Assume the following situation for all four problems below:

You are given an urn and told only that it either contains 10 red balls or 5 red and 5 white balls. You will be sampling from the urn without replacement.

1. Suppose you draw out three balls without replacement and they are all red.
    1. Let H be the hypothesis, "The urn has only red balls in it." What is your prior probability assignment $P[H|X]$?
    2. Let D represent your observed data, "All three balls drawn out are red." What is $P[D|HX]$? What is $P[D|\overline{H}X]$?
    3. What is your posterior probability assignment, $P[H|DX]$?

2. Suppose you draw a fourth ball, which is also red.
    1. Repeat the above calculation from the beginning starting from the initial background state of knowledge (that is, with the same prior probability $P[H|X]$, etc.). What is the new posterior probability $P[H|DX]$?
    2. Repeat this calculation starting from with the previously observed data as your new background state of knowledge (that is, with your previous posterior probability now as a prior probability).
    3. You should get the same answers for the two calculations above. Why is this true?

3. After having drawn out four red balls, what probability would you assign to the proposition, "The fifth ball drawn out is red."?
   [Hint: You can write $P[R_5|DX] = P[H|DX] * P[R_5|HDX] + P[\overline{H}|DX] * P[R_5|\overline{H}DX]$]

4. Assume on the fifth draw you get a **white** ball. You could reason deductively that you did not have the all-red urn all along. Show how you would reach the same conclusion by computing probabilities.