

Problem Statement: We assume that the standard input contains a sequence of non-zero integers between - 121 and 121, which ends with 0. This sequence will be given by the user.

1. Write an algorithm, called *Decomposition_Powers_Three*, which produces the **decomposition** of each integer using **powers of 3**, namely 1, 3, 9, 27, and 81, and the + and – operators. Each power of 3 should appear **at most once** in the decomposition.

Examples:

$$1 = 1$$

$$2 = 3 - 1$$

$$3 = 3$$

$$4 = 3 + 1$$

$$7 = 9 - 3 + 1$$

$$14 = 27 - 9 - 3 - 1$$

$$43 = 81 - 27 - 9 - 3 + 1$$

$$121 = 81 + 27 + 9 + 3 + 1$$

2. Show that the algorithm *Decomposition_Powers_Three* is correct using an informal proof (i.e., discussion).
3. Give a program corresponding to *Decomposition_Powers_Three*, using any of your favorite programming languages.

Observation: The intervals [-121,-41], [-40,-14], [-13,-5], [-4,-2], [-1,-1], [1,1], [2,4], [5,13], [14,40], and [41,121] play a particular role.