

(40) 1. A negative unity feedback system has the following plant:

$$G(s) = \frac{K}{s(s + 5)(s + 20)}$$

Design a first order lead compensator such that the damping ratio ζ is equal to 0.5 for a unit step input, and the settling time is less than or equal to 2/3 second. Also determine the value of the error constant. Use the root locus method.

- a) Due only one iteration.
- b) Show all design steps by hand
- c) Use MATLAB to plot the root locus for both the uncompensated and compensated systems
- d) Use MATLAB to plot the step response for the compensated system. Check the resulting values for P.O. and T_s .
- e) Include all MATLAB scripts.

(30) 2. Consider the system specified by:

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \underline{u}(t)$$

$$\underline{y}(t) = [1 \ 2] \underline{x}(t)$$

- a) Determine the state controllability and observability.
- b) Place the observer poles at $-2 \pm j4$. Use Ackermann's formula.

(30) 3. Determine the feedback gain $k > 1$ that minimizes the performance index

$$J = \int_0^{\infty} [x^2(t) + u^2(t)] dt$$

for

$$\dot{x}(t) = x(t) + u(t), \quad x(0) = \sqrt{2}$$

where

$$u(t) = -k x(t).$$

Show all steps. What is the value of J_{\min} ?