(1).

Hamiltonian of electron with mass "m" and charge '-e' in harmonic potential is

$$H = K.E + P.E$$

$$H = \frac{p^2}{2m} + \frac{1}{2} K x^2$$

if uniform electric field "E" is applied to this electron the energy releated to this electric field is $qV = qEd = qEx$

so Now Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{1}{2} kx^2 + qEx$$

$$H = \frac{p^2}{2m} + \frac{1}{2} Kx^2 - eEx$$

So, corresponding Hamiltonian operator is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{K\hat{x}^2}{2} - eE\hat{x}$$

(3)

Hamiltonian of particle with electric field

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{K\hat{x}^2}{2} + (-qE\hat{x})$$

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{K\hat{x}^2}{2} - qE\hat{x}^2$$

As we know that By unitary transformation:

for Hamiltonian with Electric field can be reduced to Hamiltonian with out Electric field

$$\hat{H}'_{old} = \hat{U}^{+}\hat{H}\hat{U}_{new}$$

As $\hat{H}$ and U comutes so $\hat{H}U = U\hat{H}$

$$\hat{H} = e^{-\frac{i}{\hbar}Ht}\left(\frac{\hat{P}^2}{2m} + \frac{K\hat{x}^2}{2} - qE\hat{x}\right)e^{\frac{i\hat{H}t}{\hbar}}$$

$$\hat{H} = \cancel{\frac{\hat{P}^2}{2m}} -$$

$$\hat{H}_{old} = \hat{H}_{new}\left(e^{-\frac{iHt}{\hbar}}e^{\frac{iHt}{\hbar}}\right)$$

$$\hat{H}_{old} = \hat{H}_{new}$$

so

$$H = \frac{\hat{P}}{2m} + \frac{K\hat{x}^2}{2}$$ ∴ Hamiltonian reduced to Hamiltonian with out electric field.

② Using operator identities from 3.2.2 prove that

$$e^{\left(\frac{i\hat{P}_x d}{\hbar}\right)} \hat{x}\, e^{\left(-\frac{i\hat{P}_x d}{\hbar}\right)} = \hat{x} + d$$

where 'd' is a real number $\hat{x}$ and $\hat{P_x}$ are momentum and position operators respectively.

∴ Let we have two operators $T_1 = \hat{x}$, and $T_2 = \hat{P_x}$

for any operator $\hat{T}$

$$\exp(\hat{T}) = \sum_{n=0}^{\infty} \frac{\hat{T}^n}{n!} \quad \text{and} \quad [\hat{T}_1, \hat{T}_2^n] = n\hat{C}\,\hat{T}_2^{n-1}$$

So; first we find the commutator of $[\hat{x}, e^{-\frac{i\hat{P}_x d}{\hbar}}]$

$$[\hat{x}, e^{-\frac{i\hat{P}_x d}{\hbar}}] = \hat{C} \sum_{n=0}^{\infty} \frac{\left(\frac{-id}{\hbar}\right)^n . n\, \hat{P}_x^{\,n-1}}{n!}$$

$$[\hat{x}, e^{-\frac{i\hat{P}_x d}{\hbar}}] = -\frac{i\hbar}{d}\hat{C} \sum_{n=0}^{\infty} \frac{\left(\frac{-id}{\hbar}\right)^{n-1} \hat{P}_x^{\,n-1}}{(n-1)!} = \left(-\frac{id}{\hbar}\right)\hat{C}\, e^{-\frac{i\hbar}{d}\hat{P}_x}$$

Now xlying $e^{\frac{i\hat{P}_x d}{\hbar}}$ from left side of Both sides.

$$e^{\frac{i\hat{P}_x d}{\hbar}} [\hat{x}, e^{-\frac{i\hat{P}_x d}{\hbar}}] = e^{+\frac{id\hat{P}_x}{\hbar}} . \frac{i\,d\hat{C}}{\hbar} e^{-\frac{id\hat{P}_x}{\hbar}}$$

<u>Solve Both sides.</u>

$$e^{\frac{i\hat{P}_x d}{\hbar}}\left(\hat{x} e^{-\frac{i\hat{P}_x d}{\hbar}} - e^{-\frac{i\hat{P}_x d}{\hbar}}\hat{x}\right) = -\frac{id}{\hbar}\left(e^{\frac{id\hat{P}_x}{\hbar}}\hat{C}\, e^{-\frac{id\hat{P}_x}{\hbar}}\right)$$

$$e^{\frac{i\hat{P}_x d}{\hbar}}\hat{x} e^{-\frac{i\hat{P}_x d}{\hbar}} - e^{0}\hat{x} = -\frac{id}{\hbar}\left(e^{id\hat{P}_x/\hbar}[\hat{x}, \hat{P}_x]\, e^{-\frac{id\hat{P}_x}{\hbar}}\right)$$

$$e^{\frac{i\hat{P}_x d}{\hbar}}\hat{x} e^{-\frac{i\hat{P}_x d}{\hbar}} - \hat{x} = -\frac{id}{\cancel{\hbar}}\left(i\cancel{\hbar}\, e^{\frac{id\hat{P}_x}{\hbar}} e^{-\frac{id\hat{P}_x}{\hbar}}\right)$$

$$e^{i\hat{P}_x d/\hbar}\hat{x} e^{-\frac{i\hat{P}_x d}{\hbar}} = \hat{x} + d \quad \underline{\text{Proved}}$$

(5)

As

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{K}{2}x^2 - qEx$$ is hamiltonian

of Particle.

Relplace

$$y = x - qE/m\omega^2$$

$$H = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2 y^2 - \frac{q^2E^2}{2m\omega^2}$$

Here $y = x - qE/m\omega^2$

As Probability of finding particle in ground state, when field is oof. is

$$P = |\langle\psi_0^0|\psi_0\rangle|^2$$

$$P = e^{-\frac{q^2E^2}{2m^2\omega^3}}$$

where if ΔE is shift in Energy of ground state that probability of finding particle in ground state when field in turned on

$$P = e^{-\frac{\Delta E}{\hbar\omega}}$$ whre ΔE is

change in energy, due to additional term.

(6):-

Heisign berg equations for Raising and lowering operators

Heisign berg equations for Any operator $\hat{A}$ is

$$\frac{d\hat{A}}{dt} = \frac{1}{i\hbar}[\hat{A}, \hat{H}] + \frac{\partial \hat{A}}{\partial t} \longrightarrow (1)$$

where $\hat{H}$ is hamiltonian of system.

$\frac{\partial \hat{A}}{\partial t}$ is explicite time dependance of operator $\hat{A}$.

for lowering and raising operator which are

$\hat{a}$, $\hat{a}^{\dagger}$.

are =

$$\hat{a}(t) = e^{\frac{i}{\hbar}\hat{H}t}\,\hat{a}\,e^{-\frac{i}{\hbar}\hat{H}t} = e^{i\omega t\hat{N}}\,a\,e^{-i\omega t\hat{N}}$$

Here $\hat{H} = \hbar\omega(\hat{a}\hat{a}^{+} + 1/2) = \hbar\omega(\hat{N} + 1/2)$

where $\hat{a}\hat{a}^{+} = \hat{N}$

Heisign berg equation for lowering operator $\hat{a}$ is

$$\frac{d(\hat{a})}{dt} = \frac{d}{dt}\left[e^{i\omega t\hat{N}}\,a\,e^{-i\omega t\hat{N}}\right]$$

$$\frac{d\hat{a}}{dt} = i\omega e^{i\omega t\hat{N}}\hat{N}\hat{a}\,e^{-i\omega t\hat{N}} - e^{i\omega t\hat{N}}\hat{a}(i\omega\hat{N})e^{-i\omega t\hat{N}}$$

$$\frac{d\hat{a}}{dt} = i\omega e^{i\omega t\hat{N}}[\hat{N}, \hat{a}]e^{-i\omega t\hat{N}}$$

$$\frac{d\hat{a}(t)}{dt} = -i\omega\, e^{i\omega t\hat{N}}\, a\, e^{-i\omega t\hat{N}}$$

$$\frac{d}{dt}\hat{a}(t) = -i\omega t\, \hat{a}(t)$$

$$\hat{a}(t) = e^{-i\omega t}\,\hat{a}$$

Similarly for Raising operator this equation becomes.

$$\hat{a}^{+}(t) = e^{i\omega t}\,\hat{a}.$$

which can be derived as.

$$\frac{d\hat{a}^{+}(t)}{dt} = \frac{d}{dt}\left(e^{i\omega t\hat{N}}\,\hat{a}^{+} e^{-i\omega t\hat{N}}\right)$$

$$\frac{d\hat{a}^{+}(t)}{dt} = e^{i\omega t\hat{N}}\, i\omega \hat{N}\hat{a}^{+} e^{-i\omega t\hat{N}} - e^{i\omega t\hat{N}}\, i\omega \hat{N}\hat{a}^{+} e^{-i\omega t\hat{N}}$$

$$\frac{d\hat{a}^{+}(t)}{dt} = i\omega\, e^{i\omega t\hat{N}}\left[\hat{N}, \hat{a}^{+}\right] e^{-i\omega t\hat{N}}$$

$$\frac{d\hat{a}^{+}(t)}{dt} = i\omega t\, \hat{a}^{+}$$

$$\hat{a}^{+}(t) = e^{i\omega t}\,\hat{a}^{+}$$

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(4):-

As Hamiltonian of un-perturbed particle is.

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2} m\omega^2 \hat{X}^2$$

So, wave function for unperturbed particle is

$$\psi_n(x) = \frac{1}{\sqrt{\sqrt{\pi}\, 2^n n!\, x_0}} e^{-\frac{x^2}{2x_0^2}} H_n\left(\frac{x}{x_0}\right)$$

for perturbed particle by Electric field E.

Hamiltonian is

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2} m\omega^2 \hat{X}^2 - qE\hat{X}$$

Replace $\hat{x} - qE/(m\omega^2)$ by a new variable "$\hat{y}$"

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2} m\omega^2 \hat{y}^2 - \frac{q^2E^2}{2m\omega^2}$$

wave function of perturbed particle in terms of wave function of unperturbed particle will be

$$\psi(y) = \psi\left(x - \frac{qE}{m\omega^2}\right) = \frac{1}{\sqrt{\sqrt{\pi}\, 2^n n!\, x_0}} e^{-y^2/2x_0^2} H_n(y/x_0)$$

$$\psi(y) = \psi(x - qE/m\omega^2) = \frac{1}{\sqrt{\sqrt{\pi}\, 2^n . n!\, x_0}} e^{-\frac{(x - qE/m\omega^2)^2}{2x_0^2}} H_n\left(\frac{x - qE/m\omega^2}{x_0}\right)$$