Q1

Equations of motion (from previous problem set solution):

$$\begin{cases} m_1\ddot{x}_1 + 7kx_1 - kx_2 - 5kx_3 = F_1 & \text{I} \\ m\ddot{x}_2 - kx_1 + 2kx_2 - kx_3 = F_2 & \text{II} \\ m\ddot{x}_3 - 5kx_1 - kx_2 + 7kx_3 = F_3 & \text{III} \end{cases}$$

First-order form:

$y_1 = x_1$ $\quad y_3 = x_2$ $\quad y_5 = x_3$

$y_2 = \dot{x}_1$ $\quad y_4 = \dot{x}_2$ $\quad y_6 = \dot{x}_3$

(1): $\dot{y}_1 = y_2$

(2): rewriting I: $m\dot{y}_2 + 7ky_1 - ky_3 - 5ky_5 = F_1$

$$\Rightarrow \dot{y}_2 = \frac{1}{m}\left[F_1 - 7ky_1 + ky_3 + 5ky_5\right]$$

(3): $\dot{y}_3 = y_4$

rewriting II: $m\dot{y}_4 - ky_1 + 2ky_3 - ky_5 = F_2$

$$\Rightarrow (4): \dot{y}_4 = \frac{1}{m}\left[F_2 + ky_1 - 2ky_3 + ky_5\right]$$

(5): $\dot{y}_5 = y_6$

rewriting III: $m\dot{y}_6 - 5ky_1 - ky_3 + 7ky_5 = F_3$

$$\Rightarrow (6): \dot{y}_6 = \frac{1}{m}\left[F_3 + 5ky_1 + ky_3 - 7ky_5\right]$$

Q2:

Equations of motion (from previous problem set solutions):

$$\begin{cases} m\ddot{x}_1 + 5kx_1 - kx_2 - 3kx_3 = 0 & \text{I} \\ 2m\ddot{x}_2 - kx_1 + 4kx_2 - kx_3 = 0 & \text{II} \\ m\ddot{x}_3 - 3kx_1 - kx_2 + 5kx_3 = 0 & \text{III} \end{cases}$$

First-order Form:

$y_1 = x_1$ $\quad y_3 = x_2$ $\quad y_5 = x_3$

$y_2 = \dot{x}_1$ $\quad y_4 = \dot{x}_2$ $\quad y_6 = \dot{x}_3$

(1) $\dot{y}_1 = y_2$

rewriting I: $m\dot{y}_2 + 5ky_1 - ky_3 - 3ky_5 = 0$

(2) $\Rightarrow \dot{y}_2 = \frac{1}{m}\left[-5ky_1 + ky_3 + 3ky_5\right]$

(3) $\dot{y}_3 = y_4$

rewriting II: $m\dot{y}_4 - ky_1 + 4ky_3 - ky_5 = 0$

$\Rightarrow$ (4): $\dot{y}_4 = \frac{1}{m}\left[ky_1 - 4ky_3 + ky_5\right]$

(5): $\dot{y}_5 = y_6$

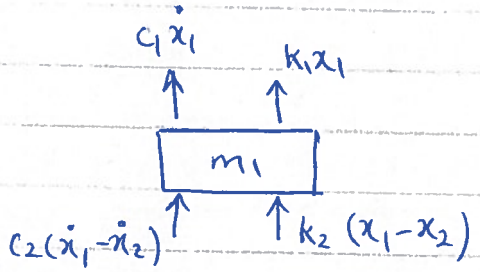
rewriting III: $m\dot{y}_6 - 3ky_1 - ky_3 + 5ky_5 = 0$

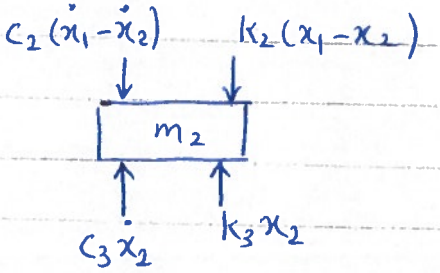
$\Rightarrow$ (6): $\dot{y}_6 = \frac{1}{m}\left[3ky_1 + ky_3 - 5ky_5\right]$

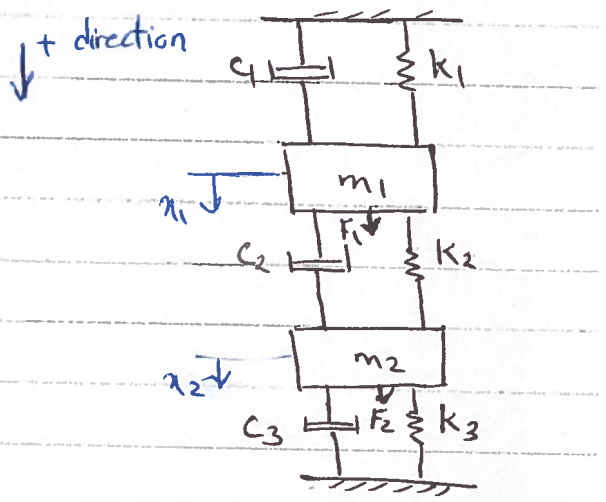
Q3 MATLAB Codes are uploaded

Q4- Page 1

(a)







$$\sum F = m_1\ddot{x}_1 \quad (1) \Rightarrow F_1 - c_1\dot{x}_1 - k_1x_1 - c_2(\dot{x}_1-\dot{x}_2) - k_2(x_1-x_2) = m_1\ddot{x}_1$$

$$\Rightarrow m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1-\dot{x}_2) + k_2(x_1-x_2) = F_1$$

$$\Rightarrow m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1+k_2)x_1 - k_2x_2 = F_1$$

$$\sum F = m_2\ddot{x}_2 \quad (2) \Rightarrow F_2 + c_2(\dot{x}_1-\dot{x}_2) + k_2(x_1-x_2) - c_3\dot{x}_2 - k_3x_2 = m_2\ddot{x}_2$$

$$\Rightarrow m_2\ddot{x}_2 - c_2\dot{x}_1 + (c_2+c_3)\dot{x}_2 - k_2x_1 + (k_2+k_3)x_2 = F_2$$

Matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2+c_3 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

(b) → $\omega_1 = 10\ \frac{rad}{s}$, $X^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ → MATLAB Codes are uploaded.

$\omega_2 = 30\ \frac{rad}{s}$, $X^{(2)} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

(c) $y_1 = x_1$

$y_2 = \dot{x}_1$

$y_3 = x_2$

$y_4 = \dot{x}_2$

$$\Rightarrow \dot{y}_1 = y_2$$

$$\dot{y}_2 = \frac{1}{m_1}\left[F_1 - (c_1+c_2)y_2 + c_2 y_4 - (k_1+k_2)y_1 + k_2 y_3\right]$$

$$\dot{y}_3 = y_4$$

$$\dot{y}_4 = \frac{1}{m_2}\left[F_2 + c_2 y_2 - (c_2+c_3)y_4 + k_2 y_1 - (k_2+k_3)y_3\right]$$

(d) & (e) → MATLAB Codes are uploaded.