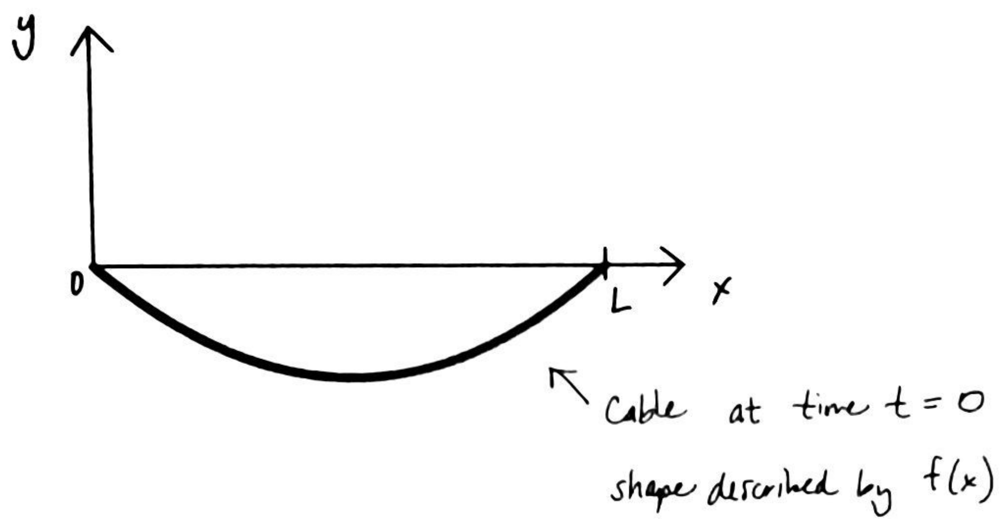
1. (10 points) Derive the partial differential equation that describes the deflection of a long flexible cable of length L with both ends fixed where the cable is subject to an initial sag due to its weight as illustrated below.



a. List the assumptions and idealizations you will use to derive the PDE.
b. List each step of your derivation and how you have applied the assumptions and idealizations from a.
c. Classify the PDE (including if it is elliptic, hyperbolic, or parabolic).

2. (10 points) The cable shown above is subjected to a small disturbance producing motion in the y-direction, changing from its initially deformed state f(x). (10 points)
   a. What do you expect to physically occur in this system upon application of the disturbance? Why?
   b. Is this a well-posed problem? Why or why not? If it is not, write conditions to make it a well-posed problem.
   c. Generate a complete solution of the well-posed problem.
   d. Does the mathematical formulation generated for your solution match your physical expectations? Please explain what you see in the solution and how it connects to or does not connect to the physical expectations.

3. (5 points) The flexible cable from question #1 has the following physical characteristics:
   - Length of 20 m
   - Diameter of 1 cm
   - Mass Density of 3.1 g/cm^3

   Its tension when hanging is 1000 N and its initial shape can be described by the function $f(x) = 0.01\,x\,(1 - \frac{x}{5})$

   a) Write the numerical form of the differential equation that describes motion due to an instantaneous vertical disturbance.
   b) Write out the numerical form of the initial and boundary conditions.
   c) Provide the solution to the problem including numerical form for the constants.

4. (5 points) An upright freezer that was set to 0°C is opened on a hot day in which the lab temperature is 30°C. The freezer holds the sides and the back of the compartment at the set temperature. You may assume that there is no variation in temperature in the vertical direction (variation may exist in x and y, but not in z)
   a) Draw a diagram depicted the scenario described.
   b) Which partial differential equation could you use to describe the change in temperature over time?
   c) Write out initial conditions and boundary conditions (either given or assumed) to make this a well-posed problem.
   d) What do you expect to happen in this system as described?
   e) Classify the PDE (including if it is elliptic, hyperbolic, or parabolic).

5. (10 points): Assume the freezer from question #4 has been left open for a very long time.

a) How would the differential equation change?
b) What would you expect to be the physical state of the freezer?
c) If the depth of the freezer is 50 cm (from back to front) and the width of the freezer is 60 cm, find the temperature at the center of the freezer.
d) How does the equation derived for your solution match your physical description in part b?

6. (5 points): Using the PDE from question 5:

a) What type of boundary conditions are given in the problem?
b) **Write boundary conditions of each type in place of what is given. For each type, what would this boundary condition physically represent?**

7. (5 points): Answer the following questions.

a) What is a Fourier series? What are the restrictions for use of a Fourier series?
b) How is the Fourier series related to the Fourier transform? What is different between them?
c) Explain how the properties of function and vector spaces enable the use of Fourier transforms?