c. $k = 0.0025$

d. $k = 0.00025$

e. List some ways of controlling the growth rate.

1.3 PROJECTS

1. You plan to invest part of your paycheck to finance your children's education. You want to have enough in the account to draw \$1000 a month every month for 8 years beginning 20 years from now. The account pays 0.5% interest each month.

   a. How much money will you need 20 years from now to accomplish the financial objective? Assume you stop investing when your first child begins college—a safe assumption.

   b. How much must you deposit each month during the next 20 years?

2. Assume we are considering the survival of whales and that if the number of whales falls below a minimum survival level m, the species will become extinct. Assume also that the population is limited by the carrying capacity M of the environment. That is, if the whale population is above M, then it will experience a decline because the environment cannot sustain that large a population level. In the following model, a_n represents the whale population after n years. Build a numerical solution for $M = 5000$, $m = 100$, $k = 0.0001$, and $a_0 = 4000$.

$$a_{n+1} - a_n = k(M - a_n)(a_n - m)$$

   Now experiment with different values for M, m, and k. Try several starting values for a_0. What does your model predict?

3. *A Killer Virus*—You have volunteered for the Peace Corps and have been sent to Rwanda to help in humanitarian aid. You meet with the World Health Organization (WHO) and find out about a new killer virus, Hanta. If just one copy of the virus enters the human body, it can start reproducing very rapidly. In fact, the virus doubles its numbers in 1 hour. The human immune system can be quite effective, but this virus hides in normal cells. As a result, the human immune response does not begin until the virus has 1 million copies floating within the body. One of the first actions of the immune system is to raise the body temperature, which in turn lowers the virus replication rate to 150% per hour. The fever and then flu-like symptoms are usually the first indication of the illness. Some people with the virus assume that they have only a flu or a bad cold. This assumption leads to deadly consequences because the immune response alone is not enough to combat this deadly virus. At maximum reaction, the immune systems alone can kill only 200,000 copies of the virus per hour. Model this initial phase of the illness (before antibiotics) for a volunteer infected with 1 copy of the virus.

   a. How long will it take for the immune response to begin?

   b. If the number of copies of the virus reaches 1 billion, the virus cannot be stopped. Determine when this happens.

c. When the number of copies of the virus reaches 1 trillion, the person will die. Determine when this occurs.

To combat this virus fully, the infected person needs to receive an injection and hourly doses of an antibiotic. The antibiotic does not affect the replication rate of the virus (the fever keeps it at 150%), but the immune system and the antibiotics together kill 500,000,000 copies of the virus per hour.

d. Model the second phase of the virus (after the antibiotics are taken). Determine the latest time at which you can start administering the antibiotic in order to save the person. Analyze your model and discuss its strengths and weaknesses. (See enclosed CD for the UMAP module.)

4. *Mercury in Fish*—Public officials are worried about the elevated levels of toxic mercury pollution in the reservoirs that provide the drinking water to your city. They have asked for your assistance in analyzing the severity of the problem. Scientists have known about the adverse affects of mercury on the health of humans for more than a century. The term *mad as a hatter* stems from the nineteenth-century use of mercuric nitrate in the making of felt hats. Human activities are responsible for most mercury emitted into the environment. For example, mercury, a by-product of coal, comes from the smokestack emissions of old, coal-fired power plants in the Midwest and South and is disseminated by acid rain. Its particles rise on the smokestack plumes and hitch a ride on prevailing winds, which often blow northeast. After colliding with mountains, the particles drop to earth. Once in the ecosystem, microorganisms in the soil and reservoir sediment break down the mercury and produce a very toxic chemical known as methyl mercury.

Mercury undergoes a process known as bioaccumulation. This occurs when organisms take in contaminants more rapidly than their bodies can eliminate them. Therefore, the amount of mercury in their bodies accumulates over time. Humans can eliminate mercury from their system at a rate proportional to the amount remaining. Methyl mercury decays 50% every 65 to 75 days (known as the half-life of mercury) if no further mercury is ingested during that time.

Officials in your city have collected and tested 2425 samples of largemouth bass from the reservoirs and provided the following data. All fish were contaminated. The mean value of the methyl mercury in the fish samples was 0.43 μg (microgram) per gram. The average weight of the fish was 0.817 kg.

a. Assume the average adult person (70 kg) eats one fish (0.817 kg) per day. Construct a difference equation to model the accumulation of methyl mercury in the average adult. Assume the half-life is approximately 70 days. Use your model to determine the maximum amount of methyl mercury that the average adult human will accumulate in her or his lifetime.

b. You find out that there is a lethal limit to the amount of mercury in the body; it is 50 mg/kg. What is the maximum number of fish per month that can be eaten without exceeding this lethal limit?

5. Complete the UMAP module "Difference Equations with Applications," by Donald R. Sherbert, UMAP 322. This module presents a good introduction to solving first- and second-order linear difference equations, including the method of undetermined coefficients for nonhomogeneous equations. Applications to problems in population and economic modeling are included.