth. Option values in the table are expressed in relative terms, as percentages of S , rather than in absolute dollars, to enable us to use the same table for both big and small projects. It's also convenient not to have to manipulate the Black-Scholes equation every time we want to value a project. The Black-Scholes model is

used once, to generate the table itself.² After that, we need only locate our project in the table and multiply by a factor of S .

Why is the option value \$19.70 less than the asset value of \$100? We've been analyzing sources of extra value associated with being able to defer an investment. The key is to remember that *extra* refers to a comparison between option value and net present value (NPV), not to a comparison between option value and present value (S). In this example, we are not expecting the option value to be greater than S ; we are expecting it to be greater than NPV, which is S minus capital expenditures (X). S equals \$100 here, but we didn't say what X was. Since NPVq equals 1.0 in this example, X must in fact be greater than \$100; otherwise NPVq would be greater than 1.0. For concreteness, suppose that this is a

Using the Black-Scholes Option-Pricing Model to "Price the Space"

Each number in the table gives the value of a European call for specified values of NPVq and $\sigma\sqrt{t}$, as a percentage of S , the value of project assets.

Example:

Suppose $S = \$100$
 $X = \$105$
 $t = 1$ year
 $r_f = 5\%$
 $\sigma = 50\%$ per year
 then NPVq = 1.0
 and $\sigma\sqrt{t} = 0.50$.

The table gives a value of 19.7%.

Interpretation:

Viewed as a call option, the project has a value of:
 Call value = $0.197 \times \$100 = \19.70 .
 Compare this to its conventional NPV:
 $NPV = S - X$
 $= \$100 - \105
 $= \$-5$.

Black-Scholes value of a European call option, expressed as a percentage of underlying asset value.

		NPVq														
		0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00	1.02	1.04	1.06	1.08
$\sigma\sqrt{t}$	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.6	1.2	2.0	3.1	4.5	6.0	7.5
	0.10	0.0	0.1	0.2	0.3	0.5	0.8	1.2	1.7	2.3	3.1	4.0	5.0	6.1	7.3	8.6
	0.15	0.5	0.7	1.0	1.3	1.7	2.2	2.8	3.5	4.2	5.1	6.0	7.0	8.0	9.1	10.2
	0.20	1.5	1.9	2.3	2.8	3.4	4.0	4.7	5.4	6.2	7.1	8.0	8.9	9.9	10.9	11.9
	0.25	2.8	3.3	3.9	4.5	5.2	5.9	6.6	7.4	8.2	9.1	9.9	10.9	11.8	12.8	13.7
	0.30	4.4	5.0	5.7	6.3	7.0	7.8	8.6	9.4	10.2	11.1	11.9	12.8	13.7	14.6	15.6
	0.35	6.2	6.8	7.5	8.2	9.0	9.8	10.6	11.4	12.2	13.0	13.9	14.8	15.6	16.5	17.4
	0.40	8.0	8.7	9.4	10.2	11.0	11.7	12.5	13.4	14.2	15.0	15.9	16.7	17.5	18.4	19.2
	0.45	9.9	10.6	11.4	12.2	12.9	13.7	14.5	15.3	16.2	17.0	17.8	18.6	19.4	20.3	21.1
	0.50	11.8	12.6	13.4	14.2	14.9	15.7	16.5	17.3	18.1	18.9	19.7	20.5	21.3	22.1	22.9
	0.55	13.8	14.6	15.4	16.1	16.9	17.7	18.5	19.3	20.1	20.9	21.7	22.4	23.2	24.0	24.8
	0.60	15.8	16.6	17.4	18.1	18.9	19.7	20.5	21.3	22.0	22.8	23.6	24.3	25.1	25.8	26.6
	0.65	17.8	18.6	19.3	20.1	20.9	21.7	22.5	23.2	24.0	24.7	25.3	26.2	27.0	27.7	28.4
	0.70	19.8	20.6	21.3	22.1	22.9	23.6	24.4	25.2	25.9	26.6	27.4	28.1	28.8	29.5	30.2
	0.75	21.8	22.5	23.3	24.1	24.8	25.6	26.3	27.1	27.8	28.5	29.2	29.9	30.6	31.3	32.0
	0.80	23.7	24.5	25.3	26.0	26.8	27.5	28.3	29.0	29.7	30.4	31.1	31.8	32.4	33.1	33.8
	0.85	25.7	26.5	27.2	28.0	28.7	29.4	30.2	30.9	31.6	32.2	32.9	33.6	34.2	34.9	35.5
0.90	27.7	28.4	29.2	29.9	30.6	31.3	32.0	32.7	33.4	34.1	34.7	35.4	36.0	36.6	37.3	

one-year option – that is, suppose we can defer the decision for one year – and that the risk-free rate of return (r_f) is 5%. Then for NPV_q to equal 1.0, X must be \$105. Recall that:

$$\begin{aligned} \text{NPV}_q &= S + \text{PV}(X) \\ &= \$100 + (\$105 \div 1.05) \end{aligned}$$

Thus the conventional NPV is actually negative:

$$\begin{aligned} \text{NPV} &= S - X \\ &= \$100 - \$105 \\ &= -\$5. \end{aligned}$$

And an option value of \$19.70 really is substantially greater than conventional NPV.

Using the Framework: An Example in Seven Steps

To illustrate how to apply the framework, consider this example of a hypothetical, but representative, capital investment. Division managers at a company we'll call Franklin Chemical are proposing a phased expansion of their manufacturing facilities. They plan to build a new, commer-

cial-scale plant immediately to exploit innovations in process technology. And then they anticipate further investments, three years out, to expand the plant's capacity and to enter two new markets. The initial investment is obviously strategic because it creates the opportunity for subsequent growth. Yet executives responsible for the company's capital budget are unimpressed by the project because its NPV is essentially zero. (Cash flow projections and NPV calculations for the projected investment are shown in the table "Franklin Chemical's Initial Calculations for a Proposed Expansion.") In fact, in the annual jockeying for funds, this program may not beat out competing alternatives. Its champions are frustrated and feel sure that the company's conventional NPV approach is missing something.

They are right.

This project has considerable option value because the initial expenditure of \$125 million buys the right to expand (or not) three years later. This is important because the expen-

ditures in the third year are large – three times the initial investment.

Step 1 is to recognize the option and describe it. It takes practice to recognize the options that may be buried in conventional projects. However, there are at least two easy ways to see the option in our example. The first is simply to look beyond the numbers and examine the project's description. It surely says something about the two-phased

Franklin Chemical's Initial Calculations for a Proposed Expansion

Discounted-cash-flow valuation: conventional NPV is our starting point.

Year	0	1	2	3	4	5	6
Operating projections							
revenues		455.0	551.0	800.0	1080.0	1195.0	1255.0
- cost of good sold		341.3	414.9	596.0	811.1	893.9	941.3
= gross profit		113.8	136.1	204.0	268.9	301.1	313.8
- SG&A expense		110.4	130.0	219.2	251.6	280.3	287.4
= operating profit		3.3	6.1	-15.2	17.3	20.8	26.3
Cash flow calculation							
EBIT (1-tax rate)		2.2	4.0	-10.0	11.5	13.7	17.4
+ depreciation		19.0	21.0	21.0	46.3	48.1	50.0
- capital expenditures	100.0	8.1	9.5	307.0	16.0	16.3	17.0
- increase NWC	25.0	4.1	5.5	75.0	7.1	8.0	9.7
= free cash flow, assets	-125.0	9.0	10.0	-371.0	34.7	37.5	40.7
+ terminal value, assets (perpetuity value with 5% per year growth)							610.3
Discount to present value							
× discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507
= PV (by year)	-125.0	8.0	8.0	-264.1	22.0	21.3	329.8

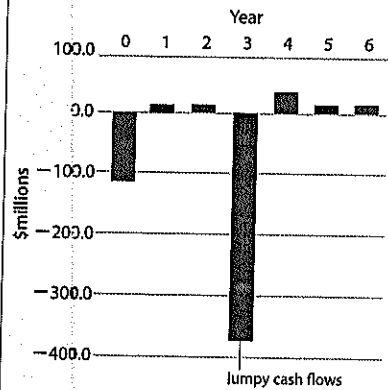
NPV (sum of all years) 0.1

Figures are in \$millions and have been rounded.

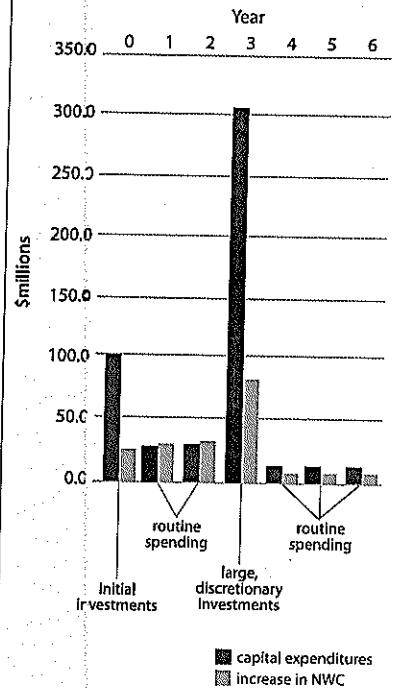
Recognizing the Option

Patterns in the total cash flows and/or in expenditure items such as capital expenditures and investments in net working capital provide clues about the structure of the option.

Free-Cash Flow



Spending Patterns



nature of the program by way of justifying the large outlays in year 3. The other is to examine the pattern of the project cash flows over time. The cash flows in the chart are very uneven: two figures are an order of magnitude larger than the other five, and both of these are negative. A graph of the capital-expenditures line would clearly show the spike in spending in year 3. Such a large sum is almost surely *discretionary*. That is, the company can choose not to make the investment, based on how things look when the time comes. This is a classic expansion option, sometimes called a *growth option*. (See the graphs "Recognizing the Option.")

Franklin's project has two major parts. The first part is to spend \$125 million now to acquire some operating assets. The second part is an option to spend an additional sum, more than \$300 million, three years from now to acquire the additional capacity and enter the new markets. The option here is a call option, owned by the company, with three years to expiration, that can be exercised by investing certain amounts in net working capital (NWC) and fixed assets. Viewing the project in this way, we want to evaluate the following: NPV (entire proposal) = NPV (phase 1 assets) + call value (phase 2 assets).

Phase 1 refers to the initial investment and the associated cash flows. It can be valued using NPV as usual. *Phase 2* refers to the opportunity to expand, which may or may not be exploited in year 3. To value phase 2, we will use the framework outlined above to synthesize a comparable call option and then value it.

Step 2 is to map the project's characteristics onto call option variables. This mapping will create the synthetic option we need and indicate where in the DCF spreadsheet we need to go to obtain values for the variables. The value of the underlying assets (S) will be the present value of the assets acquired when and if the company exercises the option. The exercise price (X) will be the expenditures required to acquire the phase 2 assets. The time to expiration (t) is three years, according to

How to Estimate Cumulative Volatility

The variable in our option-pricing model that managers are least accustomed to estimating is variance (σ^2), or standard deviation (σ), which we used to get our metric for cumulative volatility ($\sigma\sqrt{t}$). For a real option, σ cannot be found in a newspaper or in a financial statement, and most people do not have highly developed intuition about, say, the annualized standard deviation of returns on assets associated with entering a new market. In the example given in the text, we assume σ is 40% per year. Is that reasonable? Here are several sound approaches to creating or judging estimates of σ :

Take a(n educated) guess.

Assets to which you would assign higher hurdle rates because of their higher-than-average systematic risk are also likely to have higher values of σ . How high is "high" for standard deviation? Returns on broad-based U.S. stock indexes had a standard deviation of approximately 20% per year for most of the past 15 years; exceptions (upward spikes) were associated with events like the 1987 stock-market crash and the 1990-1991 Persian Gulf crisis. Individual stocks generally have a higher standard deviation than the market as a whole; returns on General Motors' stock, for example, have a σ of about 25% per year. Individual projects within companies can be expected to have a still higher σ . When I work with manufacturing assets and have no specific information at all about σ , I begin by examining a range: from 30% to 60% per year.

Gather some data.

For some businesses, we can estimate volatility using historical data on investment returns in the same or related industries. Alternatively, we might compute what is called *implied volatility* using current prices of options traded on organized ex-

changes. The idea is to observe a market price for an option whose parameters are all known except for σ . We then use a model like Black-Scholes to figure out what σ must be given all the other variables. Today we can get implied volatility for shares of a very large number of companies in many industries. Often it is possible to use implied volatilities from options on stocks to infer σ for assets in corresponding industries. The quality and availability of such data have improved enormously in the past ten years.

Simulate σ .

Spreadsheet-based projections of a project's future cash flows, together with Monte Carlo simulation techniques, can be used to synthesize a probability distribution for project returns. Once you have the synthesized distribution, the computer can quickly calculate the corresponding standard deviation. Simulation software packages for desktop computers are commercially available and work with the same popular spreadsheet applications that generate your company's DCF models. These tools also have become far more widely available and much easier to use in recent years.

the projections given in the DCF analysis, although we might want to quiz the managers involved to determine whether the decision actually could be made sooner or later. The three-year risk-free rate of interest (r_f) is 5.5% (which is the market rate of interest on a three-year U.S. government bond). Note for comparison that the risk-adjusted discount rate being applied in the spreadsheet is 12%. Finally, the standard deviation of returns on these operating assets (σ) is not given anywhere in the spreadsheet. For now, we'll assume that figure is 40% per year, a value that is neither particularly high nor low. The insert "How to Estimate Cumulative Volatility" explains ways to obtain values for σ in cases like these.

Step 3 is to rearrange the DCF projections for two purposes: to separate phase 1 from phase 2 and to isolate values for S and X. I generally find it easier to work on S and X first. That requires making a judgment about what spending is discretionary versus nondiscretionary or what spending is routine versus extraordinary. It also requires making a similar judgment about which cash inflows are associated with phase 1 as opposed to phase 2.

In this project, expenditures on net working capital and fixed assets obviously are lumpy. The very large sums in year 3 clearly are discretionary and form part of the exercise price (X). The smaller sums in other years are plausibly routine and may be netted against phase 2 cash inflows, ultimately to be discounted and form part of S, the value of the phase 2 assets.

Sometimes it is easy to separate phase 1 cash flows from phase 2 cash flows because whoever prepared the DCF analysis built it up from detailed, phase-specific operating projections. When that's the case, as it is in our example, all we have to do is use the disaggregated detail underlying the summary DCF analysis, and the table "Franklin's Projections Rearranged" presents this information for our example. At other times, we have to allocate cash flows to each phase. A common expedient is simply to break out the phase 1 cash in-

flows and terminal value. Then, phase 2 cash inflows and terminal value are whatever is left over. Note that when we discount cash flows for the two phases separately, we obtain the same NPV as before.

Step 4 is to establish a benchmark for phase 2's option value based on the rearranged DCF analysis. Having separated phases 1 and 2, we can get a conventional discounted-cash-flow NPV for each, which can be seen in the table showing Franklin Chemical's rearranged calculations. This table shows that phase 1 alone has a positive NPV of \$16.3 million while phase 2's NPV is -\$16.2 million. The sum of the two is the same NPV, \$0.1 million, that we obtained originally. Already, we have a quantitative option-related insight. The value of the whole proposal must be

at least \$16.3 million because the option value of phase 2, whatever it turns out to be, cannot be less than zero. In fact, if the option value of phase 2 is significant, the project as a whole will have a much higher value than \$16.3 million, to say nothing of the \$0.1 million we started with. This insight is available only when we separate the project's two phases and realize that we will have a choice about whether or not to undertake phase 2.

Our first DCF benchmark for phase 2 is -\$16.2 million. Actually, though, phase 2's conventional NPV is worse than that, and it's worth a digression to see why. The DCF valuation contains a common mistake. It discounted the discretionary spending in year 3 at the same 12% risk-adjusted rate that had already

Franklin's Projections Rearranged							
The DCF projections have been rearranged to separate phases 1 and 2 and isolate S and X.							
Year	0	1	2	3	4	5	6
Phase 1							
cash flow	0.0	9.0	10.0	11.0	11.6	12.1	12.7
+ terminal value							191.0
- investment	-125						
x discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507
= PV (each year)	-125.0	8.0	8.0	7.8	7.3	6.9	103.2
NPV (sum of years)	16.3 This is the conventional NPV of phase 1.						
Phase 2							
cash flow				0.0	23.1	25.4	28.0
+ terminal value							419.3
- investment				-382			
x discount factor (12%)				0.712	0.636	0.567	0.507
= PV (each year)				-271.9	14.7	14.4	226.6
NPV (sum of years)	-16.2 This is the conventional NPV of phase 2.						
Phases 1 and 2							
cash flow	0.0	9.0	10.0	11.0	34.7	37.5	40.7
+ terminal value							610.3
- investment	-125			-382			
x discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507
= PV (each year)	-125.0	8.0	8.0	-264.1	22.0	21.3	329.8
NPV (sum of years)	0.1 This is the same conventional NPV we got previously.						

Figures are in \$millions and have been rounded.

been applied to the project's cash flows. That rate is almost certainly too high because such expenditures are rarely subject to the same operating and product-market forces that make the project's cash flows risky. Construction costs, for example, may be uncertain, but they are usually much more dependent on engineering factors, weather conditions, and contractors' performance than on customers' tastes, competitive conditions, industry capacity utilization, and such. Overdiscounting future discretionary spending leads to an optimistically biased estimate of NPV. To see the magnitude of this effect, discount the year 3 expenditures of \$382 million at 5.5% instead of 12% (again, it's as if we were putting investment funds into treasury bonds between now

and year 3). Then, phase 2 has a conventional DCF value of -\$69.6 million, not -\$16.2 million, and the NPV for the whole project goes from \$0.1 million to -\$53.4 million, a very substantial difference (See the table "Getting the Right Benchmark.")

Step 5 is to attach values to the option-pricing variables. Having reformulated the DCF spreadsheet, we can now pull values for *S* and *X* from it. *X* is the amount the company will have to invest in net working capital and fixed assets (capital expenditures) in year 3 if it wants to proceed with the expansion: \$382 million. *S* is the present value of the new phase 2 operating assets. In other words, it's the DCF value now (at time zero) of the cash flows those assets are expected to

produce in the fourth year and beyond. The same table shows that to be \$255.7 million. The other option-pricing variables have already been mentioned: *t* is 3 years, *r_f* is 5.5%, and *σ* is 40% per year.

Step 6 is to combine the five option-pricing variables into our two option-value metrics: NPV_Q and $\sigma\sqrt{f}$. In this case:

$$NPV_Q = S + PV(X) = \frac{\$255.7}{382 + (1.055)^3} = 0.786$$

And

$$\sigma\sqrt{f} = 0.4 \times \sqrt{3} = 0.693.$$

The exhibit "Deriving the Option-Value Metrics for Franklin's Project" shows how all five variables were derived and how they combine to form our two metrics.

Getting the Right Benchmark

Most companies overdiscount future discretionary spending (in steps 2 and 3, we used 12%). If we discount phase 2 spending at 5.5% instead, we get a new (lower) benchmark.

Year	0	1	2	3	4	5	6
Phase 1							
cash flow	0.0	9.0	10.0	11.0	11.6	12.1	12.7
+ terminal value							191.0
- investment	-125						
x discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507
= PV (each year)	-125.0	8.0	8.0	7.8	7.3	6.9	103.2

NPV (sum of years) **16.3** This is the conventional NPV of phase 1.

Phase 2							
cash flow			0.0	23.1	25.4	28.0	
+ terminal value							419.3
- investment			-382				
x discount factor (12%)			0.712	0.636	0.567	0.507	
x discount factor (5.5%)			0.852				
= PV (cash flows 12%)			0.0	14.7	14.4	226.6	
= PV (investment 5.5%)			-325.3				

NPV (sum of years) **-69.6** This is a revised DCF benchmark for phase 2.

Phases 1 and 2							
cash flow	0.0	9.0	10.0	11.0	34.7	37.5	40.7
+ terminal value							610.3
- investment	-125			-382			
x discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507
= PV (each year)	-125.0	8.0	8.0	7.8	22.0	21.3	329.8
= PV (investment 5.5%)				-325.3			

NPV (sum of years) **-53.4** This is a revised DCF benchmark for the project as a whole.

Figures are in \$ millions and have been rounded.

Year	0	1	2	3	4	5	6
Phase 2							
cash flow				0.0	23.1	25.4	28.0
terminal value							419.3
investment				-382			
discount factor (12%)				0.712	0.636	0.567	0.507
discount factor (5.5%)				0.852			
PV (cash flows 12%)				0.0	14.7	14.4	226.6
PV (investment 5.5%)				-325.3			
NPV				-69.6			

This is X.

The sum of these present values is S.

Step 7 is to look up call value as a percentage of asset value in our Black-Scholes option-pricing table. The table does not show values that correspond exactly to our computed value for NPVq and $\sigma\sqrt{t}$, but by interpolating we can see that the value of our synthesized call option is about 19% of the value of the underlying assets (S). Accordingly, the dollar value of the option is 0.19 times \$255.7 million, which equals \$48.6 million. Recall the value of the entire proposal is given by: NPV (entire proposal) = NPV (phase 1 assets) + call value (phase 2 assets). Filling in the figures gives: NPV (entire proposal) = \$16.3 million + \$48.6 million = \$64.9 million.

Our final estimate of \$64.9 million is a long way from the original figure for NPV of \$0.1 million and even further from -\$53.4 million. Yet the option-pricing analysis uses the same inputs from the same spreadsheet as the conventional NPV. What looks like a marginal-to-terrible project through a DCF lens is in fact a very attractive one. This

seems especially so when we compare \$64.9 million with the required initial investment of \$125 million; the value associated with the option to expand the plant in year 3 is fully half again as much as the initial investment. Few projects look so good.

What should you do next? All the things you would usually do when evaluating a capital project. Perform sensitivity analyses. Check and update assumptions. Examine particularly interesting or threatening scenarios. Compare and interpret the analysis in light of other historical or contemporary investments and transactions. In addition, now that you've begun pricing synthesized real options, you may want to consider adding a few other items to your list. Some are discussed further in the insert "How Far Can You Extend the Framework?" They include checking for project features that would make it more suitable to use an American rather than a European option. They also include checking for clear disadvantages associated

with deferring investment, such as competitive preemption, which would offset some or all of the sources of value associated with waiting. Some of these concerns can be handled by straightforward modifications to the framework. Others require more sophisticated modeling than either this framework alone or conventional NPV generally can provide. Even in those cases, though, a naively formulated option value will augment whatever insight may be drawn from a DCF treatment alone. Remember that simply by recognizing the structure of the problem we gleaned an important insight about the value of the project in our example (that it had to be at least \$16.3 million) before we actually priced the option.

Does the framework really work? Yes. Even though we have taken some liberties, we know more about our project after using it than we did before. And if it seemed worthwhile, we could further refine our initial estimate of option value. But the key to getting useful insight from option

Deriving the Option-Value Metrics for Franklin's Project

Variable	Project Characteristic	Source in DCF Spreadsheet	Value	Metric
S	The present value of the project assets associated with phase 2.	Separate phase 1 from phase 2 cash flows; compute the DCF value of phase 2 alone.	\$255.7	$NPVq = \frac{\$255.7}{\$382 \div (1.055)^3} = 0.786$
X	Spending required in year 3 to obtain the phase 2 assets.	Identify large discretionary portions of capital expenditures and net-working-capital spending associated with phase 2 only, in year 3.	\$382	
t	Length of time phase 2 spending may be deferred.	Appears in spreadsheet to be three years; check with managers.	3 years	
r _f	Time value of money at same horizon (t) as option.	Obtain market rate of interest on three-year U.S. government bond, assumed here to be 5.5%.	5.5%	
σ	Standard deviation per year on phase 2 assets.	Not in DCF spreadsheet; obtain from similar traded assets, from implied volatilities on traded options, or try a range. Assume 40% per year to start.	40% per year	
				$\sigma\sqrt{t} = 0.4 \times \sqrt{3} = 0.693$

How Far Can You Extend the Framework?

Real corporate projects will present immediate challenges to some of the simplifications underlying this framework. Can the framework be souped up to handle more complex problems? Or does its very simplicity present insurmountable limitations? When might it generate seriously misleading information?

Bells and whistles can be added to the framework fairly easily, although they require extra data, and the details are beyond the scope of this article. For example, the framework assumes that the amount and timing of a project's capital expenditures (X) are certain. But what if they are not? That's usually the case with business opportunities. The framework can be adapted to handle those circumstances, but the adaptation helps only if we can describe the uncertainty. Specifically, we need to know the probability distribution of X and the joint probability distribution of S and X . That is, it matters whether X tends to be high when S tends to be high; whether the opposite is true (that is, X tends to be high when S is low and vice versa); or whether they are both not only uncertain but also unrelated.

As another example, suppose the uncertainty (or variance) associated with a project changes over time. That's fairly common, and it makes sense that it should affect our estimate of cumulative volatility. Once again, if we know *how* the variance changes over time, or if we can make plausible guesses, the framework can be adapted without much trouble. The insert "Where to Find Additional Help" cites some readings that address how such real-world problems affect option values.

Both of these examples describe situations in which the real limitation is not the framework but rather the data or our knowledge of the project's parameters. Even when we know that we lack necessary data, the framework can help by showing us what the effect on value would be if the data were one thing or another. We might conclude that it's worthwhile to gather or create better data.

Some other real-world complications are thornier. The framework does a good job of capturing the extra value associated with deferring an investment decision. But what if there are particular costs associated with deferral? For example, companies trying to be first to market with the next generation of a hot product will incur large costs if deferral allows a competitor to preempt them. Anytime

there are predictable costs to deferring, the option to defer an investment is less valuable, and we would be foolish to ignore those costs. If such additional costs were the only issue, we could easily handle them in our framework as long as we knew when the decision to invest or not to invest finally would be made. But in the real world, we often don't know. Companies may not be compelled to invest at a certain moment, but rather may have discretion to time their investments. So the problem is to decide not only whether to invest but also when.

In effect, many real options are *American* rather than *European*. American options can be exercised at any time prior to expiration; European options may be exercised only at expiration. The option-pricing table embedded in this framework prices European options. American options are more valuable than European options whenever the costs associated with deferral are predictable. When that's the case, I recommend using the framework to set the lower boundary of the option value and then supplementing that figure with another table or a spreadsheet-based algorithm to convert European option values to the corresponding American option values. That is not quite as easy as merely rewriting the pricing table because it would take a three-dimensional or possibly four-dimensional table to accommodate the extra variables needed. But it can be set up in a spreadsheet. In short, the combination of costs associated with deferral and a company's ability to time investments calls for some added analysis. But the framework needs to be augmented, not scrapped.

Finally, the Black-Scholes option-pricing model that generated the numbers in the table makes some simplifying assumptions of its own. They include assumptions about the form of the probability distribution that characterizes project returns. They also include assumptions about the *tradability* of the underlying project assets; that is, about whether those assets are regularly bought and sold. And they include assumptions about the ability of investors to continually adjust their investment portfolios. When the Black-Scholes assumptions fail to hold, this framework still yields qualitative insights but the numbers become less reliable. Consequently, it may be worthwhile to consult an expert about alternative models to improve the quantitative estimates of option value.

Where to Find Additional Help

Graduate-level corporate-finance textbooks cover the basics of option pricing, beginning from first principles:

Zvi Bodie and Robert C. Merton, *Finance* (Upper Saddle River, N.J.: Prentice-Hall, 1998).

Richard A. Brealey and Stewart Myers, *Principles of Corporate Finance*, fifth edition (New York: McGraw-Hill, 1996).

Stephen A. Ross, Randolph W. Westerfield, and Jeffrey Jaffe, *Corporate Finance*, fourth edition (Chicago: Richard D. Irwin, 1996).

Other books go beyond the basics to treat specialized problems and present more advanced models for option pricing.

Well-known titles include:

John C. Hull, *Options, Futures, and Other Derivatives*, third edition (Upper Saddle River, N.J.: Prentice-Hall, 1997).

David G. Luenberger, *Investment Science* (New York: Oxford University Press, 1998).

Paul Wilmott, Jeff Dewynne, and Sam Howison, *Option Pricing* (Oxford, England: Oxford Financial Press, 1993).

A few books focus on real options in particular. Each takes a somewhat different approach to modeling corporate opportunities:

Martha Amram and Nalin Kulatilaka, *Real Options: Managing Strategic Investment in an Uncertain World* (Boston, Mass.: Harvard Business School Press, forthcoming 1998).

Avinash K. Dixit and Robert S. Pindyck, *Investment Under Uncertainty* (Princeton, N.J.: Princeton University Press, 1994).

Lenos Trigeorgis, *Real Options: Managerial Flexibility and Strategy in Resource Allocation* (Cambridge, Mass.: MIT Press, 1996).

Shorter readings on selected related topics include:

Avinash K. Dixit and Robert S. Pindyck, "The Options Approach to Capital Investment," HBR May-June 1995.

W. Carl Kester, "Today's Options for Tomorrow's Growth," HBR March-April 1984.

Timothy A. Luehrman, "What's It Worth? A General Manager's Guide to Valuation," HBR May-June 1997.

Lenos Trigeorgis, ed., *Real Options in Capital Investment: Models, Strategies, and Applications* (Westport, Conn.: Praeger, 1995).

Finally, expanded versions of the option-pricing table published in this article are available in the Brealey and Myers text cited above and in Luehrman, "Capital Projects as Real Options: An Introduction," HBS case no. 295-074.

pricing sooner rather than later is to build on, rather than abandon, the DCF-based NPV analysis your company already uses. Had we set out to value the option from scratch, it would have been more difficult and taken longer. It also would have been hard to tell how well we had done and when to stop working on it. Option pricing should be a complement to existing capital-budgeting systems, not a substitute for them. The framework presented here is a way to start where you are and get somewhere better.

1. There are other mathematical advantages, beyond the scope of this article, associated with using the quotient instead of the difference. Also, students of economics will recognize the similarity between NPVq and the famous "Tobin's q," which measures the ratio of the value of an asset to its replacement cost.

2. This form of option-pricing table is nearly as old as the Black-Scholes model itself. I first ran into it as an M.B.A. student using Richard Brealey and Stewart Myers's text, *Principles of Corporate Finance* (New York: McGraw-Hill, 1981).

In the September-October 1998 issue of HBR, Timothy Luehrman extends the framework presented here. In "Strategy as a Portfolio of Real Options," he explores how managers can use option pricing to improve the way they make decisions about the sequence and timing of strategic investments.

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