types of learning, ranging all the way from the largely rote and mechanical, perhaps in getting started, to the predominantly creative, experimental, and pragmatic. Advancement along this spectrum or continuum will indeed involve making some "mistakes" that can be pragmatically "corrected" along the way. But one's overall evolution of knowledge must be allowed to move through this continuum, without either stagnation or abrupt interruption. That is, to dwell exclusively on the mechanistic end of this spectrum must yield a kind of mental stagnation, and to move abruptly to the "creative" or "experimental" end of the spectrum, without solid grounding in the fundamentals, denies the objective demands of a creative or experimental method and can only produce the vacuous and inane.

But what has all this to do with logic, with right reasoning, with being as rational as we can in our fixations of belief? If we employ our logical principles, our principles of rationality, only in that purely mechanistic, rote fashion such as the computing machine is pre-eminently capable of, then ironically we risk an outcome for ourselves that is irrational and unproductive. Again, if we do not actually act upon and consciously use a pragmatic, experimental method in our reasoning processes, then we are no better off than the machine, and just as capable of being led astray, of being mastered and manufactured. Some further remarks from Peirce's "Our Senses as Reasoning Machines" (1900), quoted in chapter 3, bear repetition here.

> . . . so we do when we go irreflectively by a rule of thumb, as when we apply a rule of arithmetic [or logic] the reason of which we have never been taught [or genuinely learned]. The irrationality here consists in our following a fixed method, of the correctness of which the other method [experimentalism or pragmatism] affords no assurance; so that if it [the fixed method] does not happen to be right in its application to the case at hand, we go hopelessly astray. In genuine reasoning, we are not wedded to our method. We deliberately approve it, but we stand ever ready and disposed to reexamine it and to improve upon it, and to criticize our criticism of it, without cessation (Peirce 1900, MS 831: 11).

So logic, if we are truly to make it "our own," so to speak, cannot be merely a dry husk that we mechanically pull out of our "reasoning drawer" and invoke every now and then to brush the intellectual dust off our beliefs. To quote Peirce again on the joint subject of logic and reasoning, "It is a *LIVING* process" (Peirce 1887b, my emphasis). And it is a living process that finds its way into every university classroom, onto every factory floor and, whether we clearly perceive and acknowledge it or not, into every facet of our lives.

Logic, as we have defined it, is the study of arguments, and our focus is on the larger sort we know as our pragmatic logic of events. The two large categories *within* pragmatic logical argument that we will be concerned with are the deductive area and the inductive area. Deductive logic, when applied properly, produces conclusions from the evidence supplied that are matters of certainty. Given our fallibility, we here knowingly and consciously work with *practical* certainty, but always with the possibility and aim in mind of correction of our errors against *ideal* certainty. Inductive logic, when properly applied, produces from obtained evidence conclusions that are matters of probability and, in the course of things, degrees of probability. Again, given our fallible natures, inductive logic also has both practical and ideal aspects.

We shall now examine five illustrations of deductive arguments and one broad example of inductive argumentation. Here are our five illustrative categories of deductive arguments:

(1) categorical syllogism using one combination of universal, affirmative propositions;
(2) disjunctive syllogism in two distinct versions;
(3) pure hypothetical syllogism;
(4) arguments of the form *modus ponens*;
(5) arguments of the form *modus tollens*.

Our first illustration, as its name implies, is drawn from the rather large subject of categorical syllogistic logic, while our four other illustrations are taken from propositional logic. We will spend more time on categorical syllogism than on the others. But its deductive implications are not peculiar to it; they also hold for its four propositional companions.

Actually, there are 256 different varieties of categorical syllogisms, but we shall be concerned here only with the following example. You should read and understand the three propositions composing it in a straightforward, literal way. Take care to notice that the three collections of things or sets or classes involved—namely "human beings," "mammals," and things that are "warm-blooded"—are of three different population sizes. Recalling our discussion of ***argument*** in chapter 3, the evidence section of an argument is given in a 'premise' or in 'premises'. Thus:

Syllogism 1

All human beings are mammals.
All mammals are warm-blooded.

Therefore, all human beings are warm-blooded.

The first sentence is the first premise, the second sentence is the second premise, and the final sentence, as "Therefore" implies, is the conclusion. Notice also that there are three terms at issue in the argument: "human beings," "mammals" and "warm-blooded." All categorical syllogisms are arguments reaching a conclusion through the use of two premises constructed of three terms. Readers already familiar with standard-form syllogistic logic may object that this example mixes 'mood' and 'figure' in a non-standard manner by reversing the traditional ordering of the two premises. This departure, however, provides us with a more quickly and thoroughly understood illustration of the inevitability of deductive reasoning and its necessary character (see Hurley 1994: 52). (This departure works especially well when working with Euler diagrams, as are used later, at diagram 4.3, to illustrate a *four*-term categorical deductive argument.)

The example given above is called a categorical syllogism because each of its components addresses a category or class of things: "all human beings," "all mammals," and, by implication, "all things that are warm-blooded." We see that the conclusion is supported by our two premises, and we see that the connection between premises and conclusion, the reasoning involved, ensures this support. That is, we see certain *relations* between the classes of things themselves, and we see that these relations bring us inevitably to the conclusion. When we use categorical reasoning, as we do here, we thus work with the logical relations of classes. In this case, the variety of reasoning is deductive because from two pieces of evidence it has produced a conclusion that is a matter of certainty. Some find this notion that evidence only "supports" a conclusion to be a bit elastic for deductive purposes; something a bit stronger seems needed. The relation between evidence and conclusion in a properly done deductive argument, then, might be tightened, more like a relation of cause and effect, even when the term "supports" is retained.

With regard to this issue of supporting evidence, and in contrast to the syllogism above, consider this example:

Syllogism 2

All human beings are mammals.
All warm-blooded creatures are mammals.

Therefore, all human beings are warm-blooded.

A true conclusion is offered, premises are presented that allegedly support it, some sort of an attempt at deductive reasoning seems to be in use, but something is not quite right here. The second premise, in particular, does not seem to be true. Are there not more members of the class or group called "warm-blooded" than just the members of the class or group known as "mammals," like birds? What is wrong with this illustration? How can it be that we seem to arrive at a conclusion that is true, but arrive by apparently faulty means? What is the value of such a conclusion, if it has any value at all?

This illustration is faulty in two aspects, and these two aspects must be properly handled if certainty in the conclusion of a deductive argument is our goal. These aspects are, first, the formal arrangement or pattern of the argument's parts and, second, the truth of the argument's premises. If we have a correct pattern of deductive argumentation we say that the argument form is *valid*. A valid argument form is such that it is impossible, given true premises, for the conclusion to be false. All five of the deductive argument types we are investigating have their valid forms. The first of our two syllogisms, above, has such a form. Ideally, a valid argument form that uses true premises exclusively will *necessarily* produce a true conclusion. That is, ideally, a deductive argument that so uses validity and truth together is said to be a *sound* argument. In all genuinely sound arguments, conclusions are matters of invariable truth and certainty: their truth matters not in the least on what we personally, or as groups, think of them. Thus, the conclusions of sound deductive arguments are matters of objective knowledge rather than matters of subjective opinion. Peirce put it succinctly when writing about such arguments: ". . .that from true premises they must invariably produce true conclusions" (*CP* 2.267). To deny the conclusion of a sound argument, then, asserts that the necessarily true conclusion is objectively inconsistent and in fact in contradiction with the argument's true premises.

If you think of "pattern" in much the same way that one might think of the master blueprints for a standardized series of buildings, the template used to guide the cutting of fabric toward assembling a garment, or a computer program, you will notice that there is a clear pattern to how the parts of syllogism 1 fit together. The first term of the first premise, "human beings," returns as the first term of the conclusion; the last term of the second premise, "warm-blooded," returns formally valid, although we would need to make assumptions about truth to test such an argument's validity. In fact, making assumptions about truth to determine validity is standard procedure: we assume our premises true and then see if our conclusion follows with "strict necessity" (see Hurley 1997: 42). Likewise, if we assume the premises to be true and we know that the argument is valid, we must acknowledge the truth of the conclusion. And if we can justify our assumption that the premises are actually true, the resulting sound argument will compel our confidence in the truth of the conclusion. Validity itself, therefore, is not dictated by actual truth. It is this independence of validity from truth that makes possible the model of deductive logic for computer programming: computer programs and deductive arguments process or manipulate information supplied *to* them; but completely unflawed computer programs and standard forms of valid deductive arguments function with equal precision whether the information they manipulate is true or not. Likewise, the *invalidity* of deductive argument forms is completely independent of the actual combinations of truth and falsity of premises and conclusions—with one crucial exception: a deductive argument that uses all true premises and a false conclusion is, of necessity, an instance of an invalid argument form, for how else could a false conclusion occur in a deductive argument with all true premises? That is, no such valid argument from *exists*, even as an assumption. Consequently, if, on assuming the truth of our premises, we find that our conclusion is false, our argument is *proved invalid*.

But truth and falsity of argument materials otherwise can and do result in *either* valid or invalid forms. The certainty that we expect and that to an extent limits us in deduction would be clearly contradicted should we be able to assemble a supposedly valid deductive argument with true premises that nevertheless produced a false conclusion. What would your world be like if the computers that produce your paycheck, figure your bank statement, record the grades for your transcript, or verify your driver's license used a "logical" procedure capable of producing false conclusions from true and validly organized information? Your world would be reduced to chaos. We see then that the matching of true premises with a false conclusion in the real world necessarily can produce *only* an invalid and therefore unsound argument, and that the question of actual truth or falsity of argument materials is otherwise irrelevant to questions of validity and invalidity.

The necessary nature of deductive reasoning and its critical results in the pragmatic logic of events are indispensable instruments in our acquisition and development of knowledge. Their importance cannot be overemphasized. To illustrate further the matter of logical, objective necessity, let us take another look at the subjects of 'class' and 'relation' and add to them one further item, namely, the important notion of 'algorithm'.

Using citations from the definitions of 'class' and 'relation' supplied by Peirce in Whitney's *Century Dictionary*, we can understand better the subject of the logical relations of classes. For 'class' Peirce gave the following definition:

A number of objects distinguished by common characters from all others, and regarded as a collective unit or group; a collection capable of a general definition; a kind. (Peirce 1889: 1029)

For the following four-term illustration, then, let us count pianists, keyboard players, musicians, and artists as examples of different classes. The class or set of pianists, then, would be the class or set including and limited to all pianists.

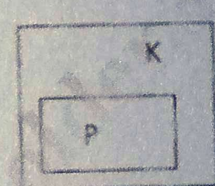
For the term 'relation' Pierce gives the following definition:

A character of a plurality of things; a fact concerning two or more things, especially and more properly when it is regarded as a predicate of one of the things connecting it with the others; the condition of being such and such with regard to something else. (Peirce 1889: 5057)

Thus could we assert the relation of the classes "pianists" and "keyboardists" as "All pianists are keyboardists" and take it as the first premise of a *four*-term deductive argument. Because syllogisms are by definition limited to three terms and two premises, a deductive argument such as we are constructing, namely, one with *four* terms and *three* premises is not, strictly speaking, a syllogism, even though the principle of inference is the same.

Taking the first letters of these class names as abbreviations, we can state our first premise more briefly, in the following form. Thus, where 'P' abbreviates "pianists" and 'K' abbreviates "keyboardists," the proposition "All P is K" abbreviates this first premise. For greater clarity, we can also render this premise again in a modification of an Euler diagram or graph, named for the eighteenth-century Swiss mathematician Leonhard Euler.

Diagram 4.3

Proposition	Graph
All P is K	

By means of the diagram given above, we immediately grasp that each and every thing which is a pianist, that is, the class represented by the rectangled P, is related to each and every thing which is a keyboardist, or the class represented by the rectangled K, and that the relation is expressed through "is." The relation expressed through "is," in this case, can be defined as "is included in." The modified Euler graph representing this relation illustrates it by means of the geometric relation "is inside of."

"All pianists are keyboardists" is, then, our first premise. Let us suppose that the investigation of our deduction is to conclude what the relation is between pianists and artists. We then add two additional premises to the first, these two being "All keyboardists are musicians" and "All musicians are artists." By employing the propositional and graphical conventions used above, we can then represent the complete deduction with its conclusion. Using the three premises in their original propositional formulations, we would begin with the following.

Syllogism 4

All pianists are keyboardists.
All keyboardists are musicians.
All musicians are artists.

Therefore: All pianists are artists.

Extending our earlier use of abbrevations, this deduction can be reduced to the following.

Diagram 4.4

All P is K
All K is M
All M is A

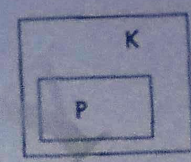
Therefore: All P is A

Its graphical equivalent, then, is given in diagram 4.5, below. If you take care to note how, at each step along the way, the relevant rectangles interlock with one another in an ascending order according to the size of the class they represent, you will see how, if the premises in fact be true, the conclusion would be utterly inescapable.

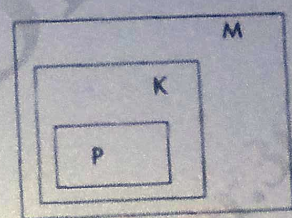
Proposition — Graph

Diagram 4.5

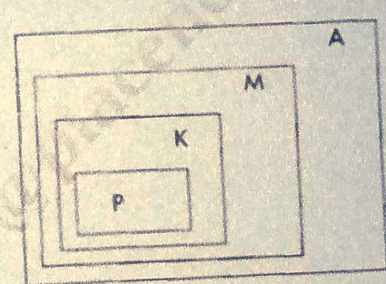
All P is K



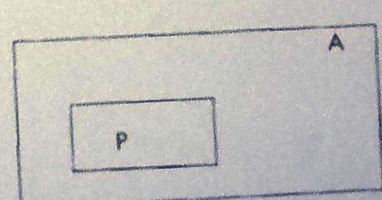
All K is M



All M is A



Therefore,
All P is A



The important aspect of this illustration, of course, is *not* the actual everyday contents of these classes, but rather the manner in which these classes are brought into relations with one another to produce a valid *form* of argument. Whether or not the argument itself is sound depends additionally, of course, on the truth of the premises. This kind of argument in the logical relations of classes is of the type that can be treated completely and correctly using the kind of logic machine proposed by Peirce and his student and colleague Allan Marquand. It can do this because it relies on and operates according to what is termed an *algorithm*, which is defined as "a mechanical procedure for carrying out, in a finite number of steps, a computation that leads from certain types of data to certain types of results" (see Brody 1967). It is the algorithm that defines the limits of a machine's computational sophistication, the limit of its "fixed method," the limit beyond which our creative faculties hold sway. To examine and illustrate more clearly what an algorithm is and how it works, we will consider the remarkable story of Peirce and Marquand's early computer design. (For a fuller account, see Ketner with Stewart 1984.)

Allan Marquand (1853–1924) was the son of the art collector and philanthropist Henry G. Marquand, and perhaps the most prominent of Peirce's students during his time at Johns Hopkins University. Peirce was a part-time faculty member in logic at the new university from 1879 through 1884 and concurrently maintained a full-time position as assistant at the U.S. Coast and Geodetic Survey. The title "assistant" required more responsibility and authority than we would, these days, ordinarily associate with it. He held the Coast Survey position from 1859 until his resignation in 1891. While at Hopkins he was active as well in the areas of mathematics, psychology, and philosophy. However, it is Peirce's activities in logic, especially with Marquand, that concern us here.

Marquand was a fellow in philosophy and ethics at Hopkins until the completion of his doctorate in 1880. His essay on the logic of

Philodemus was written under Peirce's supervision and was the only doctoral dissertation ever completed under Peirce's direction. *Studies in Logic by Members of the Johns Hopkins University*, which appeared in 1883 under Peirce's editorship, included the introduction to Marquand's dissertation, entitled "The Logic of the Epicureans," as the first part. In 1881 Marquand joined the faculty at Princeton as Tutor in Latin and lecturer in Logic. In 1883, probably as a result of a dispute with the university president, Marquand changed his position at Princeton by becoming professor of art and archaeology, a position he held until his death.

While still a student at Hopkins, Marquand had been directed, by Peirce, to the task of developing an improved version of William Stanley Jevons's mechanical instrument for performing syllogistic reasoning, his "Logical Piano," so called because, to use an analogy with modern computers, its "keyboard" resembled a section of a piano keyboard. Marquand contributed heavily to the first issue of *Studies in Logic*, including an essay entitled "A Machine for Producing Syllogistic Variations" and the following "Note on an Eight Term Logical Machine." From these contributions it is clear that Marquand had already devised three types of aids for logical computation. The first such device consisted of logical diagrams or graphs; the second was a mechanically crank-operated realization of these diagrams; and the third improved the Jevons machine, providing operation by depression of keys. Peirce was completely familiar with these developments and certainly had the upgraded Jevons device in mind when writing the "logical machine" entry for Whitney's *Dictionary and Cyclopedia*.

Logical machine, a machine which, being fed with premises, produces the necessary conclusions from them. The earliest instrument of this kind was the demonstrator of Charles, third Earl Stanhope: the most perfect is that of Professor Allan Marquand, which gives all inferences turning upon the logical relations of classes. The value of logical machines seems to lie in their showing how far reasoning is a mechanical process, and how far it calls for acts of observation. Calculating machines are specialized logical machines. (Peirce 1889: 3560)