***Show all your works including the formulas/equations to get full points**

**1.** (10 points) Identify the hyperbolic, Parabolic, and elliptic PDEs for the following equations, then name them.

| Equation | Type | Name |
|---|---|---|
| $\nabla^2 u = 0$ | | |
| $u_t = \alpha^2 u_{xx} - hu_x$ | | |
| $\nabla^2 u + \lambda^2 u = 0$ | | |
| $u_{tt} = c^2 \nabla^2 u - hu_t$ | | |
| $u_t = \alpha^2 u_{xx} - hu$ | | |
| $\nabla^2 u + k(E - V)u = 0$ | | |
| $u_{tt} = c^2 u_{xx} - hu_t - ku$ | | |
| $\nabla^2 u = k$ | | |
| $u_{tt} = c^2 u_{xx} - hu_t$ | | |
| $u_t = \alpha^2 u_{xx} - f(x,t)$ | | |

**2.** (5 points) Identify the order of the following PDE equations.

a). $u_t = u_{xx}$

b). $u_t = uu_{xxx} + \sin x$

c). $u_t = u_x$

**3.** (5 points) Identify the linear and nonlinear PDE equations.

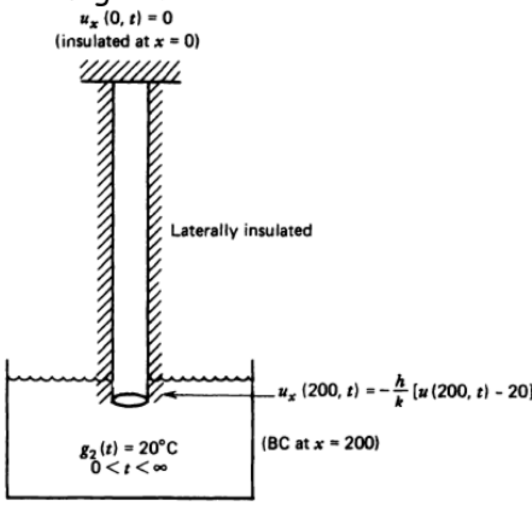
a). $u_t + uu_{xx} = 0$

b). $u_{tt} = uu_{xxx} + \sin t$

c). $xu_x + yu_y + u^2 = 0$

d). $u_{xx} + yu_{yy} = 0$

**4.** (20 points) Suppose a copper rod 200 cm long that is laterally insulted and has an initial temperature of 0° C. Suppose the top of the rod (x = 0) is insulated, while the bottom ( x = 220) is immersed in moving water that has a constant temperature of $g_2(t) = 20^o C$ as shown in the below Figure.



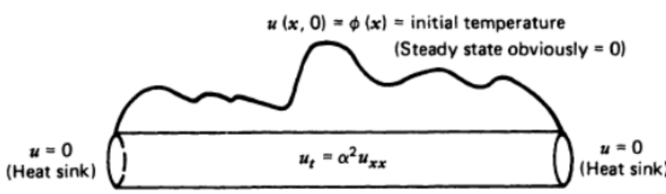

Find the four equations for this mathematical model?

**5.** (20 points) Consider IBVP (diffusion problem) of a finite rod where temperature at the ends is fixed at zero as in the below Figure with following equations.

PDE: $u_t = \alpha^2 u_{xx} \quad , \quad 0 < x < 1, \; 0 < t < \infty$

BCs: $\begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases} \quad , 0 < t < \infty$

IC: $u(x,0) = \phi(x) \quad , \quad 0 \le x \le 1$




Use the <u>separation of variables method</u> step by step to find the solution to the PDE, BCs, and the IC.

**6.** (5 points) Find the possible solution of m and n in the expression $u = \cos mt \sin nx$ such that it satisfies the wave equation $u_{tt} = c^2 u_{xx}$ .

**7.** (5 points) Show that $u = u_0 \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi c t}{L}\right)$, satisfies the one-dimensional wave equation and the conditions

(a) a given initial displacement $u(x, 0) = u_0 \sin(\pi x/L)$, and

(b) zero initial velocity, $\partial u(x, 0)/\partial t = 0$.

**8.** (5 points) Show that $u = x^4 - 2x^3y - 6x^2y^2 + 2xy^3 + y^4$ satisfies the Laplace equation.

**9.** (5 points) Show that the function $T = T_\infty + (T_m - T_\infty)e^{-U(x-Ut)/\alpha}$, where ($x \geq Ut$), satisfies the one-dimensional heat-conduction equation, together with the boundary conditions $T \to T_\infty$ as $x \to \infty$ and $T = T_m$ at $x = Ut$.

**10.** (20 points) The transmission-line equations represent the flow of current along a long, leaky wire such as a transatlantic cable. The equations take the form

$$-I_x - gv + cv_t$$
$$-V_x - rI + LI_t$$

where g, c, r and L are constants and I and v are the current and voltage respectively.

(a) Show that when r = g = 0, the equations reduce to the wave equation.

(b) Show that when L = 0, the equations reduce o a heat-conduction equation with a forcing term. Write $W = v\, e^{(gt/c)}$ to reduce to the normal form of the equation.

1. Verify the heat polynomial, $u=\frac{1}{2}x^2+t$ is a solution of the heat equation $u_t=u_{xx}$ .

2. Let $u=e^{mx+nt}$, what are $m$ & $n$ to be a solution of the heat equation $u_t=u_{xx}$.

3. Identify the transport, wave, Laplace, heat, and telegraph equations.

| Equation | Name |
|---|---|
| $u_{tt}=u_{xx}+\alpha u_t+\beta u$ | |
| $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$ | |
| $u_t=u_{xx}+u_{yy}$ | |
| $u_{tt}=u_{xx}+u_{yy}+u_{zz}$ | |
| $u_t=-cu_x$ | |

4. Show the types of the following linear PDE equations (show your work!!):

a). $u_{tx}=0$

b). $u_{xx}+u_{yy}=0$

c). $u_t=u_{xx}$