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1. (10 points) For the following problems, you must tell **WHY** (you can tell quickly that) the set $\mathcal{S}$ is not a basis of the given $\mathbb{R}^n$ space.

   (a) (4 points)

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 9 \\ -11 \\ 13 \\ 22 \\ -1 \end{bmatrix}, \begin{bmatrix} 19 \\ 31 \\ 13 \\ 12 \\ 3 \end{bmatrix} \right\} \subset \mathbb{R}^5$$

   (b) (4 points)

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ -1 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ -11 \\ 4 \\ 1 \\ 9 \end{bmatrix}, \begin{bmatrix} 9 \\ -11 \\ 13 \\ 22 \\ -1 \end{bmatrix}, \begin{bmatrix} 19 \\ 31 \\ 13 \\ 12 \\ 3 \end{bmatrix} \right\} \subset \mathbb{R}^5$$

   (c) (3 points)

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\} \subset \mathbb{R}^2$$

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2. (15 points) • (7 points) For what values of c is the following matrix **NOT** invertible (in other words, it is not injective nor surjective).

$$A = \begin{bmatrix} 2-c & 1 & 1 & 1 \\ 2-c & 2-c & c & c \\ 0 & 0 & 3+c & 0 \\ 0 & 0 & 0 & c-1 \end{bmatrix}$$

• Given the matrix

$$A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & 1 & -2 \\ -1 & 0 & 4 \end{bmatrix}$$

Determine

(a) (2 points) Dimension of $Col(A) =$

(b) (2 points) Dimension of $Null(A) =$

(c) (2 points) Dimension of $Row(A) =$

(d) (2 points) Dimension of $Null(A^T) =$

Do not forget to show your work!

3. (15 points) (a) (10 points) Find a basis for the subspace U of $\mathbb{R}^4$ (given below)

$$U = \left\{ \begin{bmatrix} r+s \\ r-s \\ 3r+2s \\ r \end{bmatrix} \in \mathbb{R}^4 | r, s \in \mathbb{R} \right\}.$$

Do NOT forget to show that the basis you found is indeed a basis!!! (I mean, stating the set without proving it is a basis of U has no credit. Thus, as any problem in this exam, prove your claim).

(b) (5 points) Prove or give a counterexample: If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a basis of V and U is a subspace of V such that $\mathbf{v}_1, \mathbf{v}_2 \in U$ but $\mathbf{v}_3, \mathbf{v}_4 \notin U$, then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis of U.

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4. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 + 2x_3 \\ -x_1 + 2x_2 - 3x_3 \\ 2x_1 + x_3 \end{bmatrix}.$$

Show that T is one-to-one and that T is surjective. Is T an isomorphism?

5. (10 points) (Extra-credit) This problem is extra-credit for the first attempt of this midterm but an obligatory one for the reviewed version (the 2nd attempt).
Suppose $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_m\}$ is a linearly dependent set of vectors in V (where V is a vector space). Suppose also that W is a non-zero vector space. Prove that there exists $\{\mathbf{w}_1, \mathbf{w}_2, ... \mathbf{w}_m\} \subset W$ such that no $T \in \mathcal{L}(V, W)$ satisfies $T(\mathbf{v}_k) = \mathbf{w}_k$ for each $k = 1, 2, ..., m$.