1) Find these terms of the sequence $\{a_n\}$, where $a_n = 2\cdot(-3)^n + 5^n$

a) a_0 (d) a_5

3) What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals 2^{n-1}

5) List the first 10 terms of each of these sequences

(c) The sequence that lists the odd positive integers in increasing order, listing each odd integer twice

f) The sequence whose first term is 2, second term is 4. and each succeeding term is the sum of the two preceding terms

9) Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

(b) $a_n = a_{n-1}^2$, $a_1 = 2$

7) Find ~~the~~ solution to each of these recurrence relations and initial conditions. Use an iterative approach such as that used.

a) $a_n = 3a_{n-1}$, $a_0 = 2$

e) $a_n = 2a_{n-1} - 1$, $a_0 = 1$

3) Exercises 1.7

) Show that the square of an even number is an even number using a direct proof.

) Show that if n is an integer and n^3+5 is odd, then n is even using

a) Set up a recurrence relation for the salary of this employee n years after 2009.
b) What will the salary of this employee be in 2017?
c) Find an explicit formula for the salary of this employee n years after 2009.

23. Find a recurrence relation for the balance $B(k)$ owed at the end of k months on a loan of \$5000 at a rate of 7% if a payment of \$100 is made each month. [*Hint:* Express $B(k)$ in terms of $B(k-1)$; the monthly interest is $(0.07/12)B(k-1)$.]

24. a) Find a recurrence relation for the balance $B(k)$ owed at the end of k months on a loan at a rate of r if a payment P is made on the loan each month. [*Hint:* Express $B(k)$ in terms of $B(k-1)$ and note that the monthly interest rate is $r/12$.]
b) Determine what the monthly payment P should be so that the loan is paid off after T months.

25. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

a) $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, \ldots$
b) $1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, \ldots$
c) $1, 0, 2, 0, 4, 0, 8, 0, 16, 0, \ldots$
d) $3, 6, 12, 24, 48, 96, 192, \ldots$
e) $15, 8, 1, -6, -13, -20, -27, \ldots$
f) $3, 5, 8, 12, 17, 23, 30, 38, 47, \ldots$
g) $2, 16, 54, 128, 250, 432, 686, \ldots$
h) $2, 3, 7, 25, 121, 721, 5041, 40321, \ldots$

26. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

a) $3, 6, 11, 18, 27, 38, 51, 66, 83, 102, \ldots$
b) $7, 11, 15, 19, 23, 27, 31, 35, 39, 43, \ldots$
c) $1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, \ldots$
d) $1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, \ldots$
e) $0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, \ldots$
f) $1, 3, 15, 105, 945, 10395, 135135, 2027025, 34459425, \ldots$
g) $1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, \ldots$
h) $2, 4, 16, 256, 65536, 4294967296, \ldots$

27. Show that if a_n denotes the nth positive integer that is not a perfect square, then $a_n = n + \{\sqrt{n}\}$, where $\{x\}$ denotes the integer closest to the real number x.

28. Let a_n be the nth term of the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots$, constructed by including the integer k exactly k times. Show that $a_n = \lfloor \sqrt{2n} + \frac{1}{2} \rfloor$.

29. What are the values of these sums?

a) $\sum_{k=1}^{5} (k+1)$
b) $\sum_{j=0}^{4} (-2)^j$
c) $\sum_{i=1}^{10} 3$
d) $\sum_{j=0}^{8} (2^{j+1} - 2^j)$

30. What are the values of these sums, where $S = \{1, 3, 5, 7\}$?

a) $\sum_{j \in S} j$
b) $\sum_{j \in S} j^2$
c) $\sum_{j \in S} (1/j)$
d) $\sum_{j \in S} 1$

31. What is the value of each of these sums of terms of a geometric progression?

a) $\sum_{j=0}^{8} 3 \cdot 2^j$
b) $\sum_{j=1}^{8} 2^j$
c) $\sum_{j=2}^{8} (-3)^j$
d) $\sum_{j=0}^{8} 2 \cdot (-3)^j$

32. Find the value of each of these sums.

a) $\sum_{j=0}^{8} (1 + (-1)^j)$
b) $\sum_{j=0}^{8} (3^j - 2^j)$
c) $\sum_{j=0}^{8} (2 \cdot 3^j + 3 \cdot 2^j)$
d) $\sum_{j=0}^{8} (2^{j+1} - 2^j)$

33. Compute each of these double sums.

a) $\sum_{i=1}^{2} \sum_{j=1}^{3} (i + j)$
b) $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j)$
c) $\sum_{i=1}^{3} \sum_{j=0}^{2} i$
d) $\sum_{i=0}^{2} \sum_{j=1}^{3} ij$

34. Compute each of these double sums.

a) $\sum_{i=1}^{3} \sum_{j=1}^{2} (i - j)$
b) $\sum_{i=0}^{3} \sum_{j=0}^{2} (3i + 2j)$
c) $\sum_{i=1}^{3} \sum_{j=0}^{2} j$
d) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$

35. Show that $\sum_{j=1}^{n} (a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, \ldots, a_n$ is a sequence of real numbers. This type of sum is called **telescoping**.

36. Use the identity $1/(k(k+1)) = 1/k - 1/(k+1)$ and Exercise 35 to compute $\sum_{k=1}^{n} 1/(k(k+1))$.

37. Sum both sides of the identity $k^2 - (k-1)^2 = 2k - 1$ from $k = 1$ to $k = n$ and use Exercise 35 to find

a) a formula for $\sum_{k=1}^{n} (2k-1)$ (the sum of the first n odd natural numbers).
b) a formula for $\sum_{k=1}^{n} k$.

***38.** Use the technique given in Exercise 35, together with the result of Exercise 37b, to derive the formula for $\sum_{k=1}^{n} k^2$ given in Table 2. [*Hint:* Take $a_k = k^3$ in the telescoping sum in Exercise 35.]

39. Find $\sum_{k=100}^{200} k$. (Use Table 2.)

40. Find $\sum_{k=99}^{200} k^3$. (Use Table 2.)

***41.** Find a formula for $\sum_{k=0}^{m} \lfloor \sqrt{k} \rfloor$, when m is a positive integer.

***42.** Find a formula for $\sum_{k=0}^{m} \lfloor \sqrt[3]{k} \rfloor$, when m is a positive integer.

There is also a special notation for products. The product of $a_m, a_{m+1}, \ldots, a_n$ is represented by $\prod_{j=m}^{n} a_j$, read as the product from $j = m$ to $j = n$ of a_j.

Exercise 1-8

(3) Prove that if x and y are real numbers, then $Max(x,y) + Min(x,y) = x+y$. (Hint: Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x < y$, respectively.

29) Prove that there is no positive integer n such that $n^2 + n^3 = 100$

Exercise 5.1

3) Let $P(n)$ be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/$ for the positive integer n.

(a) what is the statement $P(1)$?

(b) Show that $P(1)$ is true, completing the basis step of the proof.

c) what is the inductive hypothesis?

d) What do you need to prove in the inductive step?

(e) Complete the inductive step, identifying where you use the inductive hypothesis.

f) Explain why these steps show that this formula is true whenever n is a positive integer.