ASK (OOK) signal at $t = 1$ msec for the logic 0 to logic 1 and at $t = 2$ msec logic 1 to logic 0 binary data transition and the nominal a carrier frequency $f_c = 20$ kHz.

The binary ASK (OOK) signal can be decomposed into a baseband PAM signal $s_{PAM}(t)$ and the sinusoidal carrier, as given by Equation 3.15. The optimum receiver for binary ASK (OOK) is the correlation receiver for asymmetrical signals, as described in Chapter 2 and shown in Figure 2.37 for binary baseband asymmetrical PAM, but modified for bandpass demodulation, as shown here in Figure 3.4.

The communication channel is represented by the *AWGN* block from the *Channels, Communications Blockset*. The *Product* block multiplies the input signal $s_i(t)$ with the coherent reference signal $s_{ref}(t)$, the difference signal $s_1(t) - s_0(t)$, as given by Equation 3.9 and Equation 3.14. Since for binary ASK (OOK) here $s_0(t) = 0$ and $s_{ref}(t) = s_1(t)$.

The reference signal $s_{ref}(t)$ then is the *Sine Wave* block, with parameters of an amplitude $A = 5$ V peak, a frequency $f_c = 20$ kHz and 0° phase offset. The output of the multiplier is processed by the *Integrate and Dump* block from the *Comm Filters, Communications Blockset* synchronized to the input signal and whose parameters are specified as an integration time equal to the number of samples in a bit time T_b of 1 msec or 500, derived from the 500 k samples/sec simulation rate.

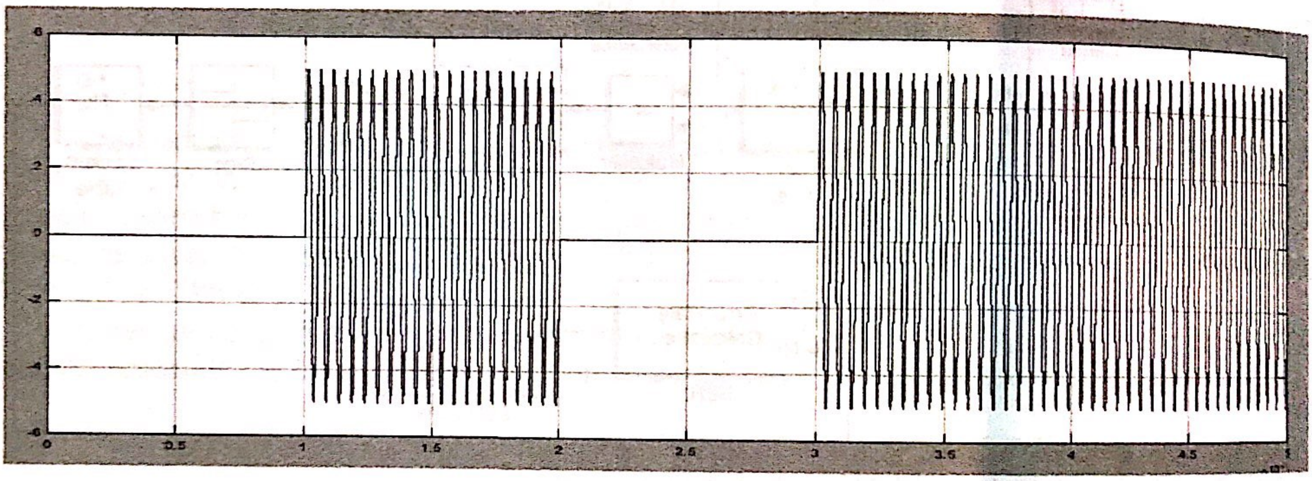

Figure 3.5 A binary ASK (OOK) signal with amplitude $A = 5$ V peak, carrier frequency $f_c = 20$ kHz and bit time $T_b = 1$ msec for the data bits $b_i = 01011$ (*Fig34.slx*).

The optimum threshold τ_{opt} is given by Equation 3.8, since the binary ASK (OOK) signals even though asymmetrical are equally probable. The *Integrate and Dump* block displays a nominal gain equal to the number of samples in the integration period (500 = 500 k samples/sec × 1 msec) and does not scale by the term $1/T_b$ as in Equation 3.2. The optimum threshold τ_{opt} then is 3125 here and set by the *Constant* block from the *Sources, Simulink Blockset*, since $z_0(iT_b) = 0$ and $z_1(iT_b) = 6250$, as given by Equation 3.2.

Note that the *Constant* block sets the optimum threshold τ_{opt} with the use of the *Sum* block from the *Math Operations, Simulink Blockset* in Figure 3.4. The output of the *Sum* block is inputted to the *Sign* block both from the *Math Operations, Simulink Blockset* whose non-binary output (−1, 0, 1) is converted to binary (0, 1) by the *Lookup Table* block. The input values [−1, 0, 1] use the output table data [0, 0, 1] and produces the binary receive data.

Since the binary ASK (OOK) received signal is sampled at a 500 kHz simulation rate with $T_b = 1$ msec, each signal has 500 simulation samples per bit time T_b. However, it is not necessary to *downsample* the 500 kHz simulation rate to the binary data rate $r_b = 1$ kb/sec for the BER analysis. The integration time specification of the *Integrate and Dump* block provides data at this rate for the simulation.

The two comparison inputs to the BER analysis provided by the *Error Rate Calculation* block from the *Comm Sinks, Communications Blockset* are single simulation samples per bit time T_b derived from the binary data source and the binary received data. The *Error Rate Calculation* block parameters are set with a received data delay of two binary bits to compensate for the processing inherent in the binary ASK (OOK) transmitter and receiver.

The *Display* block from the *Simulink, Sinks Blockset* displays the rate error, the total number of errors and the total number of data bit comparisons. The *Simulation Stop Time* is set to 10 sec which processes 10 000 information bits in possible error. Setting the normalized energy difference per bit to noise ratio E_b/N_o dB in the *AWGN* block to a value greater than 50 dB should produce 0 observed errors.

Binary ASK Power Spectral Density

The magnitude of the frequency domain representation $P_{BASK\,(OOK)}(f)$ (derived from the Fourier transform) of a binary ASK (OOK) signal of width T_b seconds, peak amplitude A volts, and carrier frequency f_c Hz is given by Equation 3.16 [Proakis14].

$$P_{BASK\,(OOK)}(f) = \frac{1}{2}\left[\frac{AT_b}{2}\,\mathrm{sinc}\big(2\pi(T_b/2)(f-f_c)\big)+\frac{A}{2}\delta(f-f_c)\right]- \frac{1}{2}\left[\frac{AT_b}{2}\,\mathrm{sinc}\big(2\pi(T_b/2)(f+f_c)\big)-\frac{A}{2}\delta(f+f_c)\right] \quad (3.16)$$

This is the bi-sided voltage spectrum, extending from all negative to all positive frequencies ($-\infty < f < +\infty$). The analysis for the voltage spectrum utilizes that for the rectangular PAM pulse, as given by Equation 2.1. The *unipolar* PAM pulse $s_{PAM}(t)$ in Equation 3.15 has an amplitude 0 V and $A = 5$ V, but can be decomposed into a *polar* PAM pulse of $\pm A/2 = \pm 2.5$ V and a constant (DC) level of $A/2 = 2.5$ V in Equation 3.16 [Stern09].

The polar PAM pulse is represented by the sinc ($\sin x/x$) function and the constant term by the impulse ($\delta(f)$) in the frequency domain. The *modulation property* of the Fourier transform shifts the sinc and impulse representations in the frequency domain by replacing the argument f by $f - f_c$ and $f + f_c$ and scaling by 1/2 in Equation 3.16.

The bi-sided energy spectral density $E_{BASK\,(OOK)}(f)$ of n pseudonoise binary ASK (OOK) signals is n times the square of the magnitude of the voltage spectrum divided by the load resistance R_L, as given by Equation 3.17.

$$E_{BASK\,(OOK)}(f) = \frac{n}{4R_L}\left[\frac{A^2T_b^2}{4}\,\mathrm{sinc}^2\big(2\pi(T_b/2)(f-f_c)\big)+\frac{A^2T_b}{4}\delta(f-f_c)\right]+ \frac{n}{4R_L}\left[\frac{A^2T_b^2}{4}\,\mathrm{sinc}^2\big(2\pi(T_b/2)(f+f_c)\big)+\frac{A^2T_b}{4}\delta(f+f_c)\right] \quad (3.17)$$

The bi-sided power spectral density (PSD) $PSD_{BASK\,(OOK)}(f)$ is the energy spectral density divided by the time to transmit the n binary ASK (OOK) signals or nT_b, as given by Equation 3.18. The impulse terms are discrete components and not affected by the PSD transformation, the energy spectral density averaged over multiple bit times nT_b.

$$PSD_{BASK\,(OOK)}(f) = \frac{1}{4R_L}\left[\frac{A^2T_b}{4}\,\mathrm{sinc}^2\big(2\pi(T_b/2)(f-f_c)\big)+\frac{A^2}{4}\delta(f-f_c)\right]+ \frac{1}{4R_L}\left[\frac{A^2T_b}{4}\,\mathrm{sinc}^2\big(2\pi(T_b/2)(f+f_c)\big)+\frac{A^2}{4}\delta(f+f_c)\right] \quad (3.18)$$

Increasing the *Simulation Stop Time* to 5.24288 sec results in 524 288 (2^{19}) samples at a 500 kHz sampling rate and a spectral resolution of approximately 0.95 Hz (the sampling rate divided by the number of samples or 500 kHz/524 288). The estimated, normalized ($R_L = 1\ \Omega$) PSD of the transmitted binary ASK (OOK) signal without AWGN is shown unscaled as a single-sided spectrum in Figure 3.6 in dBm/Hz. The PSD is obtained by the *Spectrum Scope* block from the *Signal Processing Sinks*, *Signal Processing Blockset*, derived from the *Simulink* model *Fig34.slx*.

The maximum frequency displayed is 250 kHz, the Nyquist frequency or half of the system simulation rate (500 kHz/2). There are approximate nulls in the PSD that occur at the data rate $r_b = 1/T_b$ (1 kb/sec or 1 kHz) and multiples of the data rate due to the sinc^2 term in Equation 3.18. The carrier component is shown as a spectral impulse ($\delta(f)$) at $f_c = 20$ kHz, as given by Equation 3.18 and shown in Figure 3.6, derived from the *Simulink* model *Fig36.slx*. However details of the PSD are difficult to discern at this spectral resolution and display.

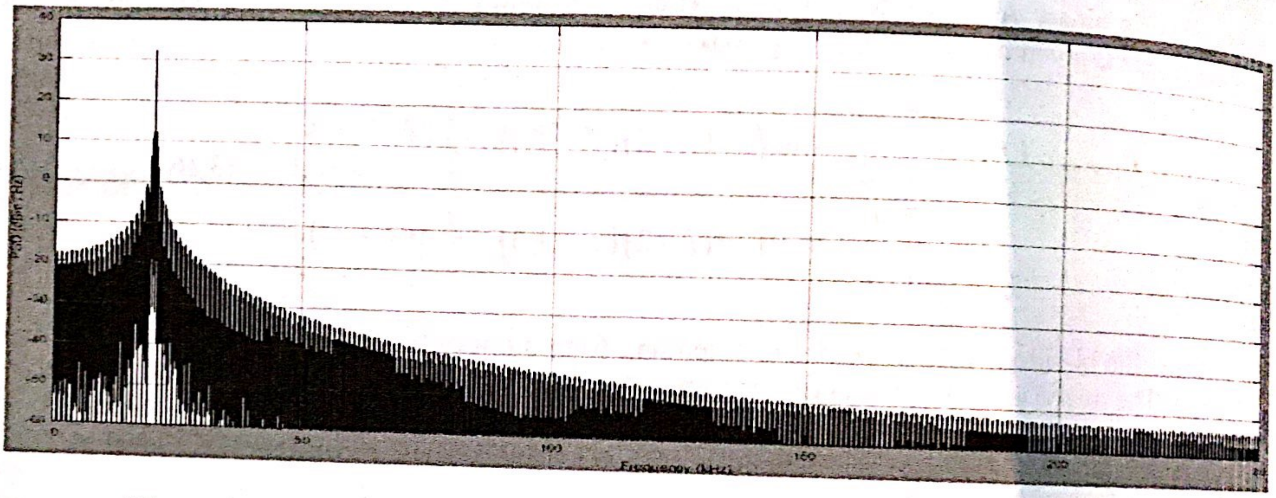

Figure 3.6 Power spectral density of a 1 kb/sec random binary ASK (OOK) signal, $f_c = 20$ kHz, $r_b = 1$ kb/sec (*Fig36.slx*).

The normalized PSD can be scaled to resolve greater spectral detail, as shown in Figure 3.7, with a center frequency of 20 kHz and a span of ± 10 kHz. Since the rectangular PAM pulse $s_{PAM}(t)$ of the binary ASK (OOK) signal has a width τ equal to the bit time T_b, there are approximate nulls in the PSD that occur at the bit rate $r_b = 1/T_b$ (1 kb/sec or 1 kHz) and multiples of the bit rate centered about the carrier frequency $f_c = 20$ kHz due to the sinc^2 term in Equation 3.18. The PSD is not smooth because, although there are a large number of data, the ensemble is finite, but the spectral envelope clearly demonstrates the sinc^2 behavior.

The PSD of a bandpass signal is often referred to as a double sideband spectrum (DSB), with an upper sideband (USB) and lower sideband (LSB) centered about the carrier frequency, as shown in Figure 3.7. Since the bandwidth of a bandpass ASK signal is twice that of the baseband PAM signal, the transmission bandwidth of data at a rate r_b b/sec requires r_b Hz. The theoretical minimum for the binary transmission of data at a rate r_b b/sec is a bandwidth of $r_b/2$ Hz as postulated by Nyquist [Lathi09].

The carrier frequency is deliberately chosen to be relatively low to facilitate the *MATLAB* and *Simulink* simulation, but this causes *aliasing* to occur. The amplitude of the PSD centered about the carrier frequency should be symmetrical, as given by Equation 3.18. However, the amplitude of the PSD below the carrier frequency is elevated due to this aliasing. The alias affect, although noticeable, can be alleviated to a degree by a bandpass filter centered at the carrier frequency and with a bandwidth sufficient to pass the information [Proakis14].

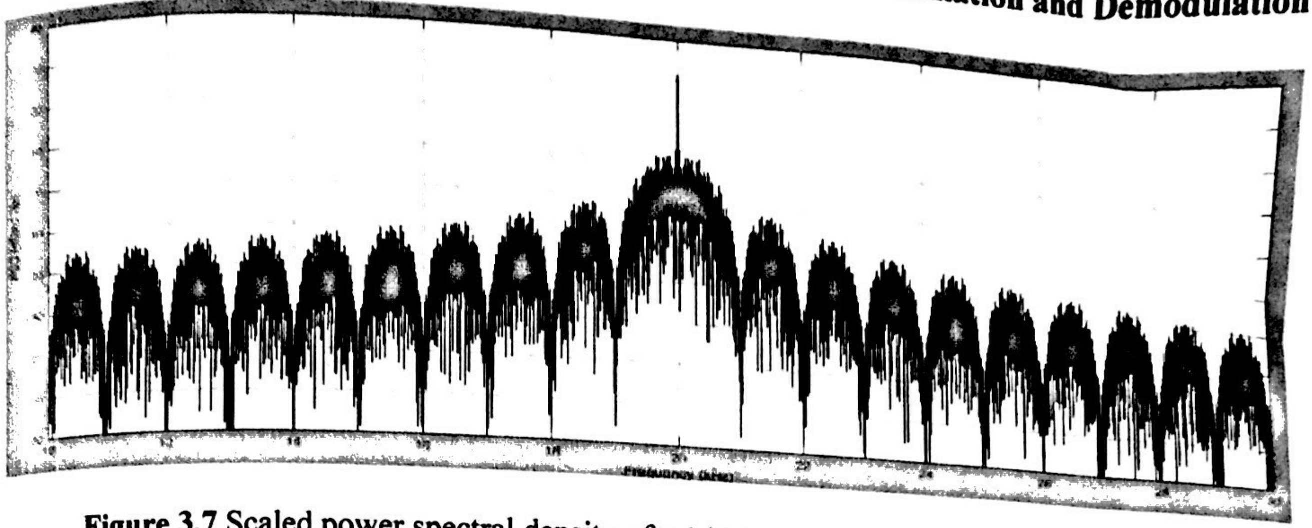

Figure 3.7 Scaled power spectral density of a 1 kb/sec random binary ASK (OOK) signal with $f_c = 20$ kHz, $r_b = 1$ kb/sec (*Fig36.slx*).

The bandwidth of the binary ASK signal is somewhat problematical, as it is for binary PAM in Chapter 2. Defining the bandwidth as a percentage of the total power of the binary ASK signal again is the usual course, since all of the available power would require an infinite bandwidth. The total power is easily calculated in the temporal domain. The power P in the binary ASK signal per bit time T_b, derived from Equation 3.14 is given by Equation 3.19.

$$P = \frac{A^2}{T_b\, R_L} \int_{(i-1)T_b}^{i\,T_b} m_j^2(t)\, \sin^2(2\pi\, f_c t)\, \mathrm{d}t = \frac{A^2}{2R_L} \quad \text{if } m_j(t) = 1 \quad (i-1)T_b \le t \le i\,T_b \qquad (3.19)$$

$$P = 0 \quad \text{if } m_j(t) = 0 \quad (i-1)T_b \le t \le i\,T_b$$

Equation 3.19 assumes that the ratio of the carrier frequency to the bit rate f_c/r_b is either an integer or much greater than 1. The integration then over one bit time T_b is either for a whole number of cycles of the square of the carrier sinusoid or a whole number of cycles and an insignificant residual term at twice the carrier frequency. If the *apriori* probabilities are equal ($P_0 = P_1 = 0.5$) then the total power P_T is given by Equation 3.20.

$$P_T = \frac{A^2}{2\,R_L}\, P_1 + 0\, P_0 = \frac{A^2}{4\,R_L} \qquad (3.20)$$

Table 3.1 Bandwidth of a binary ASK (OOK) signal as a percentage of the total power.

Bandwidth (Hz)	Percentage of Total Power
$2/T_b$	90%
$3/T_b$	93%
$4/T_b$	95%
$6/T_b$	96.5%
$8/T_b$	97.5%
$10/T_b$	98%

The binary ASK (OOK) signal has an amplitude $A = 5$ V and the total normalized ($R_L = 1\ \Omega$) power $P_T = 6.25$ W = 37.95 dBm (10 log$_{10}$[6.25/0.001]). Numerically integrating the power spectral density, Equation 3.18, from the carrier frequency to an arbitrary but symmetrical frequency on either side of the carrier frequency and comparing the resulting power to the expected total power provides the data in Table 3.1. Table 3.1 for binary bandpass ASK (OOK) is similar to Table 2.1 for binary baseband rectangular PAM. For the same percentage of the total power the bandwidth of the bandpass ASK signal is double that for the baseband PAM signal.

Performance of Binary ASK for the Optimum Receiver in AWGN

The binary ASK (OOK) transmitter and optimum bandpass receiver in Figure 3.4 can be evaluated for its performance in the presence of AWGN. A bandpass BER analysis is conducted with the same parameters as the baseband BER analyses in Chapter 2. The *System Stop Time* is set to 10 sec for convenience, which would process 10 000 information bits in possible error. However, here the 500 kHz *System Time* rate results in 5×10^6 simulation sample points.

Table 3.2 is a tabular list of the observed BER in a single trial of 10 000 information bits as a function of E_d/N_o to the theoretical P_b for binary ASK (OOK), as given by Equation 3.11 for asymmetrical signals. Binary ASK (OOK) does not satisfy Equation 3.1 and is asymmetric. The optimum receiver is implemented as the correlation receiver for asymmetrical signals, as shown in Figure 3.4.

Table 3.2 Observed BER and theoretical P_b as a function of E_d/N_o in a binary ASK (OOK) digital communication system with optimum receiver.

E_d/N_o dB	BER	P_b
∞	0	0
12	2.9×10^{-3}	2.53×10^{-3}
10	1.38×10^{-2}	1.25×10^{-2}
8	3.66×10^{-2}	3.75×10^{-2}
6	7.85×10^{-2}	7.93×10^{-2}
4	1.335×10^{-1}	1.318×10^{-1}
2	1.863×10^{-4}	1.872×10^{-1}
0	2.387×10^{-1}	2.394×10^{-1}

The *AWGN* block from the *Channels, Communications Blockset* facilitates the BER analysis with the input of a desired E_d/N_o dB directly. The normalized energy difference per bit E_d is determined by the number of bits per symbol (1), the normalized (R_L = 1 Ω) signal power (12.5 W) in terms of the square of the difference of the signals and the symbol period (T_b = 1 msec). However, note that the *AWGN* block parameter is labeled E_b rather than E_d.

The normalized energy difference per bit here is $E_d \approx 1.25 \times 10^{-2}$ V²-sec, as given by Equation 3.10 and Equation 3.15. The attenuation of the communication channel is taken to be zero ($\gamma = 1$). The input signal power $P = 12.5$ W which is used to set the E_d/N_o ratio in the *AWGN* block.

Binary ASK (OOK) provides the largest energy difference per bit E_d for a given power in the sinusoidal carrier and therefore the lowest P_b for any binary ASK digital communication system. A comparison of Table 3.2 and Table 2.8 in Chapter 2 shows the same BER performance for binary ASK using the bandpass optimum receiver as that for rectangular PAM using the baseband optimum receiver.

Binary Frequency Shift Keying

Binary frequency shift keying (FSK) *shifts* the carrier frequency to one of two discrete frequencies during the bit time T_b for the representation of binary logic signals for the transmission of information. The modulated sinusoidal carrier signal $s_f(t)$ has a constant amplitude of A V, a frequency of f_c Hz, and a 0° reference phase angle, as given by the analytical expression in Equation 3.21 [Lathi09].

$$s_j(t) = A\sin(2\pi(f_c + k_f m_j(t))\,t) \quad (i-1)T_b \le t \le iT_b \quad j = 0, 1 \quad (3.21)$$

The information signal or data source is $m_j(t)$ ($j = 0, 1$) and for binary FSK $m_j(t) = \pm 1$ V for one bit time T_b. The factor k_f, whose units are Hz/V, is the *frequency deviation factor* (or the modulation gain). The *frequency deviation* Δf is given by Equation 3.22.

$$|\Delta f| = |k_f m_j(t)| \qquad (3.22)$$

Since $m_j(t) = \pm 1$ V the magnitude of the frequency deviation Δf is equal on either side of the carrier frequency f_c.

Simulation of Binary FSK

A binary frequency shift keying (FSK) coherent digital communication system with AWGN is shown in Figure 3.8, derived from the *Simulink* model *Fig38slx*. The *MATLAB* and *Simulink* simulation is similar to that of the binary ASK communication system in Figure 3.4 but utilizes the alternative structure for the asymmetrical correlation receiver as shown in Figure 3.3.

The data source is the *Random Integer Generator* block from the *Comm Sources, Communications Blockset* which produces random, uniformly distributed binary integers (0 and 1) at a data bit time T_b = 1 msec or a data rate $r_b = 1/T_b$ = 1 kb/sec. The bit rate is the same as that used for baseband modulation and demodulation systems in Chapter 2, which facilitates the comparison of power spectral densities.

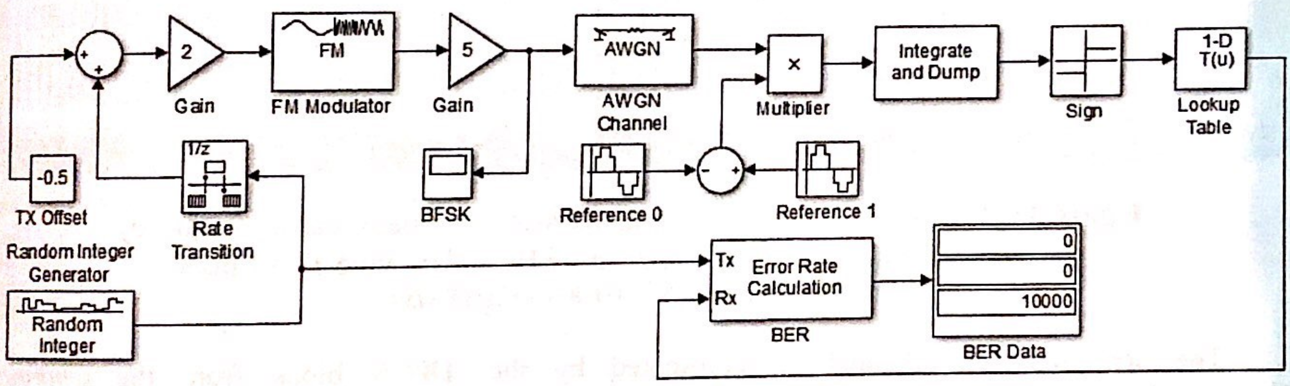

Figure 3.8 A binary FSK digital communication system with BER analysis and the optimum correlation receiver (*Fig38.slx*).

The *Rate Transition* block from the *Signal Attributes, Simulink Blockset* produces data at a simulation sample time $T_{simulation}$ = 2 μsec or a simulation rate of 500 kHz. The unipolar binary data (0 and 1) is offset by −0.5 with the *Constant* block and the *Sum* block and processed by the *Gain* block from the *Simulink, Source* and *Math Operations* blockset with a parameter of a gain G = 2 to generate polar data with a simulated amplitude of ± 1.

The non-linear FSK modulator is the *FM Modulator Passband* block from the *Modulation, Communications Blockset* with parameters of an amplitude of 1 V, a carrier frequency f_c = 20 kHz, 0° phase offset and an initial modulation gain (or frequency deviation factor k_f) of 2000 Hz/V. The *FM Modulator Passband* block is used for the *MATLAB* and *Simulink* simulation of FSK for bandpass simulation, rather than the *M-FSK Modulator Baseband* block from the *Communication Blockset* for adherence to the analytical description of an FM signal, as given by Equation 3.21 and described in Chapter 1.

The output of the *FM Modulator Passband* block has a fixed amplitude of ± 1 V. A *Gain* block from the *Simulink, Source* and *Math Operations* blockset with a parameter of a gain G = 5 generates the FSK signal with a simulated amplitude of ± 5 V, the same as that for binary ASK.

The resulting initial frequency deviation Δf here is ± 1 V × 2000 = ± 2 kHz, as given by Equation 3.22. A binary FSK signal is asymmetrical, since the bandpass signals $s_j(t)$ (j = 0, 1) do not satisfy Equation 3.1 and are given by Equation 3.23.

$$\begin{aligned} s_0(t) &= 5\sin(2\pi(f_c - \Delta f)t) \quad (i-1)T_b \le t \le iT_b \quad b_i = 0 \\ s_1(t) &= 5\sin(2\pi(f_c + \Delta f)t) \quad (i-1)T_b \le t \le iT_b \quad b_i = 1 \end{aligned} \tag{3.23}$$

The binary FSK signal is shown in Figure 3.9, derived from the *Simulink* model *Fig38.slx*, for an arbitrary data bit sequence b_i = 01001. The carrier frequency f_c is 20 kHz, the frequency deviation Δf initially is 2 kHz, the information bit is b_i and the bit time is T_b. Note the frequency shift that occurs for the FSK signal at t = 1 msec of +4 kHz for the logic 0 to logic 1 and at t = 2 msec of −4 kHz for the logic 1 to logic 0 binary data transition.

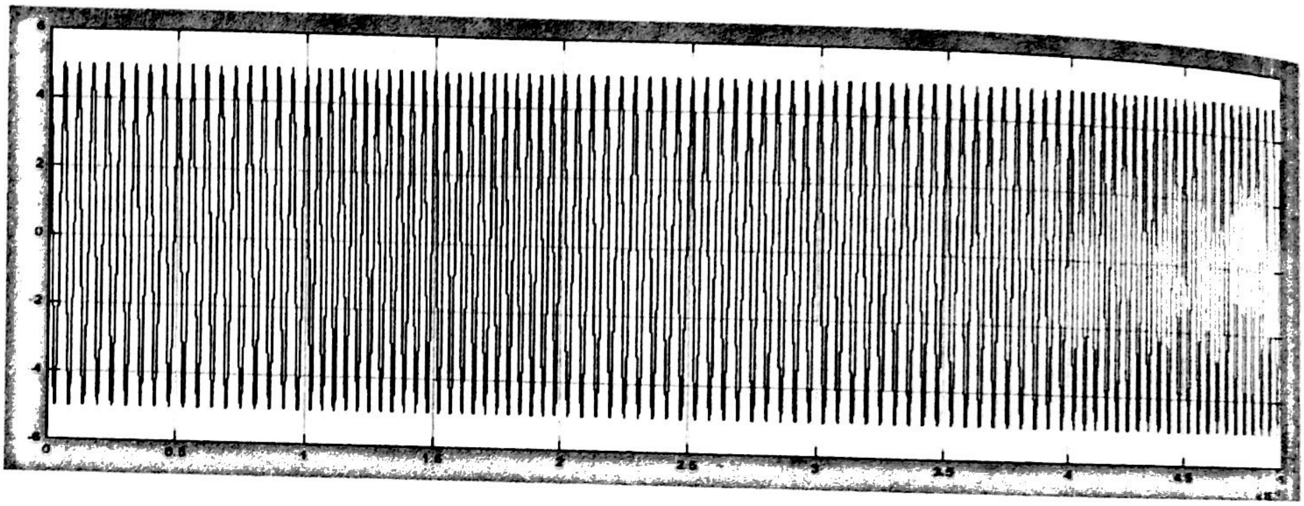

Figure 3.9 A binary FSK signal with amplitude A = 5 V peak, carrier frequency f_c = 20 kHz, frequency deviation Δf = ± 2 kHz and bit time T_b = 1 msec for the data bits b_i = 01001 (*Fig38.slx*).

The communication channel is represented by the *AWGN* block from the *Channels, Communications Blockset*. The optimum receiver for FSK is the correlation receiver for asymmetrical signals, as described in Chapter 2 and shown in Figure 2.34 for binary baseband PAM, but modified for passband demodulation, as shown in Figure 3.8. The *FM Demodulator Passband* block from the *Modulation, Communications Blockset* utilizes the Hilbert Transform filter and is not used as the optimum receiver, as described in Chapter 1.

The *Product* block multiplies the input signal $s_j(t)$ with the coherent reference signal $s_{ref}(t)$, the difference signal $s_1(t) - s_0(t)$, as given by Equation 3.9 and Equation 3.23. The reference signals are two *Sine Wave* blocks, with parameters of an amplitude A = 5 V peak, frequencies $f_c \pm \Delta f$ (20 ± 2 kHz) and 90° phase offsets. The *FM Modulator Passband* block outputs a cosine signal and the reference signal $s_{ref}(t)$ should be aligned with a +90° phase shift. The two sinusoids are then processed by the *Sum* block to provide the coherent reference signal $s_{ref}(t)$.

The output of the multiplier is processed by the *Integrate and Dump* block from the *Comm Filters, Communications Blockset* synchronized to the input signal. The parameters of the *Integrate and Dump* block are specified as an integration time equal to the number of samples in a bit time T_b = 1 msec or 500, derived from the 500 k samples/sec simulation rate.

The optimum threshold τ_{opt} is given by Equation 3.8 since the binary FSK signals even though asymmetrical are equally probable.. The *Integrate and Dump* block displays a nominal gain equal to the number of samples in the integration period (500 = 500 k samples/sec × 1 msec) and does not scale by the term $1/T_b$ as in Equation 3.2. Thus the optimum threshold τ_{opt} = 0 V, since $z_0(iT_b)$ = –6250 and $z_1(iT_b)$ = 6250, as given by Equation 3.2.

The output of the *Integrate and Dump* block is inputted to the *Sign* block both from the *Math Operations, Simulink Blockset* whose non-binary output (–1, 0, 1) is converted to binary (0, 1) by the *Lookup Table* block from the *Lookup Tables, Simulink Blockset*. The input values [–1, 0, 1] use the output table data [0, 0, 1] and produces the binary receive data.

Since the binary FSK received signal is sampled at a 500 kHz simulation rate with T_b = 1 msec, each signal has 500 simulation samples per bit time T_b. However, it is not necessary to *downsample* the 500 kHz simulation rate to the binary data rate r_b = 1 kb/sec for the BER analysis. The integration time specification of the *Integrate and Dump* block provides data at this rate.

The two comparison inputs to the *BER* analysis provided by the *Error Rate Calculation* block from the *Comm Sinks, Communications Blockset* are single simulation samples per bit time T_b derived from the binary data source and the binary received data. The *Error Rate Calculation* block parameters are set with a received data delay of two binary bits to compensate for the processing inherent in the binary FSK transmitter and receiver.

The *Display* block from the *Simulink, Sinks Blockset* displays the rate error, the total number of errors and the total number of data bit comparisons. The *Simulation Stop Time* is set to 10 sec which processes 10 000 information bits in possible error. Setting the normalized energy difference per bit to noise ratio E_b/N_o dB in the *AWGN* block to a value greater than 50 dB should produce 0 observed errors.

Binary FSK Power Spectral Density

The power spectral density (PSD) $PSD_{BFSK}(f)$ of a binary FSK signal of width T_b seconds, peak amplitude A volts, and carrier frequency f_c Hz is somewhat difficult to develop. An approximate method is to decompose the binary FSK signal as a sum of two ASK signals with two data sources, as given by Equation 3.24 [Stern04].

$$s_j(t) = A\, m_{1,j}(t)\sin(2\pi(f_c + \Delta f)\,t) + A\, m_{2,j}(t)\sin(2\pi(f_c - \Delta f)\,t) \qquad (i-1)T_b \le t \le iT_b \quad j = 0, 1 \tag{3.24}$$

The data sources or information signals $m_{1,j}(t)$ and $m_{2,j}(t)$ (j = 0, 1) are given in Equation 3.25.

$$\begin{aligned} m_{1,j}(t) &= 1 \quad (i-1)T_b \le t \le iT_b \quad b_i = 1 \\ m_{1,j}(t) &= 0 \quad (i-1)T_b \le t \le iT_b \quad b_i = 0 \\ \\ m_{2,j}(t) &= 1 \quad (i-1)T_b \le t \le iT_b \quad b_i = 0 \\ m_{2,j}(t) &= 0 \quad (i-1)T_b \le t \le iT_b \quad b_i = 1 \end{aligned} \tag{3.25}$$

The decomposed FSK signal is then two ASK signals with carrier frequencies of $f_c \pm \Delta f$ and the pulse modulation signals $s_{PAM}(t) = m_{k,j}(t)$ (k,j = 0, 1), as given by Equation 3.15. The bi-sided PSD for a binary FSK signal $PSD_{BFSK}(f)$ can be approximated as two PSDs from the two ASK signals from Equation 3.18, as given by Equation 3.26.

$$
\begin{aligned}
&PSD_{BFSK}(f) = \\
&\frac{A^2T_b}{16R_L}\operatorname{sinc}^2\left(2\pi(T_b/2)(f-f_c-\Delta f)\right)+\frac{A^2}{16R_L}\delta(f-f_c-\Delta f)+ \\
&\frac{A^2T_b}{16R_L}\operatorname{sinc}^2\left(2\pi(T_b/2)(f-f_c+\Delta f)\right)+\frac{A^2}{16R_L}\delta(f-f_c+\Delta f)+ \\
&\frac{A^2T_b}{16R_L}\operatorname{sinc}^2\left(2\pi(T_b/2)(f+f_c+\Delta f)\right)+\frac{A^2}{16R_L}\delta(f+f_c+\Delta f)+ \\
&\frac{A^2T_b}{16R_L}\operatorname{sinc}^2\left(2\pi(T_b/2)(f+f_c-\Delta f)\right)+\frac{A^2}{16R_L}\delta(f+f_c-\Delta f)
\end{aligned}
\tag{3.26}
$$

This approach is a useful approximation but not rigorous [Stern04]. A rigorous analysis for the binary FSK PSD would require the addition of the voltage spectral density using complex numbers prior to squaring to obtain the energy and power spectral densities. However, the approximation for the binary FSK PSD in Equation 3.26 is adequate for the analysis here [Lathi09].

The minimum frequency deviation Δf is $1/2T_b = r_b/2$ Hz. With this value of Δf the peak of the PSD of each of the decomposed ASK signals in binary FSK would be at a null of the PSD of the other signal, as given by Equation 3.26. FSK using this value of Δf is known as *minimum frequency shift keying* (MFSK).

In the temporal domain, binary FSK signals with $\Delta f = nr_b/2$ Hz (multiples of the minimum frequency deviation, n is an integer) are *orthogonal* to each other over one bit time T_b. An orthogonal signal o(t) is described by Equation 3.27, which is similar to the correlation receiver process in Equation 3.2.

$$
\begin{aligned}
o(t) &= \int_{(i-1)\,T_b}^{i\,T_b} s_j(t)\,s_k(t)\,\mathrm{d}t = 0 \quad j \neq k \quad (i-1)T_b \leq t \leq i\,T_b \\
o(t) &= \int_{(i-1)\,T_b}^{i\,T_b} s_j(t)\,s_k(t)\,\mathrm{d}t = A_o \quad j = k \quad (i-1)T_b \leq t \leq i\,T_b
\end{aligned}
\tag{3.27}
$$

Since $s_j(t)$ and $s_k(t)$ are sinusoidal signals at a carrier frequency f_c, Equation 3.27 assumes that the ratio of the carrier frequency to the bit rate f_c/r_b is either an integer or much greater than 1. The integration then over one bit time T_b is either for a whole number of cycles of the square of the carrier sinusoid or a whole number of cycles and an insignificant residual term at twice the carrier frequency.

The initial frequency deviation Δf here is $2/T_b = 2r_b$ Hz. Although not the minimum value, this Δf is a multiple (4) of the minimum and the peak of one PSD still would be at a null of the other PSD. This allows the resulting PSD to clearly demonstrate that of the decomposed ASK signals in binary FSK.

Increasing the *Simulation Stop Time* to 5.24288 sec results in 524 288 (2^{19}) samples at a 500 kHz sampling rate and a spectral resolution of approximately 0.95 Hz (the sampling rate divided by the number of samples or 500 kHz/524 288). The estimated, normalized ($R_L = 1\ \Omega$) PSD of the transmitted binary FSK signal without AWGN is shown scaled as a single-sided spectrum in Figure 3.10 in dBm/Hz. The frequency deviation Δf is 2 kHz here.

The carrier components are shown as a spectral impulses ($\delta(f)$) at $f_c \pm \Delta f = 20 \pm 2$ kHz, as given by Equation 3.26 and shown in Figure 3.10. The PSD is obtained by the *Spectrum Scope* block from the *Signal Processing Sinks, Signal Processing Blockset* derived from the *Simulink* model *Fig310.slx*.

There are approximate nulls in the PSD that occur at the bit rate $r_b = 1/T_b$ (1 kb/sec or 1 kHz) and multiples of the bit rate centered about the two decomposed ASK carrier frequencies of $f_c = 20 \pm 2$ kHz due to the sinc^2 term in Equation 3.26. The PSD is not smooth because, although there are a large number of data, the ensemble is still finite, however the spectral envelope clearly demonstrates the sinc^2 behavior. The frequency deviation $\Delta f = 2$ kHz is also a multiple of the bit rate $r_b = 1$ kHz, so that the PSD of the decomposed ASK signals can be demonstrated.

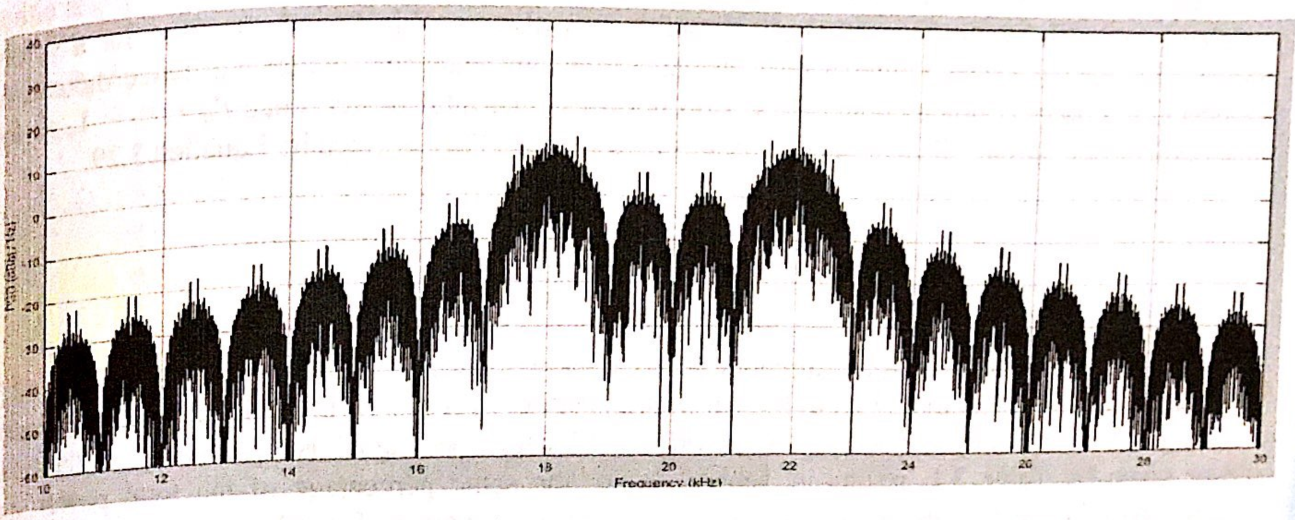

Figure 3.10 Scaled power spectral density of a 1 kb/sec random binary FSK signal, $f_c = 20$ kHz, $\Delta f = \pm 2$ kHz, $r_b = 1$ kb/sec (*Fig310.slx*).

Minimum frequency shift keying (MFSK) occurs when $\Delta f = 1/2T_b = r_b/2$ Hz. Setting the modulation gain $k_f = 500$ Hz/V results in $\Delta f = \pm 1$ V $\times$ 500 = $\pm$ 500 Hz, since $r_b/2 = 500$ Hz. The resulting normalized PSD is shown scaled in dBm/Hz in Figure 3.11.

The approximate nulls in the PSD remain at the bit rate r_b and multiples of the bit rate centered about the two decomposed ASK carrier frequencies due to the sinc^2 term in Equation 3.26. The carrier components are shown as a spectral impulses ($\delta(f)$) at $f_c \pm \Delta f = 20 \pm 0.5$ kHz, as given by Equation 3.26 and shown in Figure 3.11. This carrier frequency separation is the minimum possible, since each carrier spectral impulse is at the null of the PSD of the other decomposed ASK signal.

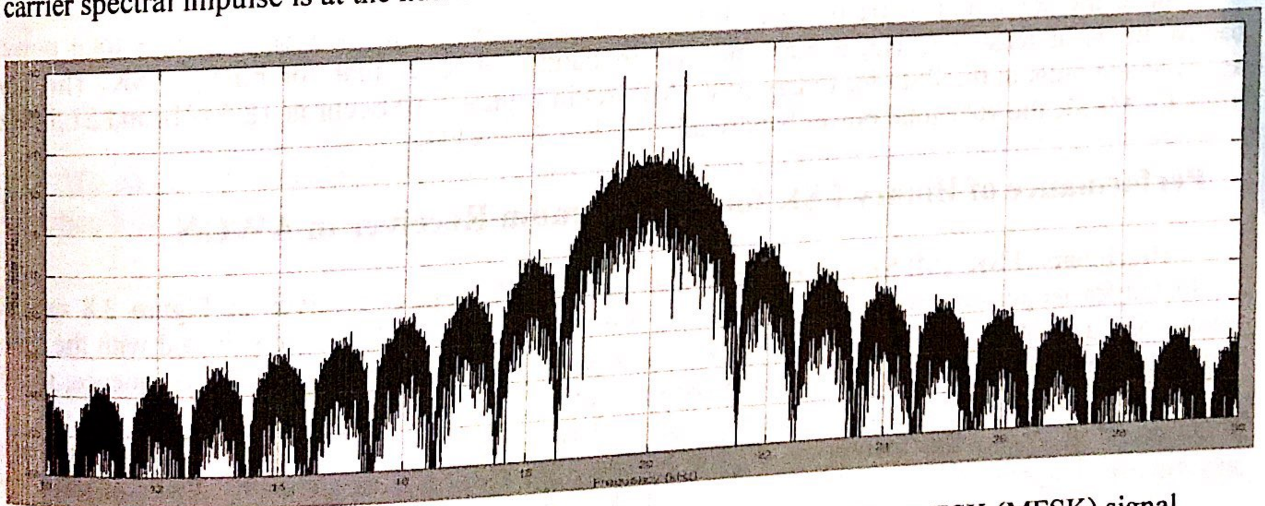

Figure 3.11 Scaled power spectral density of a 1 kb/sec random binary FSK (MFSK) signal, $f_c = 20$ kHz, $\Delta f = \pm$ 500 Hz, $r_b = 1$ kb/sec (*Fig310.slx*).

The bandwidth of the binary FSK signal is problematical, as it is for binary ASK. Defining the bandwidth as a percentage of the total power of the binary FSK signal again is again usual course. The total power is easily calculated in the temporal domain. The power P in the binary FSK signal per bit time T_b, derived from Equation 3.21 and Equation 3.22, is given by Equation 3.28.

$$P = \frac{A^2}{T_b R_L} \int_{(i-1)T_b}^{iT_b} \sin^2(2\pi(f_c \pm \Delta f)t)\,dt = \frac{A^2}{2R_L} \quad (i-1)T_b \le t \le iT_b \qquad (3.28)$$

Equation 3.28 assumes that the ratio of the carrier frequencies to the bit rate $(f_c \pm \Delta f)/r_b$ is either an integer or much greater than 1. The integration then over one bit time T_b is either for a whole number of cycles or a whole number of cycles and an insignificant residual term at twice the carrier frequency. The *apriori* probabilities (P_0, P_1) do not affect the total power since $P_0 + P_1 = 1$, as for binary ASK in Equation 3.20, and the total power P_T for binary FSK is given by Equation 3.29.

$$P_T = \frac{A^2}{2R_L} P_1 + \frac{A^2}{2R_L} P_0 = \frac{A^2}{2R_L} \qquad (3.29)$$

The binary FSK signal has an amplitude $A = 5$ V and the normalized ($R_L = 1\ \Omega$) power $P_T = 12.5$ W = 40.97 dBm (10 log$_{10}$[12.5/0.001]). The decomposition of a binary FSK signal into two binary ASK signals separated by $2\Delta f$, with the bandwidth of a binary ASK signal as a percentage of the total power given by Table 3.1, results in Table 3.3. For the same percentage of the total power the bandwidth of the binary FSK signal is greater than that for the binary ASK (OOK) signal by $2\Delta f$ Hz.

Table 3.3 Bandwidth of a binary FSK signal as a percentage of the total power.

Bandwidth (Hz)	Percentage of Total Power
$2\Delta f + 2/T_b$	90%
$2\Delta f + 3/T_b$	93%
$2\Delta f + 4/T_b$	95%
$2\Delta f + 6/T_b$	96.5%
$2\Delta f + 8/T_b$	97.5%
$2\Delta f + 10/T_b$	98%

Since the minimum value of Δf is $1/2T_b$ Hz in FSK (MFSK), the resulting 90% total power bandwidth is at least $3/T_b$ Hz, a 50% increase in bandwidth over that for binary ASK. The first approximate nulls in the PSD for binary FSK (MFSK) in Figure 3.10 occur at 18.5 kHz and 21.5 kHz. Thus for MFSK the 90% total power bandwidth here is $3/T_b = 3$ kHz.

Performance of Binary FSK for the Optimum Receiver in AWGN

The binary FSK (MFSK) transmitter and optimum bandpass receiver in Figure 3.8 can be evaluated for its performance in the presence of AWGN. A BER analysis is conducted with the same parameters as that for binary ASK. The *Simulation Stop Time* is set to 10 sec for convenience, which would process 10 000 information bits in possible error.

Table 3.4 is a tabular list of the observed BER in a single trial of 10 000 information bits as a function of E_d/N_o to the theoretical P_b for binary FSK (MFSK), as given by Equation 3.11 for asymmetrical signals. Binary FSK does not satisfy Equation 3.1 and is asymmetric.

The *AWGN* block from the *Channels, Communications Blockset* facilitates the BER analysis with the input of a desired E_d/N_o dB directly. The normalized energy difference per bit E_d is determined by the number of bits per symbol (1), the symbol period ($T_b = 1$ msec) and the normalized ($R_L = 1\ \Omega$) input signal power (P = 25 W) which is used to set the E_d/N_o ratio in the *AWGN* block. However, the *AWGN* block parameter is labeled E_b rather than E_d.

The normalized energy difference per bit here is $E_d \approx 2.5 \times 10^{-2}$ V^2-sec, as given by Equation 3.10 and Equation 3.23. The attenuation of the communication channel is taken to be zero ($\gamma = 1$). A