Project 1 – Solutions

8.4

$$\dot{x} = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$

Clearly, $\Delta(\lambda) = (\lambda - 1)^3 = \lambda^3 - 3\lambda^2 + 3\lambda - 1 \rightarrow \begin{cases} \alpha_1 = -3 \\ \alpha_2 = 3 \\ \alpha_3 = -1 \end{cases}$

The desired poles are $-2, -1 \pm j$:

$$\Delta_f(\lambda) = (\lambda+2)(\lambda+1+j)(\lambda+1-j) = s^3 + 4\lambda^2 + 6\lambda + 4 \rightarrow \begin{cases} \bar{\alpha}_1 = 4 \\ \bar{\alpha}_2 = 6 \\ \bar{\alpha}_3 = 4 \end{cases}$$

Now, $\bar{k} = [\bar{\alpha}_3 - \alpha_3 \quad \bar{\alpha}_2 - \alpha_2 \quad \bar{\alpha}_1 - \alpha_1]$

$= [5 \quad 3 \quad 7]$

To find P^{-1}, we can use

$$\mathcal{C} = [B \quad AB \quad A^2B] = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\bar{\mathcal{C}}^{-1} = \begin{bmatrix} \alpha_2 & \alpha_1 & 1 \\ \alpha_1 & 1 & \\ 1 & & 0 \end{bmatrix} \rightarrow P^{-1} = \mathcal{C}\bar{\mathcal{C}}^{-1} = \begin{bmatrix} 4 & -4 & 1 \\ -1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

Then, $k = \bar{k}P = [15 \quad 47 \quad -8]$

To verify, compute

$$A_{CL} = A - bk = \begin{bmatrix} -14 & -46 & 6 \\ 0 & 1 & 1 \\ -15 & -47 & 9 \end{bmatrix}$$

$$\text{eig}(A_{CL}) = \begin{cases} -2 \\ -1+j \\ -1-j \end{cases} \quad \checkmark$$

8.5 Can $g(s) = \frac{(s-1)(s+2)}{(s+1)(s-2)(s+3)}$ be transformed into $g_f(s) = \frac{s-1}{(s+1)(s+3)}$

using state feedback?

$\Rightarrow$ Yes, i) we place the poles (eigenvalues) at $-2, -2, -3$.

Is $g_F(s)$ BIBO stable?

$\Rightarrow$ Yes, since its poles are in the LHP.

Is the feedback system asymptotically stable?

$\Rightarrow$ Yes, because the closed-loop eigenvalues are in the LHP.

.6 Now, $g_f(s) = \frac{1}{s+3}$ and the same $g(s)$.

$\Rightarrow$ Can do it via state feedback, i) we place the poles at

$1, -2, -3$

$\Rightarrow$ BIBO stable, since $g_f(s)$ has pole at -3, in LHP.

$\Rightarrow$ <u>But</u>: system is <u>not</u> asymptotically stable, since

the closed-loop eigenvalue at 1 results in the

mode $e^t \to \infty$ as $t \to \infty$.