5.4 Suggested Problems

In Problems 1-8 determine whether the given vectors are linearly dependent or linearly independent. If they are linearly dependent, find a nontrivial linear combination of the vectors that has sum **0**.

1. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

2. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

4. $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$

5. $\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

6. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -7 \\ 2 \\ 11 \end{bmatrix}$

7. $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

8. $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -2 \\ 4 \end{bmatrix}$

9. If **a**, **b**, **c** are three vectors in 3-space, their scalar triple product is $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$. Use the standard volume interpretation of the scalar triple product to give a geometric justification of Theorem 1 when $n = 3$.

10. Let
$$\mathbf{a} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}.$$

    (a) Give a geometric argument that shows that the vectors are linearly dependent in $\mathbb{R}^2$.

    (b) Give an algebraic argument that shows the vectors are linearly dependent in $\mathbb{R}^2$.

11. Let

$$\mathbf{v}_1 = \begin{bmatrix} a \\ c \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} b \\ d \end{bmatrix}, \text{ and } \quad \mathbf{v}_3 = \begin{bmatrix} e \\ f \end{bmatrix}$$

be vectors in in $\mathbb{R}^2$.

(a) Give a geometric argument that shows that the vectors are linearly dependent in $\mathbb{R}^2$.

(b) Give an algebraic argument that shows the vectors are linearly dependent in $\mathbb{R}^2$. *Hint.* First express

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{0}$$

in the form $A\mathbf{c} = \mathbf{b}$ where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

What is $\mathbf{b}$? Now use what you know about solvability of linear systems.