

1) Derive the 2D Laplacian operator in polar coordinates using the following relationships:
 $x = R \cos \theta$ and $y = R \sin \theta$.

2) Solve Laplace's equation in polar coordinates for the equilibrium temperature $T(r, \theta)$ in a circular disk of radius $r = 1$ with $T(1, \theta) = f(\theta)$, which denotes a prescribed temperature on the boundary. (Use separation of variables).

3) Complete Trim pg 174 #2

4) Complete Trim pg 40, #2-4.

On the Region what form (a) Dirichlet; (b) Neumann and (c) Robin boundary conditions take for the PDE?

$$2. \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + V = F(x, y, z) \quad 0 < x < L; y > 0; z > 0$$

$$3. \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} = F(r, \theta) \quad 0 < r < r_0; -\pi < \theta < \pi$$

$$4. \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} = F(r, \theta) \quad 0 < r < r_0; 0 < \theta < \pi$$

A cylindrical, homogeneous isotropic rod with insulated sides has Temperature $f(x)$, $0 \leq x \leq L$; at time $t=0$. For time $t>0$; its ends (at $x=0$ and $x=L$) are held at temperature 0°C . Find a Formula for the Temperature $u(x, t)$ in the rod for $0 < x < L$ and $t > 0$.