8

ELECTROMAGNETIC WAVES

8.1. INTRODUCTION

Oersted, experimentally predicted that the electric current produces a magnetic field. Faraday from his experiments on electromagnetic induction found that a changing magnetic field produces an electric field. The question arises ; is the reverse true ? Can a changing electric field produce a magnetic field ? This answer was given by Clerk Maxwell, stating that the magnetic field is not only produced by electric current but also by a time varying electric field. Maxwell concluded his idea on the basis of his experiment, when he was finding the magnetic field at a point outside and inside a capacitor connected to a time-varying current, using Ampere's circuital law. He found that the results were not consistent. Maxwell pointed out that Ampere's circuital law becomes consistent only if along with conduction current, there is an additional current called by him, the **displacement current.**

Maxwell formulated a set of equations involving electric and magnetic fields and their sources ; the charge and current. These equations are known as **Maxwell's equations.** These equations along with Lorentz force formula explain mathematically all the basic laws of electromagnetism.

Maxwell through his equations came out with a most important prediction, *i.e.*, **the existence of electromagnetic waves,** which can propagate in space with a speed very close to the speed of light (3×10^8 ms^{-1}). This led Maxwell to conclude that light is an electromagnetic wave. In 1885 Hertz demonstrated experimentally, the existence of electromagnetic waves. Later on Marconi used these waves in establishing a communication between two locations at a distance of about 50 km, without using the wires between the two locations. This led to the discovery of **wireless communication** and ***Marconi was considered as a father of wireless communication*** Now-a-days the wireless communication has revolutionised our life.

In this unit, we shall study the production, properties and uses of electromagnetic waves.

8.2. DISPLACEMENT CURRENT

According to Ampere circuital law,

the line integral of magnetic field $\vec{B}$ around any closed path is equal to μ_0 times the total current threading the closed path, i.e.,

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DISPLACEMENT CURRENT

According to Ampere circuital law,

the line integral of magnetic field $\vec{B}$ ***around any closed path is equal to*** μ_0 ***times the total current threading the closed path, i.e.,***

$$\oint_C \vec{B}.d\vec{l} = \mu_0 I$$

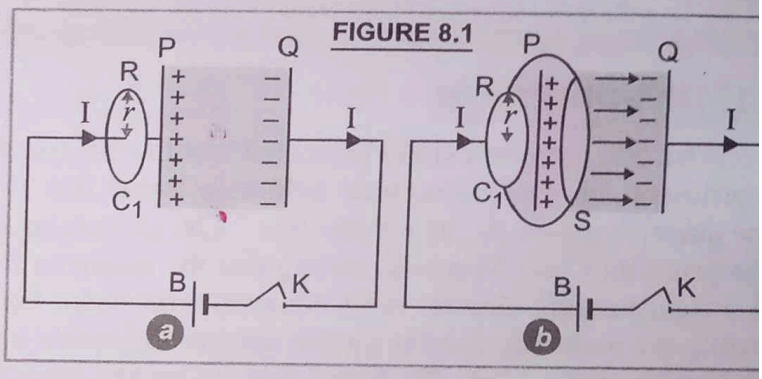
e I is the net current threading the surface bounded by a closed path C. Maxwell in 1864 argued th on (1) is logically inconsistent. He explained the same with the help of following observations.

Consider a parallel plate capacitor having plates P and Q connected to a battery B, through a tapp K. When key K is pressed, the conduction current flows through the connecting wires. The capac ts storing charge. As the charge on the capacitor grows, the conduction current in the wires decrea en the capacitor is fully charged, the conduction current stops flowing in the wires. During charging acitor, there is no conduction current between the plates of capacitor. During charging, let at an instar he conduction current in the wires. This current will produce magnetic field around the wires which detected by using a compass needle.

Let us find the magnetic field at point R which is at a perpendicular distance r from connecting wire gion outside the parallel plate capacitor. For this we consider a plane circular loop C_1, of radius r, wh tre lies on the wire and its plane is perpendicular to the direction of current carrying w g. 8.1(a)). The magnitude of the magnetic field is same at all points on the loop and is acting tangentia ng the circumference of the loop. If B is the magnitude of magnetic field at R, then using Amper uital law, for loop C_1, we have

$$\oint_{C_1} \vec{B}.d\vec{l} = \oint_{C_1} B\,dl\cos 0^\circ = B\,2\pi r = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r} \qquad \ldots$$

Now, we consider a different surface, *i.e.*, a n box shaped surface without lid with its circular which has the same boundary as that of loop The box does not touch to the connecting wire plate P of capacitor. The flat circular bottom S ne tiffin box lies in between the capacitor plates. 8.1(b). No conduction current is passing ugh the tiffin box surface S, therefore $I = 0$. On lying Ampere's circuital law to loop C_1 of this n box surface, we have



$$\oint_{C} \vec{B}.d\vec{l} = \mathrm{B}\,2\pi r = \mu_0 \times 0 = 0 \qquad \text{or} \qquad B = 0 \qquad \ldots(3$$

From (2) and (3), we note that there is a magnetic field at R calculated through one way and no magnetic fie calculated through another way. Since this contradiction arises from the use of Ampere's circuital law, hen **ere's circuital law is logically inconsistent.** Maxwell argued that the above inconsistency of Ampere ital law must be due to **something missing.** The missing term must be such that one gets the same magnet with whatsoever surface is used. This missing term must be related with a changing electric field which passe gh the surface S between the plates of capacitor, during charging.

If at the given instant of time, q is the charge on the plate of capacitor and A is the plate area of capacito agnitude of the electric field between the plates of capacitor is

$$E = \frac{q}{\epsilon_0 A}$$

This field is perpendicular to surface S. It has the same magnitude over the area A of the capacitor plate ecomes zero outside the capacitor.

The electric flux through the surface S is, $\phi_E = \vec{E}.\vec{A} = EA\cos 0^\circ = \frac{1}{\epsilon_0}\frac{q}{A} \times A = \frac{q}{\epsilon_0}$ ...(4

f $\frac{dq}{dt}$ is the rate of change of charge with time on the plate of the capacitor, then

$$\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{q}{\epsilon_0}\right) = \frac{1}{\epsilon_0}\frac{dq}{dt} \qquad \text{or} \qquad \frac{dq}{dt} = \epsilon_0 \frac{d\phi_E}{dt}$$

ere, $\frac{dq}{dt}$ = current through surface S corresponding to changing electric field = I_D, called **Maxwell's** ement current. Thus

***displacement current** is that current which comes into play in the region in which the electric field and hence the electric flux is changing with time.*

This displacement current is given by $$I_D = \epsilon_0 \frac{d\phi_E}{dt} \quad ...(5)$$

This displacement current is the missing term in Ampere's circuital law. Maxwell pointed out that for istency of Ampere's circuital law, there must be displacement current I_D $(= \epsilon_0 \, d\phi_E/dt)$ along with luction current I in the closed loop as $(I + I_D)$ has the property of continuity, although individually they not be continuous.

Maxwell modified Ampere's circuital law in order to make the same logically consistent. He stated ere's circuital law to the form,

$$\oint \vec{B}.d\vec{l} = \mu_0 (I + I_D) = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right) \quad ...(6)$$

This is called **Ampere Maxwell's Law.**

Important Properties of Displacement Current

1. Displacement current comes into play in a region or space where the electric field or electric flux i changing with time.
2. Displacement current does not exist when there is a steady condition in a region or where the electri field is not changing with time.
3. Displacement current does not flow through the conducting wires like conduction current, *i.e.* displacement current in conducting wire is zero.
4. The magnitude of displacement current in a region or space is equal to ϵ_0 times the rate of change o electric flux in that region or space, *i.e.*, $I_D = \epsilon_0 \frac{d\phi_E}{dt}$.
5. The displacement current incombination with conduction current satisfies the property of continuity
6. The displacement current produces magnetic field in a region, like conduction current flowing i wires produces in and around the wires.

CONTINUITY OF CURRENT

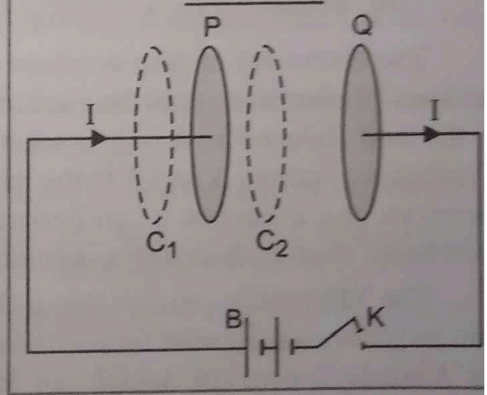
Maxwell's modification of Ampere's circuital law gives that $\oint_C \vec{B}.d\vec{l} = \mu_0 (I + I_D)$

e $I_D = \epsilon_0 \frac{d\phi_E}{dt}$, called displacement current, I is the conduction current and ϕ_E is the electric flu s the loop C.

The sum of the conduction current and displacement current (i.e., $I + I_D$) has the important propert ntinuity along any closed path although individually they may not be continuous.

To prove it, consider a parallel plate capacitor having plates P), being charged with battery B. During the time, charging is place, let at an instant, I be the conduction current flowing gh the wires. Let C_1 and C_2 be the two loops, which have exactly me boundary as that of the plates of capacitor. C_1 is little towards nd C_2 is a little towards right of the plate P of parallel plate tor, Fig. 8.2.

FIGURE 8.2



Due to battery B, let the conduction current I be flowing through d wires at any instant, but there is no conduction current across pacitor gap, as no charge is transported across this gap.

For loop C_1, there is no electric flux, *i.e.*, $\phi_E = 0$ and $\frac{d\phi_E}{dt} = 0$

$$I + I_D = I + \epsilon_0 \frac{d\phi_E}{dt} = I + \epsilon_0 (0) = I \quad ...(7$$

For loop C_2, there is no conduction current, *i.e.*, $I = 0$

$$\therefore \quad I + I_D = 0 + I_D = I_D = \epsilon_0 \frac{d\phi_E}{dt} \qquad ...(8)$$

At the given instant if q is the magnitude of charge on the plates of the capacitor of area A, then electric E in the gap between the two plates of this capacitor is given by

$$E = \frac{q}{\epsilon_0 A} \qquad \left(\because E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}\right)$$

$\therefore$ Electric flux, $$\phi_E = EA = \frac{q}{\epsilon_0 A} A = \frac{q}{\epsilon_0}$$

Thus from (8), we have $$I + I_D = \epsilon_0 \frac{d}{dt}\left(\frac{q}{\epsilon_0}\right) = \frac{dq}{dt} = I \qquad ...(9$$

From (7) and (9), we conclude that the sum $(I + I_D)$ has the same value on the left and right side of plat the parallel plate capacitor. Hence $(I + I_D)$ has the property of continuity although individually they ma be continuous.

It is to be noted that in any general medium, both conduction current and displacement current will b ent, giving rise to the total current. In a conducting medium, conduction current dominates ov lacement current, whereas in an insulating medium, displacement current dominates over the conductic ent. It is important to note that the displacement current has the same physical effects as the conductic ent. For example, during the charging of capacitor, if we measure the magnetic field at a point veen the two plates of capacitor, it is found to be the e as that just outside the capacitor at point R. The electric d $\vec{E}$ and magnetic field $\vec{B}$ between the plates of acitor at the point S are shown in Fig. 8.3(a). The direction $\vec{E}$ is from positive plate to the negative plate whereas direction of $\vec{B}$ at S is perpendicular to the plane of the er. The cross-sectional view of the electric and magnetic ls are shown in Fig. 8.3(b), where electric field is shown rosses and the magnetic field is shown by the tangents circle in the same plane.

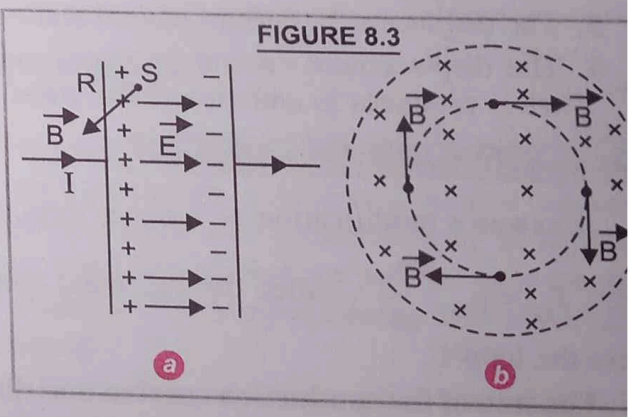


FIGURE 8.3

. CONSEQUENCES OF DISPLACEMENT CURRENT OR PREDICTION OF ELECTROMAGNETIC WAVE

The discovery of displacement current is of great importance as it has established a symmetry betw laws of electricity and magnetism. Faraday's law of electromagnetic induction states that the magnit ne emf induced in a coil is equal to the rate of change of magnetic flux linked with it. Since, the veen two points A and B is the measure of maximum workdone in taking a unit charge from point A t efore, the existence of an emf shows the existence of an electric field. It is due to this fact, ***Fara cluded that a changing magnetic field with time gives rise to an electric field.***

The Maxwell's concept that a changing electric field with time gives rise to displacement current wh produces a magnetic field similar to that of conduction current. It is infact, *a symmetrical counterpa Faraday's concept, which led Maxwell to conclude that the displacement current is also a sourc netic field.* It means ***the time varying electric and magnetic fields give rise to each other.*** From t cepts, Maxwell concluded **the existence of electromagnetic wave in a region where electric gnetic fields were changing with time.**

Sample Problem **How would you establish an intantaneous displacement current of 1·0 A i**
ıe space between the parallel plates of 1 μF capacitor.

Sol. $I_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right)$

or $I_D = \frac{\epsilon_0 A}{d} \times \frac{dV}{dt} = \frac{CdV}{dt}$ or $\frac{dV}{dt} = \frac{I_D}{C} = \frac{1{\cdot}0}{10^{-6}} = 10^6\ \text{Vs}^{-1}$

Thus an instantaneous displacement current of 1·0 A can be set up by changing the potential differen
ross the parallel plates of capacitor at the rate of $10^6\ \text{Vs}^{-1}$.

.5. MAXWELL'S EQUATIONS AND LORENTZ FORCE

In the absence of any dielectric or magnetic material, the four Maxwell's equations are given below

(*i*) $\oint_S \vec{E}.d\vec{s} = q/\epsilon_0$. This equation is **Gauss's Law in electrostatics.** This gives the value of to
ectric flux in terms of the charge enclosed by surface *S*. This equation gives an *electric field due to discr*
harge or due to certain charge distribution and tells that the electric lines of force start from some posit
harge and end at some other negative charge, *i.e.,* ***the electric lines of force do not form continuous clo***
uth. This law predicts that isolated charge exists.

(*ii*) $\oint_S \vec{B}.d\vec{s} = 0$. This equation is **Gauss's Law in magnetostatics,** where $\oint_S \vec{B}.d\vec{s}$ represents
rface integral of $\vec{B}$ over a closed surface (*i.e.*, the magnetic flux over the surface *S*). This law shows
ıe number of magnetic lines of force entering a closed surface is equal to number of magnetic lines of f
aving it. It means, there is no starting or end point of magnetic lines of force. Thus, the ***magnetic lines of f***
lways form closed paths. If it were not so, then the isolated magnetic monopoles would have existed. T
ıe Maxwell's equation (*ii*) confirms the ***absence of magnetic monopole.***

(*iii*) $\oint \vec{E}.d\vec{l} = -\frac{d}{dt}\int_S \vec{B}.d\vec{s}$. This equation is **Faraday's law of electromagnetic induction** wh

$\vec{B}.d\vec{s}$ represents the surface integral of $\vec{B}$ over a surface bounded by a closed curve over which the
tegral for electric field is taken. This law gives a relation between electric field and a changing mag
ux. This law shows that ***the line integral of electric field around any closed path (i.e. the emf) is equ***
e time rate of change of magnetic flux through the surface bounded by the closed path.

(*iv*) $\oint \vec{B}.d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt}\int_S \vec{E}.d\vec{s}$. This equation is generalised form of **Ampere's law as Mo**
Maxwell and is also known as ***Ampere-Maxwell law.*** This law shows that the line integral of the ma
eld around any closed path is related to the conduction current (*I*) and displacement cu
$_D = \epsilon_0 \frac{d}{dt}\int_s \vec{E}.d\vec{s}$) through that path. This law predicts that the magnetic field can be produce
nduction current as well as by displacement current.

(*v*) **Lorentz Force,** $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

The above five equations give a complete description of all electromagnetic interactions.

Importance of Maxwell's equations

On the basis of these equations, Maxwell predicted the following :

) An accelerated charge is the source of electromagnetic waves.

) The electromagnetic waves can propagate through the space with the speed of light ($= 3 \times 10^8$ m/s).

) The electromagnetic wave is of transverse nature.

) Light itself is an electromagnetic wave as it is transverse in nature and it moves in space with a spee $= 3 \times 10^8$ m/s.

. HISTORY OF ELECTROMAGNETIC WAVES

Faraday from his experimental study of electromagnetic induction concluded that a *magnetic fiel ıging with time at a region produces an electric field in that region.* Maxwell in 1865 from his theoretic y pointed out "**there is a great symmetry in nature**", *i.e., an electric field changing with time in on produces a magnetic field there.* It means a change in either field (electric or magnetic) with tim luces the other field. This idea led Maxwell to conclude that the variation in electric and magnetic fiel ors perpendicular to each other leads to the production of electromagnetic disturbances in space. Thes ırbances have the properties of wave and can travel in space even without any material medium. Thes es are called **electromagnetic waves.**

According to Maxwell,

> ***the electromagnetic waves** are those waves in which there are sinusoidal variation of electric and magnetic field vectors at right angles to each other as well as at right angles to the direction of wave propagation.*

Both these fields vary with time and space and have the same frequency. In Fig. 8.4., the electric fiel or $(\vec{E})$ and magnetic field vector $(\vec{B})$ are vibrating along Y and Z directions and propagation of electro netic wave is shown in X-direction.

Maxwell also found that the electromagnetic wave should travel in free space (or vacuum) with a speed n by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad ...(10)$$

where μ_0 and ϵ_0 are permeability and ittivity of the free space respectively.

We know, $\mu_0 = 4\pi \times 10^{-7}$ Wb A^{-1} m^{-1} ;

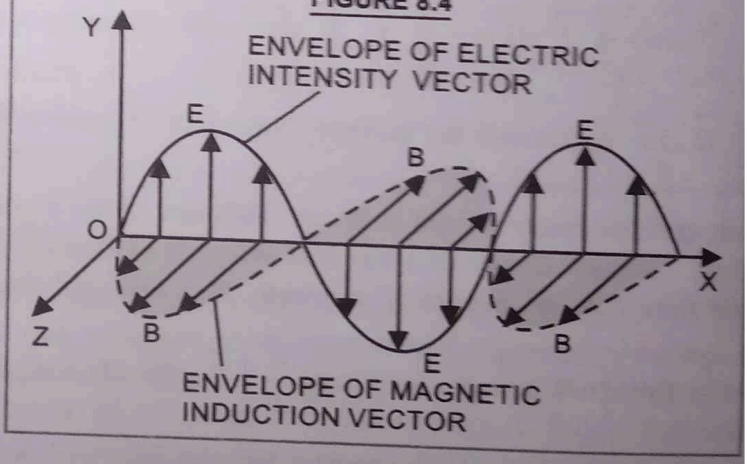
$\epsilon_0 = 8{\cdot}85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

Putting these values in (10), we have

$c = 3.00 \times 10^8$ ms^{-1}

The velocity of electromagnetic wave in a ım is given by $v = \frac{1}{\sqrt{\mu \epsilon}}$

FIGURE 8.4



where μ, ϵ are the absolute permeability and absolute permittivity of the medium. We also know that) μ_r and $\epsilon = \epsilon_0 \epsilon_r$ where μ_r, ϵ_r are the relative permeability and relative permittivity of the medium.

fore, $$v = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \qquad \left[\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}\right]$$

Maxwell also concluded that electromagnetic wave is transverse in nature and light is magnetic wave.

Examples of electromagnetic waves are radio waves, microwaves, infrared rays, light waves, ultraviole s, X-rays and γ-rays.

In 1888, **Hertz** demonstrated experimentally the production of electromagnetic waves using a spark illator and then detecting them successfully. The wavelength of electromagnetic waves produced with the p of Hertz experiment was 6 m. In recognition to the successful demonstration for the production o tromagnetic waves by Hertz, the *SI* unit of frequency was named as hertz (Hz).

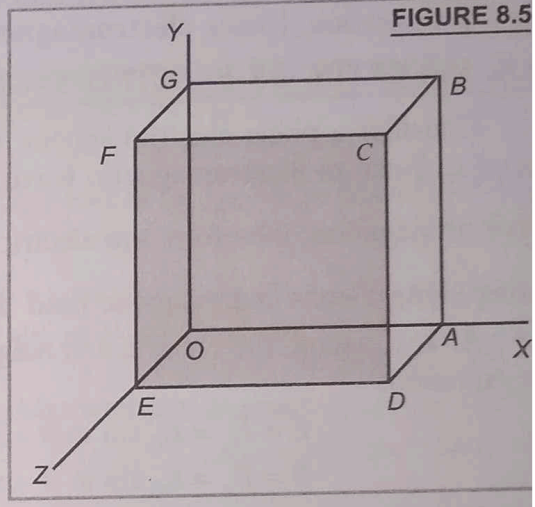
After 7 years in 1895, an Indian Physicist **Jagdish Chander Bose** was able to produce electromagneti ves of wavelength 5 mm to 25 mm. But his experiment was confined to laboratory only.

In 1896, **Guylielmo Marconi** discovered that if one of the spark gap terminals of Hertz oscillator i nected to an antenna and the other terminal is earthed, the electromagnetic waves produced can go up t eral kilometer. These waves could be detected at a large distance by making use of antenna-earth syster letector. Using these arrangements, in 1899, Marconi was the first to establish a wireless communicatio oss the English channel, a distance of about 50 km.

7. TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES

Consider a plane electromagnetic wave travelling in the irection and an elementary rectangular parallelopiped *BCDEFG* placed with its edges parallel to the three axes, 8.5. The wave front of electromagnetic wave is in the *YZ* ne and *ABCD* is a portion of it at time *t*. The values of electric magnetic fields will be zero to the right of the face *ABCD*, to the left of this face the values of the two fields depend on nd *t* but not on *y* and *z*, since we are considering a plane ve.

FIGURE 8.5



To establish transverse nature of electromagnetic waves, will establish that $\vec{E}$ and $\vec{B}$ are perpendicular to the ction of wave propagation.

If the rectangular parallelopiped does not enclose any charge, the total electric flux across it must b , according to Gauss's law, *i.e.*, $\oint \vec{E}.d\vec{s} = 0$

$$r\left(\int_{ABCD} \vec{E}.d\vec{s} + \int_{OGFE} \vec{E}.d\vec{s}\right) + \left(\int_{BGFC} \vec{E}.d\vec{s} + \int_{OEDA} \vec{E}.d\vec{s}\right) + \left(\int_{DCFE} \vec{E}.d\vec{s} + \int_{OABG} \vec{E}.d\vec{s}\right) = ($$

...(1

Since $\vec{E}$ does not depend on *y* and *z*, therefore, the contribution to the total electric flux coming fro aces normal to *y* and *z* axes will cancel out in pairs *i.e.*

$$\left(\int_{BGFC} \vec{E}.d\vec{s} + \int_{OEDA} \vec{E}.d\vec{s}\right) = 0 \qquad ...(1$$

and

$$\left(\int_{DCFE} \vec{E}.d\vec{s} + \int_{OABG} \vec{E}.d\vec{s}\right) = 0 \qquad ...(1$$

From equations (11), (12) and (13), we have

$$\int \vec{E}.d\vec{s} + \int \vec{E}.d\vec{s} = 0 \qquad ...(1$$

ABCD OGFE

If E_x and E_x' are the values of x-component of electric field on faces $ABCD$ and $OGFE$ respectively and be the area of each of these faces, then

$$\int_{ABCD} \vec{E}.d\vec{s} = E_x S \qquad \text{and} \qquad \int_{OGFE} \vec{E}.d\vec{s} = -E_x' S$$

The negative sign arises due to the fact that the area vector at the face $OGFE$ is in opposite direction to t of face $ABCD$.

From Eq. (14), we have $E_x S - E_x' S = 0$ or $(E_x - E_x') S = 0$

This relation is true only if either, $E_x - E_x' = 0$, *i.e.*, $E_x = E_x'$ or $E_x = E_x' = 0$

Since $S \neq 0$, the possibility, $E_x = E_x'$ predicts that the field is static. But a static field cannot propagate a ve of finite wavelength. Therefore, $E_x = E_x' = 0$, *i.e.*, no component of electric field is parallel to the ection of wave propagation. It means, ***the electric field is perpendicular to the direction of propagation electromagnetic wave.***

A similar argument holds true for the magnetic field. We thus conclude that ***in the propagation o tromagnetic waves, the electric and magnetic fields are transverse*** *i.e.* perpendicular to the direction o ve propagation. Hence **electromagnetic waves are transverse in nature.**

3. VELOCITY OF ELECTROMAGNETIC WAVES

Consider a plane electromagnetic wave propagating along positive direction of X-axis in space wit ed c. Since in electromagnetic wave, the electric and magnetic fields are transverse to the direction o ve propagation, therefore, the electric and magnetic fields are in Y–Z plane. Let the electric field $\vec{E}$ b ng along Y-axis and magnetic field $\vec{B}$ along Z-axis.

At any instant, the electric and magnetic fields varying sinusoidally with x and t can be represented b equations

$$E = E_y = E_0 \sin \omega (t - x/c) \quad ...(15$$

$$B = B_z = B_0 \sin \omega (t - x/c) \quad ...(16$$

Here E_0 and B_0 are the amplitudes of electric and magnetic s along Y-axis and Z-axis respectively. Consider a rectangular $PQRS$ in X-Y plane as shown in Fig. 8.6.

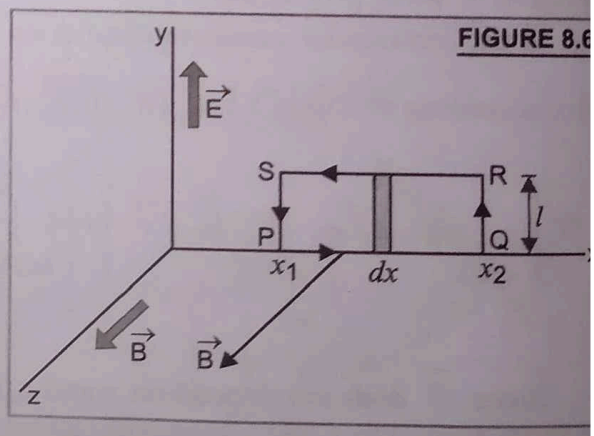


FIGURE 8.6

The line integral of $\vec{E}$ over the closed path $PQRS$ will be

$$\oint_{PQRS} \vec{E}.d\vec{l} = \int_P^Q \vec{E}.d\vec{l} + \int_Q^R \vec{E}.d\vec{l} + \int_R^S \vec{E}.d\vec{l} + \int_S^P \vec{E}.d\vec{l}$$

$$= 0 + E_{(x_2)}\, l + 0 + E_{(x_1)}\, (-l)$$

$$= E_0\, l \left[\sin \omega \left(t - \frac{x_2}{c} \right) - \sin \omega \left(t - \frac{x_1}{c} \right) \right] \quad ...(1$$

Magnetic flux linked with surface surrounded by rectangular path $PQRS$ will be

$$\phi_B = \int_{x_1}^{x_2} B(x)\, l\, dx = \int_{x_1}^{x_2} B_0\, l \left[\sin \omega \left(t - \frac{x}{c} \right) \right] dx = \frac{B_0\, lc}{\omega} \left[\cos \omega \left(t - \frac{x_2}{c} \right) - \cos \omega \left(t - \frac{x_1}{c} \right) \right]$$

$$\frac{d\phi_B}{dt} = \frac{B_0\, lc}{\omega} \left[-\omega \sin \omega \left(t - \frac{x_2}{c} \right) + \omega \sin \omega \left(t - \frac{x_1}{c} \right) \right] = -B_0\, lc \left[\sin \omega \left(t - \frac{x_2}{c} \right) - \sin \omega \left(t - \frac{x_1}{c} \right) \right]$$

...(1

Using Faraday's law of electromagnetic induction, we have

$$\oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$$

Putting the values from (17) and (18), we get

$$E_0 = c\, B_0 \qquad \text{...(19)}$$

Since E and B are in phase, we can write

$E = c\, B$ at any point in space.

Consider a rectangular path $PUTQ$ in the X-Z plane as n in Fig. 8.7. The line integral of $\vec{B}$ over the closed path Q, we have

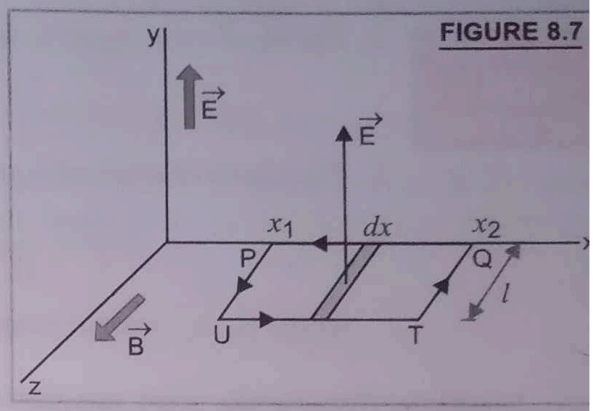


FIGURE 8.7

$$\oint_{PUTQ} \vec{B}.\vec{dl} = \int_P^U \vec{B}.\vec{dl} + \int_U^T \vec{B}.\vec{dl} + \int_T^Q \vec{B}.\vec{dl} + \int_Q^P \vec{B}.\vec{dl} = B_{(x_1)}\, l + 0 - B_{(x_2)}\, l + 0$$

$$= B_0\, l\left[\sin\omega\left(t - \frac{x_1}{c}\right) - \sin\omega\left(t - \frac{x_2}{c}\right)\right] \qquad \text{...(20}$$

The electric flux linked with the surface surrounded by rectangular path $PUTQ$ is

$$\phi_E = \int_{x_1}^{x_2} \vec{E}.\vec{ds} = \int_{x_1}^{x_2} E(x)\, l\, dx = E_0\, l \int_{x_1}^{x_2} \sin\omega\left(t - \frac{x}{c}\right) dx = -\frac{c}{\omega} E_0\, l\left[-\cos\omega\left(t - \frac{x_2}{c}\right) + \cos\omega\left(t - \frac{x_1}{c}\right)\right.$$

or $$\frac{d\phi_E}{dt} = -c\, E_0\, l\left[\sin\omega\left(t - \frac{x_2}{c}\right) - \sin\omega\left(t - \frac{x_1}{c}\right)\right] = c\, E_0\, l\left[\sin\omega\left(t - \frac{x_1}{c}\right) - \sin\omega\left(t - \frac{x_2}{c}\right)\right] \qquad \text{...(2}$$

In space, there is no conduction current. According to Ampere Maxwell law in space

$$\oint_{PUTQ} \vec{B}.\vec{dl} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}.$$

Putting values from (20) and (21), we get

$$B_0 = \mu_0 \epsilon_0\, c\, E_0 = \mu_0 \epsilon_0\, c\, (c\, B_0) \quad \text{or} \quad 1 = \mu_0 \epsilon_0\, c^2 \quad \text{or} \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad \text{...(2}$$

Which is the speed of electromagnetic waves in vacuum.

For vacuum, $\mu_0 = 4\pi \times 10^{-7}\ \text{T mA}^{-1}$

and $$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \text{Nm}^2\,\text{C}^{-2} \quad \text{or} \quad \epsilon_0 = \frac{1}{4\pi \times 9 \times 10^9}\ \text{N}^{-1}\,\text{m}^{-2}\,\text{C}^2$$

Putting the values in (22), we get $$c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 1/(4\pi \times 9 \times 10^9)}} = 3 \times 10^8\ \text{m/s}$$

which is exactly the speed of light in vacuum.

This shows that ***light is an electromagnetic wave.***

ETAIN IN MORY

1. The speed (c') of electromagnetic wave in a dielectric of dielectric constant K is given b

$$c' = \frac{1}{\sqrt{K\mu_0\epsilon_0}} = \frac{c}{\sqrt{K}}$$

TAIN IN MORY

2. For an electromagnetic waves, the magnitude of the propagation vector $(\vec{k})$ is,

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

3. The speed of electromagnetic wave through a medium of permeability μ and permittivity ϵ is,

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \qquad \left[\because \mu = \mu_0 \mu_r \text{ and } \epsilon = \epsilon_0 \epsilon_r\right]$$

where μ_r, ϵ_r are the relative permeability and relative permittivity of medium.

Sample Problem **The magnetic field in a plane electromagnetic wave is given by $2 \times 10^{-7} \sin (0{\cdot}5 \times 10^3 x + 1{\cdot}5 \times 10^{11} t)$ T. (a) What is the wavelength and frequency of the wave ? Write an expression for the electric field.**

NCERT Solved Example

Sol. *(a)* Comparing the given equation with the equation of magnetic field varying sinusoidally with x as

$$B_y = B_0 \sin \omega\left(t + \frac{x}{c}\right) = B_0 \sin \frac{2\pi}{T}\left(t + \frac{x}{c}\right) = B_0 \sin\left(\frac{2\pi}{T} t + \frac{2\pi}{T}\frac{x}{c}\right) = B_0 \sin\left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T}\right)$$

we get, $\frac{2\pi}{\lambda} = 0{\cdot}5 \times 10^3$ or $\lambda = \frac{2\pi}{0{\cdot}5 \times 10^3} m = \mathbf{1{\cdot}26 \times 10^{-2}\, m}$ and $\frac{2\pi}{T} = 1{\cdot}5 \times 10^{11}$

or $2\pi \nu = 1{\cdot}5 \times 10^{11}$ or $\nu = \frac{1{\cdot}5 \times 10^{11}}{2\pi} = \mathbf{2{\cdot}39 \times 10^{10}\ Hz}$

(b) $B_0 = 2 \times 10^{-7}$ T, $E_0 = c\, B_0 = (3 \times 10^8) \times 2 \times 10^{-7} = 60\ \text{Vm}^{-1}$

In electromagnetic wave, the electric field is varying sinusoidally with x and t, perpendicular to the direction of magnetic field as well as the direction of propagation of electromagnetic wave. So electric field varying along z-direction. Thus, $E_z = E_0 \sin\left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T}\right) = \mathbf{60 \sin (0{\cdot}5 \times 10^3 x + 1{\cdot}5 \times 10^{11} t)\ V/m}$

PRODUCTION OF ELECTROMAGNETIC WAVES

We know that an electric charge at rest has electric field in the region around it, but no magnetic field. Moving charge produces both the electric and magnetic fields. If a charge is moving with a constant velocity (if current is not changing with time), the electric and magnetic fields will not change with time, hence no electromagnetic wave can be produced. But if the charge is moving with a non-zero acceleration (*i.e., charge accelerated*) both the magnetic field and electric field will change with space and time, it then produces electromagnetic wave. This shows that an **accelerated charge emits electromagnetic waves.**

We have already explained in unit 4, chapter 7 that *L–C* circuit is an oscillatory circuit where the charge oscillating across the capacitor plates. An oscillating charge in *L–C* circuit has a non-zero acceleration hence it emits electromagnetic waves which have the same frequency as that of the oscillating charge.

In an atom, an electron while orbiting around the nucleus in a stable orbit, although accelerating, does not emit electromagnetic waves. Electromagnetic waves are emitted only when it falls from higher energy orbit to lower energy orbit.

Electromagnetic waves (*i.e.* X-rays) are also produced when fast moving electrons are suddenly stopped by a metal target of high atomic number.

IMPORTANT CHARACTERISTICS AND PROPERTIES OF THE ELECTROMAGNETIC WAVES

1. The electromagnetic waves are produced by accelerated or oscillating charge.
2. These waves do not require any material medium for propagation.