Chapter 10

Problems

10.1 Suppose you estimate the following distributed lag model with the independent variable lagged three times:

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{1,t-1} + \beta_3 x_{1,t-2} + \beta_4 x_{1,t-3} + \varepsilon_t$$

a. Assume that there is a random shock of size c to $x_{1,t}$, such that the shock was temporary (affected $x_{1,t}$ and then went away). Assume that no other shocks occur. What affect will this have on the y_t variable in the current period? Next period? Two periods from now? Three periods from now? Four periods from now? Five periods from now?

b. Now assume that the shock is permanent. How does this affect y_t in the current period? Next period? Two periods from now? Three periods for now? Four periods from now? Five periods from now?

10.2 How do the time-series assumptions differ from the multiple linear-regression assumptions? Why are they different?

10.3 Suppose you thought your data have a trend.

a. Describe how you would correct for it within the regression equation.

b. What transformation would you apply to the data to account for the trend?

10.4 What is seasonality? If you had monthly data and only a dependent variable, how would your correct for it? Section 10.5 explains how to de-trend data. How would you recommend you de-seasonalize data? Your answer should include a step-by-step explanation.

10.5 Suppose you thought your data had a structural break at a specific time period. What implications would this have on the coefficient estimates? How would you correct for the structural break?

10.6 Explain how a regression of y_t on x_t could result in a statistically significant relationship between the two variables when they are not actually related. What is this type of relationship called?

10.7 What is out-of-sample prediction? Say you had 500 time-series observations and would like to determine how well your model is performing. How would you suggest obtaining an out-of-sample prediction?

Exercises

E10.1 Use the data set for this problem. These data contain the number of shipments sent out for a shipping company each month and the number of calls they get each month.

a. Create two time-series graphs with shipments and calls as well as a scatterplot with shipments on the y-axis.

b. Estimate the static regression model of shipments against calls.

c. Create lagged version of calls with calls lagged one, two, and three periods.

d. Estimate distributed lagged model by regressing shipping on calls, calls lagged once, calls lagged twice, and calls lagged three times.

e. Compare and contrast the results from parts (b) and (d). Which model do you prefer?

E10.2 The file has monthly beer sales for a beer manufacturer over a number of years.

a. Graph beer sales over time. Does it look like the series has a time trend or seasonality associated with it?

b. Run a regression that accounts for a time trend, and statistically test whether the time trend is statistically significant at a 5 percent level. What do you conclude now about whether these data have a time trend?

c. Run a regression that accounts for seasonality in these data, keeping in mind that these data are monthly. Statistically test if there is seasonality in these data at a 5 percent level. What do you conclude about if these data have seasonality?

Essential Chapter 10

10.6. What are the various methods of detecting autocorrelation? State clearly the assumptions underlying each method.

10.7. Although popularly used, what are some limitations of the Durbin-Watson d statistic?

10.8. State whether the following statements are true or false. Briefly justify your answers.

a. When autocorrelation is present, OLS estimators are biased as well as inefficient.

b. The Durbin-Watson d is useless in autoregressive models like the regression (10.7) where one of the explanatory variables is a lagged value(s) of the dependent variable.

c. The Durbin-Watson d test assumes that the variance of the error term u_t is homoscedastic.

d. The first difference transformation to eliminate autocorrelation assumes that the coefficient of autocorrelation ρ must be -1.

e. The R^2 values of two models, one involving regression in the first difference form and another in the level form, are not directly comparable.

10.9. What is the importance of the Prais-Winsten transformation?

PROBLEMS

10.10. Complete the following table:

Sample size	Number of explanatory variables	Durbin-Watson d	Evidence of autocorrelation
25	2	0.83	Yes
30	5	1.24	—
50	8	1.98	—
60	6	3.72	—
200	20	1.61	—

10.11. Use the runs test to test for autocorrelation in the following cases. (Use the Swed-Eisenhart tables. See Appendix 10A.)

Sample size	Number of +	Number of −	Number of runs	Autocorrelation (?)
18	11	7	2	—
30	15	15	24	—
38	20	18	6	—
15	8	7	4	—
10	5	5	1	—

10.12. For the Phillips curve regression Equation (5.29) given in Chapter 5, the estimated d statistic would be 0.6394.

a. Is there evidence of first-order autocorrelation in the residuals? If so, is it positive or negative?

b. If there is autocorrelation, estimate the coefficient of autocorrelation from the *d* statistic.

c. Using this estimate, transform the data given in Table 5-6 and estimate the generalized difference equation (10.15) (i.e., apply OLS to the transformed data).

d. Is there autocorrelation in the regression estimated in part (*c*)? Which test do you use?

10.13. In studying the movement in the production workers' share in value added (i.e., labor's share) in manufacturing industries, the following regression results were obtained based on the U.S. data for the years 1949 to 1964[20] (t ratios in parentheses):

$$\text{Model A: } \hat{Y}_t = 0.4529 - 0.0041t; \; r^2 = 0.5284; \; d = 0.8252$$
$$t = \qquad (-3.9608)$$

$$\text{Model B: } \hat{Y}_t = 0.4786 - 0.00127t + 0.0005t^2; \; R^2 = 0.6629; \; d = 1.82$$
$$t = \qquad (-3.2724) \quad (2.7777)$$

where Y = labor's share and t = the time.

a. Is there serial correlation in Model A? In Model B?

b. If there is serial correlation in Model A but not in Model B, what accounts for the serial correlation in the former?

c. What does this example tell us about the usefulness of the *d* statistic in detecting autocorrelation?

10.14. *Durbin's two-step method of estimating* ρ.[21] Write the generalized difference equation (10.14) in a slightly different but equivalent form as follows:

$$Y_t = B_1(1 - \rho) + B_2X_t - \rho B_2X_{t-1} + \rho Y_{t-1} + v_t$$

In *step 1* Durbin suggests estimating this regression with Y as the dependent variable and X_t, X_{t-1}, and Y_{t-1} as explanatory variables. The coefficient of Y_{t-1} will provide an estimate of ρ. The ρ thus estimated is a *consistent* estimator; that is, in large samples it provides a good estimate of true ρ.

In *step 2* use the ρ estimated from step 1 to transform the data to estimate the generalized difference equation (10.14).

Apply Durbin's two-step method to the U.S. import expenditure data discussed in Chapter 7 and compare your results with those shown for the original regression.

10.15. Consider the following regression model:[22]

$$\hat{Y}_t = -49.4664 + 0.88544X_{2t} + 0.09253X_{3t}; \qquad R^2 = 0.9979; d = 0.8755$$
$$t = (-2.2392) \quad (70.2936) \quad (2.6933)$$

[20] See Damodar N. Gujarati, "Labor's Share in Manufacturing Industries," *Industrial and Labor Relations Review*, vol. 23, no. 1, October 1969, pp. 65–75.

[21] *Royal Statistical Society*, series B, vol. 22, 1960, pp. 139–153.

[22] See Dominick Salvatore, *Managerial Economics*, McGraw-Hill, New York, 1989, pp. 138, 148.

where Y = the personal consumption expenditure (1982 billions of dollars)
X_2 = the personal disposable income (1982 billions of dollars) (PDI)
X_3 = the Dow Jones Industrial Average Stock Index

The regression is based on U.S. data from 1961 to 1985.

a. Is there first-order autocorrelation in the residuals of this regression? How do you know?

b. Using the Durbin two-step procedure, the preceding regression was transformed per Eq. (10.15), yielding the following results:

$$Y_t^* = -17.97 + 0.89X_{2t}^* + 0.09X_{3t}^*; \qquad R^2 = 0.9816;\ d = 2.28$$

$$t = \qquad (30.72) \qquad (2.66)$$

Has the problem of autocorrelation been resolved? How do you know?

c. Comparing the original and transformed regressions, the t value of the PDI has dropped dramatically. What does this suggest?

d. Is the d value from the transformed regression of any value in determining the presence, or lack thereof, of autocorrelation in the transformed data?

10.16. *Durbin h statistic.* In autoregressive models like Eq. (10.7):

$$Y_t = B_1 + B_2X_t + B_3Y_{t-1} + v_t$$

the usual d statistic is not applicable to detect autocorrelation. For such models, Durbin has suggested replacing the d statistic by the h statistic defined as

$$h \approx \hat{\rho}\sqrt{\frac{n}{1 - n \cdot \text{var}(b_3)}}$$

where n = the sample size
$\hat{\rho}$ = the estimator of the autocorrelation coefficient ρ
var (b_3) = the variance of the estimator of B_3, the coefficient of lagged Y variable

Durbin has shown that for *large samples,* and given the null hypothesis that true $\rho = 0$, the h statistic is distributed as

$$h \sim N(0, 1)$$

It follows the standard normal distribution, that is, normal distribution with zero mean and unit variance. Therefore, we would reject the null hypothesis that $\rho = 0$ if the computed h statistic exceeds the critical h value. If, e.g., the level of significance is 5%, the critical h value is -1.96 or 1.96. Therefore, if a computed h exceeds $|1.96|$, we can reject the null hypothesis; if it does not exceed this critical value, we do not reject the null hypothesis of no (first-order) autocorrelation. Incidentally, $\hat{\rho}$ entering the h formula can be obtained from any one of the methods discussed in the text.

Now consider the following demand for money function for India for the periods 1948 to 1949 and 1964 to 1965:

$$\widehat{\ln M_t} = 1.6027 - 0.1024 \ln R_t + 0.6869 \ln Y_t + 0.5284 \ln M_{t-1}$$
$$\text{se} = (1.2404) \quad (0.3678) \quad (0.3427) \quad (0.2007) \quad R^2 = 0.9227$$
$$d = 1.8624$$

where M = real cash balances
R = the long-term interest rate
Y = the aggregate real national income

a. For this regression, find the h statistic and test the hypothesis that the preceding regression does not suffer from first-order autocorrelation.
b. As the regression results show, the Durbin-Watson d statistic is 1.8624. Tell why in this case it is inappropriate to use the d statistic. But note that you can use this d value to estimate $\rho(\hat{\rho} \approx 1 - d/2)$.

10.17. Consider the data given in Table 10-7 (on the textbook's Web site) relating to stock prices and GDP for the period 1980–2006.

a. Estimate the OLS regression

$$Y_t = B_1 + B_2X_t + u_t$$

b. Find out if there is first-order autocorrelation in the data on the basis of the d statistic.
c. If there is, use the d value to estimate the autocorrelation parameter ρ.
d. Using this estimate of ρ, transform the data per the generalized difference equation (10.14), and estimate this equation by OLS (1) by dropping the first observation and (2) by including the first observation.
e. Repeat part (*d*), but estimate ρ from the residuals as shown in Eq. (10.20). Using this estimate of ρ, estimate the generalized difference equation (10.14).
f. Use the first difference method to transform the model into Eq. (10.17) and estimate the transformed model.
g. Compare the results of regressions obtained in parts (*d*), (*e*), and (*f*). What conclusions can you draw? Is there autocorrelation in the transformed regressions? How do you know?

10.18. Consider the following model:

$$Y_t = B_1 + B_2X_{2t} + B_3X_{3t} + B_4X_{4t} + u_t$$

Suppose the error term follows the AR(1) scheme in Eq. (10.6). How would you transform this model so that there is no autocorrelation in the transformed model? (*Hint:* Extend Eq. [10.15].)

10.19. Establish Eq. (10.8). (*Hint:* Expand Eq. [10.5] and use Eq. [10.9]. Also, note that for a large sample size Σe_{t-1}^2 and Σe_t^2 are approximately the same.)

10.20. *The Theil-Nagar* ρ *based on d statistic.* Theil and Nagar have suggested that in small samples instead of estimating ρ as $(1 - d/2)$, it should be estimated as

$$\hat{\rho} = \frac{n^2(1 - d/2) + k^2}{n^2 - k^2}$$

where n = the sample size
d = the Durbin-Watson d
k = the number of coefficients (including the intercept) to be estimated

Show that for large n, this estimate of ρ is equal to the one obtained by the simpler formula $(1 - d/2)$.

10.21. Refer to Example 7.3 relating expenditure on imports (Y) to personal disposable income (X). Now consider the following models:

	Model 1	Model 2	Model 3
Intercept	−136.16	22.69	12.18
X	0.2082	0.2975	0.0382
Time	—	−18.525	−3.045
$Y(-1)$	—	—	0.9659
R^2	0.969	0.984	0.994
d	0.216	0.341	1.611

a. What do these results suggest about the nature of autocorrelation in this example?

b. How would you interpret the time and lagged Y terms in Model 3?
Note: The estimated coefficients in all the models, except for the X and Time coefficients in Model 3, were statistically significant at the 5% or lower level of significance.

10.22. *Monte Carlo experiment.* Consider the following model:

$$Y_t = 1.0 + 0.9X_t + u_t \tag{1}$$

where X takes values of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Assume that

$$\begin{aligned} u_t &= \rho u_{t-1} + v_t \\ &= 0.9u_{t-1} + v_t \end{aligned} \tag{2}$$

where $v_t \sim N(0, 1)$. Assume that $u_0 = 0$.

a. Generate 10 values of v_t and then 10 values of u_t per Equation (2).

b. Using the 10 X values and the 10 u values generated in the preceding step, generate 10 values of Y.

c. Regress the Y values generated in part (*b*) on the 10 X values, obtaining b_1 and b_2.

d. How do the computed b_1 and b_2 compare with the true values of 1 and 0.9, respectively?

e. What can you conclude from this experiment?

10.23. Continue with Problem 10.22. Now assume that $\rho = 0.1$ and repeat the exercise. What do you observe? What general conclusion can you draw from Problems 10.22 and 10.23?