Suppose that $Y_1, Y_2, \ldots, Y_n, \ldots$ are i.i.d. random variables with $E(Y_1) = 0$ and $Var(Y_1) = \sigma^2$. Let $X_{n,i} \equiv a_{n,i}Y_i$ for $1 \leq i \leq n$ where $\{a_{n,i} : 1 \leq i \leq n\}$ are constants. Consider $S_n \equiv \sum_{i=1}^n X_{n,i}$.

(a) Compute $E(S_n)$ and $Var(S_n) \equiv \sigma_n^2$.

(b) Show that if $A_n^2 \equiv \max_{1\leq i\leq n} |a_{ni}|^2 / \sum_1^n a_{ni}^2 \to 0$, then $S_n/\sigma_n \to_d Z \sim N(0,1)$.

(c) Now suppose that $a_{n,i} = n^{-1/2}(i/n)^\alpha$ for some $\alpha \in \mathbb{R}$. For what values of α does $S_n \to_d v_\alpha Z \sim N(0, v_\alpha^2)$ for some $v_\alpha^2 < \infty$? Compute v_α^2 as a function of α.