

1. Suppose Mr. Smith claims that he can read your mind, in the sense that if you write down a whole number from 1-10 and keep it hidden from him, he claims he can tell you what the number was. Describe the experiment you would use to verify this claim, such that the only possible hypotheses are:
 H : "Mr. Smith can read your mind."
 C : "Mr. Smith is guessing numbers purely by chance."
Let D be the proposition "Mr. Smith correctly identifies n out of n numbers."
What value of n would make you uncertain about H (defined as having a posterior probability assignment on the order of $\frac{1}{2}$) after observing D ? What does this imply about your prior probability assignment for H ?
2. Suppose, with the same setup as above, that I claim to have performed your experiment exactly as you described it and that Mr. Smith did, in fact, correctly identify n out of n numbers, for the value of n you specified. Let L be the proposition "I am lying to you." What is your prior probability assignment for L , and what does this mean regarding the posterior probability you assign to H ?
3. The Sanhedrin (court of law in ancient Israel) consisted of 23 judges and had the following rule for determining guilt:

If, during a trial, a majority of the judges voted to convict, then the suspect was convicted, *except* if the decision to convict was *unanimous* (23-0), in which case the suspect was set free.

The reason for this exception was to reduce the possibility of the judges acting out of bias and unfairly ignoring arguments in defense of the accused.

Let B denote the hypothesis "The court is unfairly biased." and assume that if B is true all the judges will certainly vote to convict.

Assume that before a particular trial begins you assign equal probability to the suspect being guilty or innocent, and further assume that, if B is *not* true, you assign each judge independently a 75% probability

of correctly determining guilt or innocence, and that this suspect is unanimously found guilty. Assuming a prior probability for B equal to 1%, what is your posterior probability for B ?

4. The eighth planet in the Solar System, Neptune, was discovered in 1846 after astronomers had noticed deviations in the orbit of the neighboring planet, Uranus, away from its theoretically predicted path and speculated that the cause was the gravitational pull of a previously unobserved planet. However, they *could*, just as logically, have concluded that there were only *seven* planets and that Newton's theory of gravity was incorrect. What led them to reject one hypothesis instead of the other? Can you explain this situation in terms of prior and posterior probabilities? What further observations might have led them to reject Newton's laws instead?