

## Question 2

(a) Consider a game with two players **A** and **B**. Player **A** has strategies  $A_1$  and  $A_2$ , while player **B** has strategies  $B_1$  and  $B_2$ . The payoffs are shown in the table below.

	$B_1$	$B_2$
$A_1$	$(a, \tilde{a})$	$(b, \tilde{b})$
$A_2$	$(c, \tilde{c})$	$(d, \tilde{d})$

In the above, the first number  $x$  in

$$(x, y)$$

refers to the payoff for **A**, while the second number  $y$  is the payoff for **B**. Here  $a, \tilde{a}, b, \tilde{b}, c, \tilde{c}, d$  and  $\tilde{d}$  are fixed numbers. Assume that

$$a > \tilde{d} \quad \text{and} \quad \tilde{c} \geq c.$$

Is it possible that BOTH

$$(A_1, B_1) \quad \text{and} \quad (A_2, B_2)$$

are Nash equilibria? Justify your answer.

(b) In a hypothetical pandemic, two countries **C** and **D** are in discussion for opening up their boundaries for mutual tourism (so call *travel bubbles*). The factors to be considered are shown in the following (\*<sub>1</sub>–\*<sub>3</sub>):

\*<sub>1</sub>  $V_C$  – the number of percentage (%) of the population in country **C** that have received the vaccine. In particular,

$$0 \leq \frac{V_C}{100} \leq 1.$$

Likewise,  $V_D$  – the number of % of the population in country **D** that have received the vaccine.

\*<sub>2</sub>  $N_C$  – the average number (for last 7 days) of local infected cases per day, in country **C**. It is assumed that  $N_C \leq 10$ , so that

$$0 \leq \frac{N_C}{10} \leq 1.$$

Likewise,  $N_D$  – the average number (for last 7 days) of local infected cases per day, in country **D**. It is also assumed that  $N_D \leq 10$ ,

\*<sub>3</sub>  $Q_C$  – the number of day(s) for required quarantine for visitors (from country **D**) entering country **C**. It is assumed that  $Q_C \leq 7$ , so that

$$0 \leq \frac{Q_C}{7} \leq 1.$$

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Likewise,  $Q_D$  – the number of day(s) for required quarantine for visitors (from country **C**) entering country **D**. It is also assumed that  $Q_D \leq 7$ .

(There is NO required quarantine for people returning to their home country from the travel bubbles.)

Each one of the countries **C** and **D** has two strategies at its disposal, namely,

EITHER

(**I & O**) - opening up for incoming and outgoing tourists (between countries **C** and **D**),

OR

(**O no I**) - allowing outgoing tourists only, and banning incoming tourists (between countries **C** and **D**).

Consider the following payoffs.

•<sub>1</sub> Both countries **C** and **D** apply the (**I & O**) strategy:

$$\text{Payoff } \mathbf{C} = 1 + \frac{V_D}{100} - \frac{N_D}{10} - \frac{Q_D}{7},$$

$$\text{Payoff } \mathbf{D} = 1 + \frac{V_C}{100} - \frac{N_C}{10} - \frac{Q_C}{7}.$$

•<sub>2</sub> Only country **C** applies the (**I & O**) strategy, and country **D** applies the (**O no I**) strategy:

$$\text{Payoff } \mathbf{C} = 1 + \frac{V_D}{100} - \frac{N_D}{10} - \frac{Q_D}{7},$$

$$\text{Payoff } \mathbf{D} = \frac{1}{4} + \frac{V_C}{100} - \frac{N_C}{10}.$$

•<sub>3</sub> Only country **D** applies the (**I & O**) strategy, and (**O no I**) strategy:

$$\text{Payoff } \mathbf{C} = \frac{1}{4} + \frac{V_D}{100} - \frac{N_D}{10},$$

$$\text{Payoff } \mathbf{D} = 1 + \frac{V_C}{100} - \frac{N_C}{10} - \frac{Q_C}{7}.$$

•<sub>4</sub> Both countries **C** and **D** apply the (**O no I**) strategy:

$$\text{Payoff } \mathbf{C} = \text{Payoff } \mathbf{D} = 0.$$

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(i) Under the conditions that

$$\frac{V_D}{100} \geq \frac{N_D}{10}, \quad Q_C \leq 5 \quad \text{and} \quad Q_D \leq 5,$$

determine all the Nash equilibria (um) (if any) of the game. Justify your answer.

(ii) Find the conditions on  $V_C$ ,  $N_C$ ,  $Q_C$ ,  $V_D$ ,  $N_D$  and  $Q_D$ , so that the situation described in  $\bullet_4$  is a Nash equilibrium of the game. Your answer should be in its simplest form. Justify your answer.