Simon Fraser University
Department of Economics
ECON 302

9. Moral Hazard and Explicit Contracting

9.1 Hidden Action

Hidden action problem, often called moral hazard, happen when one party in a transaction can undertake action that are not observable by other agent(s) involved in the transaction. The moral hazard problem happen agents involved in the transaction don't all have the same desired behaviours.

As oppose the implicit contracting environment we studied before, actions by an agent are not observable, even with delay. However, those actions are correlated with observable outcomes and can be contracted on with the help of a third party like the courts.

Examples:

- Lawyers and clients; lawyer's effort is not observable, but winning is.
- Managers and a firm: effort is not observable, but sales or profits are.
- Insured agents and insurance providers: effort at lowering risk is not observable, but accidents are.
- Entrepreneurs and investors: effort or risk taking is not observable, but profits are.
- Politician and Voters: effort is not observable, but economic measures are (unemployment rate, GDP).
- Renter and capital owner: care is not observable; depreciation is.
- Contractors and clients: effort is not observable, but outcomes are.

9.2 Principal Agent Model

The principal agent model describes contracts that take place when moral hazard problem are present.

The principal

- The principal own some technology, or capital, or is a client.
- Must hire or interact with an agent who will perform tasks.
- Examples: Clients; Firms; Insurance providers; Investor; Voters.
- Is assumed to be risk neutral.

Two outcomes model:

- Q^s is payoff for the principal if success.
- Q^f is payoff for the principal if failure

The Agent

- The agent can perform tasks that the principal cannot (or don't want to)
- Examples: Lawyers; Managers; Insured; Renter; Entrepreneurs; Politicians.
- Is assumed to be risk neutral or risk averse.
- We start with risk neutral Agent.

Two effort model:

- $e = 1$ is high effort by the agent and cost c.
- $e = 0$ is low effort by the agent and cost zero.

- Call p_1 the probability of success if $e = 1$.
- Call $p_0 < p_1$ the probability of success if $e = 0$.

The big question: When effort is desirable, how do the principal make sure the agent picks $e = 1$?

Effort is desirable if:

$$\underbrace{p_1 Q^s + (1 - p_1)Q^f - c}_{\text{if } e=1} \geq \underbrace{p_0 Q^s + (1 - p_0)Q^f}_{\text{if } e=0}$$

.

$$[p_1 - p_0][Q^s - Q^f] \geq c.$$

MC OF EFFORT

INCREASE IN PROBABILITY OF SUCCESS IF $e=1$

INCREASE IN PAYOFF IS S INSTEAD OF F

MB EFFORT

Fixed wages or payment

Imagine the Principal was to pay the agent w no matter what happen.

Agent's payoff depending of effort:

- $w - c$ if pick $e = 1$.
- w if pick $e = 0$.

Obviously the agent will pick $e = 0$.

Full Delegation Solution

The Principal give Q^s or Q^f away to the agent in exchange of a fee R.

Agent's payoff depending of effort:

- $p_1 Q^s + (1-p_1)Q^f - c - R$ if pick $e = 1$.
- $p_0 Q^s + (1-p_0)Q^f - R$ if pick $e = 0$.

The agent will pick $e = 1$ (incentive compatibility constraint) if:

$$p_1 Q^s + (1-p_1)Q^f - c - R \geq p_0 Q^s + (1-p_0)Q^f - R;$$

$$[p_1 - p_0][Q^s - Q^f] \geq c.$$

MBe > MCe

- The Agent get the full return and bear the cost;
- The Agent then makes the right decision, $e = 1$;
- The Principal can extract the surplus via R;
- The agent will accept any take-or-leave it offer where (participation constraint)

$$p_1 Q^s + (1-p_1)Q^f - c - R \geq 0.$$

$$R = p_1 Q^s + (1-p_1)Q^f - c.$$

· IF AGENT HAS AN OUTSIDE OPTION $\bar{U}$ THEN REPLACE 0 WITH $\bar{U}$

Full Delegation Solution

Advantages:

- When possible works great because incentives are aligned.
- Simple to setup and enforced.
- Works even if Q^s and Q^f are not observable to a third party (court).

Limitation:

- Not possible when Q^s and Q^f are non-monetary: criminal charges;
- Note that the Agent bears all the risk;
- If the agent is risk averse, the principal will have to pay a risk premium;
- Participation constraint with risk averse Agent:

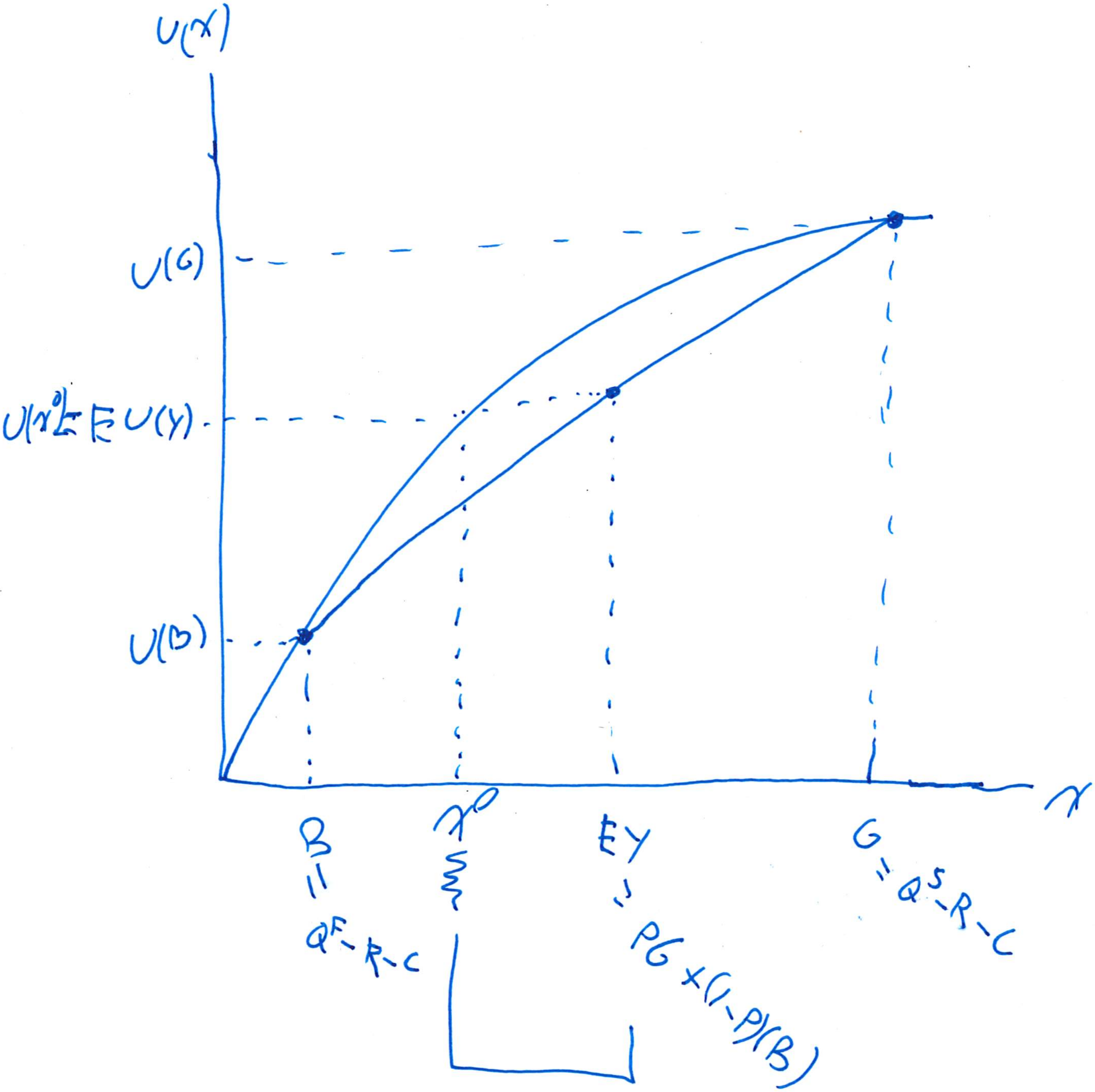
$$p_1 u(Q^s - R - c) + (1 - p_1)u(Q^f - R - c) \geq 0.$$

- With concave $u(Q)$: we have that

$$u(x^e) = p_1 u(Q^s - R - c) + (1 - p_1)u(Q^f - R - c);$$

- The certain equivalent is $x^e < p_1 Q^s + (1 - p_1)Q^f - R - c$
- Conclusion, could pay the Agent less if there was less risk.

$$R < \underbrace{p_1 Q^s + (1-p_1)Q^f} - c - x^e$$



$E(Y) > x^0$

Pay According to Outcome Solution

The Principal set up a contract, where the payments are:

- w^s if Q^s;
- w^f if Q^f;

Agent's payoff depending of effort (risk neutral):

- $p_1 w^s + (1 - p_1) w^f - c$ if pick $e = 1$.
- $p_0 w^s + (1 - p_0) w^f$ if pick $e = 0$.

The agent will pick $e = 1$ (incentive compatibility constraint) if:

$$p_1 w^s + (1 - p_1) w^f - c \geq p_0 w^s + (1 - p_0) w^f$$

$$\underbrace{[p_1 - p_0]}_{\text{increase in probability of ws}}\underbrace{[w^s - w^f]}_{\text{increase in wage}} \geq c. \quad \leftarrow \text{MCe}$$

MBe

- The agent will accept any take-or-leave it offer where (participation constraint)

$$p_1 w^s + (1 - p_1) w^f - c \geq 0.$$

$$p_1 w^s + (1 - p_1) w^f = c.$$

	$e=1$	$e=0$
Q^S	$P_S =$ BIG	P_S : SMALL
Q^M	$P_M = \phi$	P_M BIG
Q^F	$P_F =$ SMALL	$P_F = \phi$

Pay According to Outcome Solution

Properties:

- The Principal pay more when success (some lawyers get paid only if wins).
- It is not because they deserve it, it is because it gives the incentives to pick $e = 1$.

Risk aversion:

- The principal still have to pay a risk premium;
- Participation constraint with risk averse Agent:

$$p_1 u(w^s - c) + (1 - p_1)u(w^f - c) \geq 0.$$

- With concave $u(Q)$: we have that

$$u(x^e) = p_1 u(w^s - c) + (1 - p_1)u(w^f - c);$$

- The certain equivalent is $x^e < p_1 w^s + (1 - p_1)w^f - c$;
- The risk premium grows as $w^s - w^f$ grows;
- With risk averse agent, the principal will want to minimizes the risk so the incentive compatibility will be satisfied with equality:

$$p_1 u(w^s - c) + (1 - p_1)u(w^f - c) = p_0 u(w^s \quad) + (1 - p_0)u(w^f \quad);$$

9.3 Limited liability contract

The legal protection available to the shareholders of privately and publicly owned corporations under which the financial liability of each shareholder for the company's debts and obligations is limited to the par value of his or her fully paid-up shares.

- Limited Liability Corporation (LLC): owner(s) is only liable up the the amount invested or earned.

- Sole Proprietor: Liable for all dept.

Limited liability creates moral hazard problems.

Model with Borrowing

An entrepreneur can start a project:

- Investment Needed is K;
- Entrepreneur has no liquid capital;
- Entrepreneur is risk neutral;
- Output depends on effort:
  - Q^s with probability $p(e)$
  - Q^f with probability $1 - p(e)$
  - Probability $p(e)$ is such that $p'(e) > 0$ and $p''(e) < 0$.
- Cost of effort is ce
- Interest rate is r
- Assumption:
  - $Q^s > (1+r)K$;
  - $Q^f < (1+r)K$;

Sole Proprietor

Imagine the entrepreneur is a Sole Proprietor. She borrow K to start the project. If default, the entrepreneur needs to liquidate $(1 + r)K - Q^f$ of non-liquid assets (house, car) to pay the full loan back.

Choice of effort

- Expected payoff for the entrepreneur:

$$E[\pi] = p(e)[Q^s - (1 + r)K] + [1 - p(e)][Q^f - (1 + r)K] - ce.$$

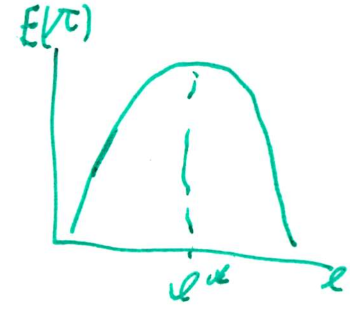
$$E[\pi] = p(e)Q^s + [1 - p(e)]Q^f - (1 + r)K - ce.$$

The entrepreneur will choose the effort that maximizes expected payoff $E[\pi]$. The equilibrium effort e^* is given by the FOC:

$$\underbrace{p'(e^*)}_{\downarrow}\underbrace{[Q^s - Q^f]}_{\text{GAIN FROM SUCCESS}} = c. \quad \leftarrow \text{MC } e$$

$\underbrace{\text{INCREASE IN}}_{\text{MB } e}$ PROB OF SUCCESS WITH e

The investor gets repaid with probability $p(e^*)$



Example: $p(e) = \ln e$.

- Effort is given by $e^* = \frac{Q^s - Q^f}{c}$;
- Investor SUCCESS probability $p(e^*) = \ln\left(\frac{Q^s - Q^f}{c}\right)$.

Limited Liability Corporation

Imagine the entrepreneur is a Limited Liability Corporation. She borrow K to start the project. In case of default, the entrepreneur only loses Q^f.

Choice of effort

- Expected payoff for the entrepreneur:

$$E[\pi] = p(e)[Q^s - (1+r)K] + [1-p(e)]0 - ce.$$

The entrepreneur will choose the effort that maximizes expected payoff $E[\pi]$. The equilibrium effort e^ℓ is given by the FOC:

$$p'(e^\ell)[\underbrace{Q^s - (1+r)K}_{\text{RETURN OF SUCCESS}}] = c. \quad \text{EMC}$$

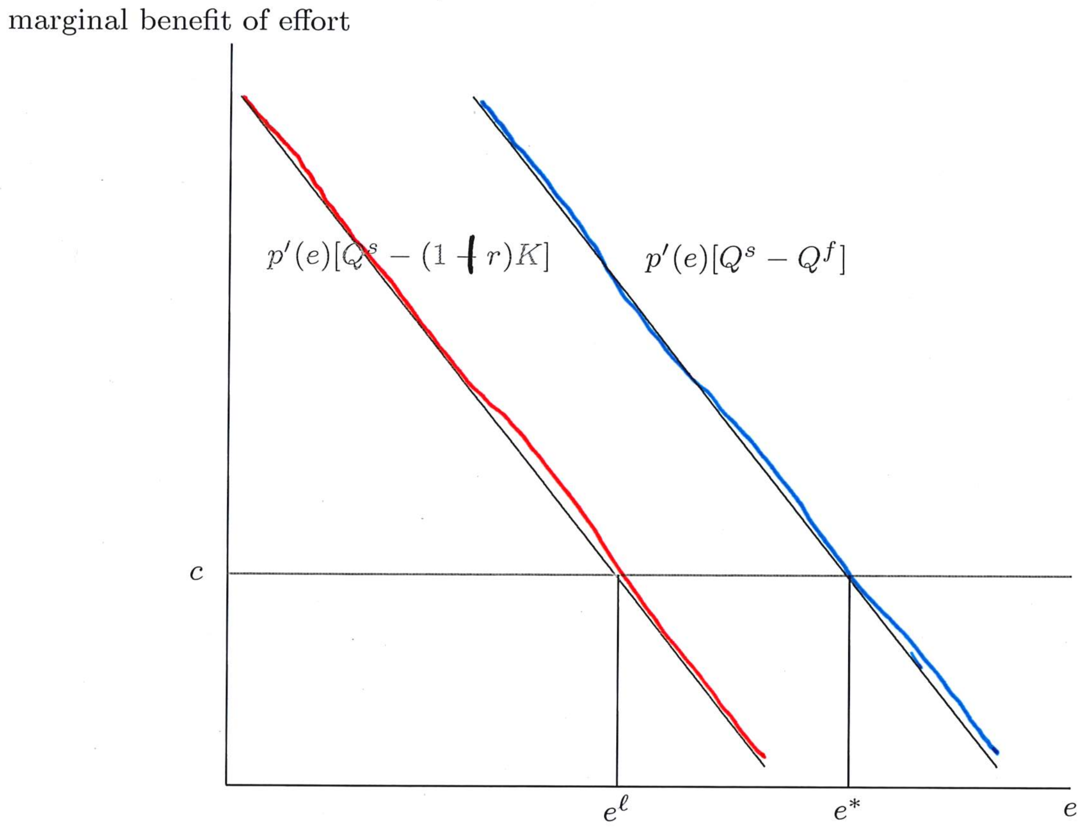
↑ INCREASE IN PROBABILITY

MBe

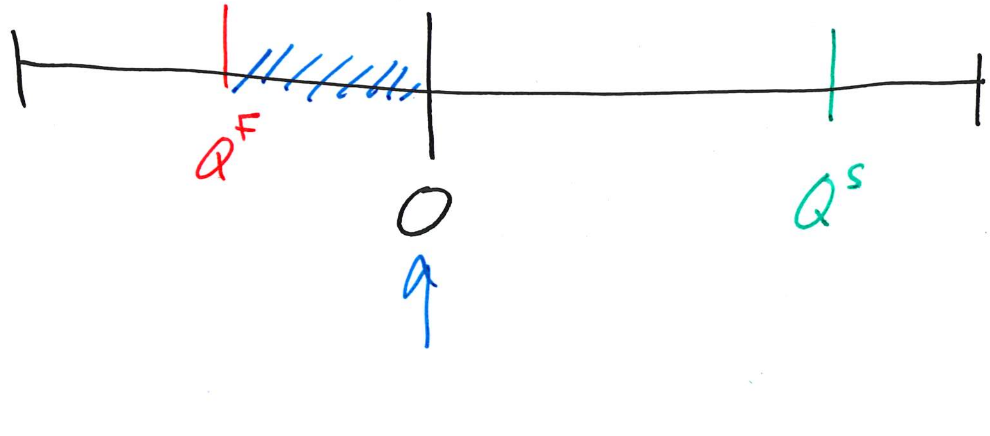
Effort is lower because marginal benefit of effort is lower $[Q^s - (1+r)K] < [Q^s - Q^f]$. The investor gets repaid with a lower probability $p(e^\ell)$

Example: $p(e) = \ln e$.

- Effort is given by $e^\ell = \frac{Q^s - (1+r)K}{c}$;
- Investor is repaid with probability $p(e^\ell) = \ln\left(\frac{Q^s-(1+r)K}{c}\right)$.



marginal benefit of effort
$p'(e)[Q^s - (1+r)K]$
$p'(e)[Q^s - Q^f]$
c
e^ℓ
e^*
e



Q^F
0
q
Q^S

Collateral

The investor can ask for a collateral to the entrepreneur of a Limited Liability Corporation. Imagine collateral X that must be given in case of default.

Choice of effort

- Expected payoff for the entrepreneur:

$$E[\pi] = p(e)[Q^s - (1+r)K] + [1 - p(e)][0 - X] - ce.$$

The entrepreneur will choose the effort that maximizes expected payoff $E[\pi]$. The equilibrium effort e^x is given by the FOC:

$$p'(e^x)[Q^s - (1+r)K + X] = c.$$

Effort is increasing with the collateral. If X is such that $Q^s - (1+r)K + X = Q^s - Q^f$. then $e^x = e^*$. The investor gets repaid with a lower probability $p(e^*)$

Example: $p(e) = \ln e$.

- Effort is given by $e^x = \frac{Q^s-(1+r)K+X}{c}$;
- Investor is repaid with probability $p(e^x) = \ln\left(\frac{Q^s-(1+r)K+X}{c}\right)$.
- If $X = (1+r)K - Q^f$ then $e^x = e^*$.