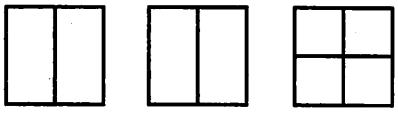
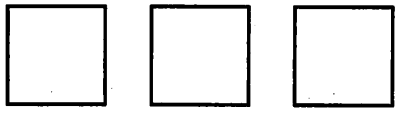
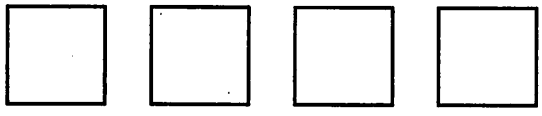
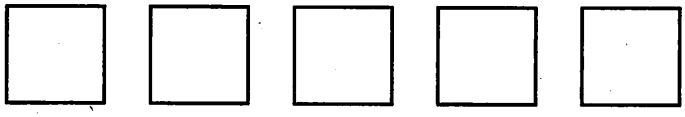
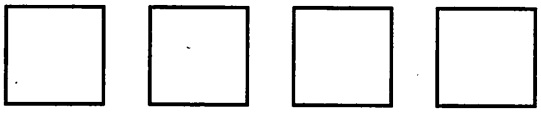
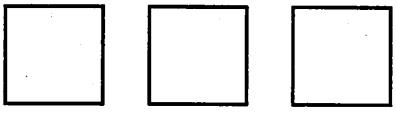
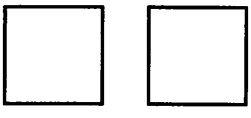
EXPLORATION 4.6 Sharing Brownies

A powerful way to have children explore fraction ideas is to pose problems involving sharing. Problems like these have been found in many articles in *Teaching Children Mathematics,* other journals, and elementary school textbooks. One interesting historical note is that many children's strategies mirror the ancient Egyptians' approach toward fractions, which is discussed in the text. For example, when children are asked how four people could share three brownies, a common solution path looks like the illustration below. Rather than give each person $\frac{1}{4}$ of a brownie, they draw solutions where the pieces will be as big as possible. That is, they prefer the solution in which each person gets $\frac{1}{2}$ of one brownie and $\frac{1}{4}$ of another to the solution of each person getting three $\frac{1}{4}$ brownies. Please stay within this mindset as you solve the following problems.



In each case, determine how to divide the brownies and write the amount each child will get. Do not add the amounts together. For example, in the problem above, the solution is $\frac{1}{2} + \frac{1}{4}$.

Brownies	Children	
3	6	
4	6	
5	6	
4	5	
3	5	
2	5	

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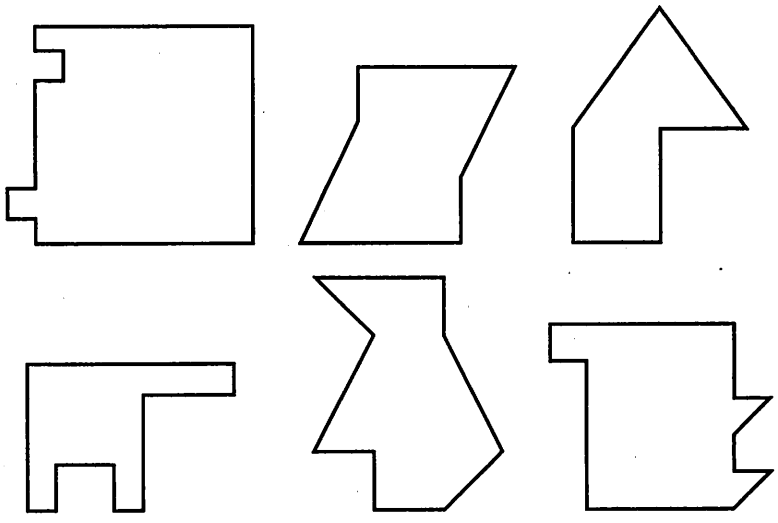
EXPLORATION 4.7 Partitioning

PART 1: Halves

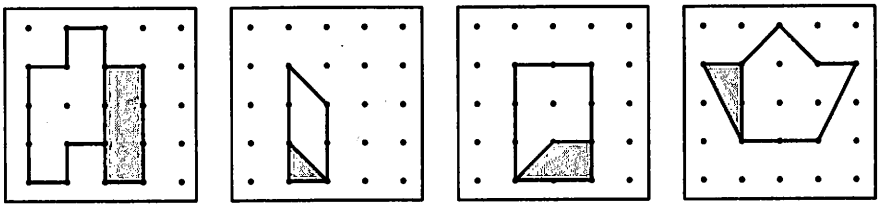
These problems are similar to ones in the fourth-grade book of an elementary textbook series! Because the figures are not simple polygons, they require the learner to apply various fraction ideas to answer the questions. In each case, shade $\frac{1}{2}$ of the polygon.



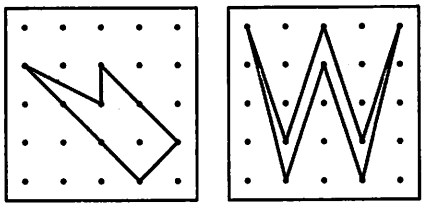
PART 2: Determining parts on a square Geoboard

These kinds of problems are also found in elementary schools and bring out the relationship between fractions and measurement. *Hint:* What do you remember about finding the area of triangles?

1. Determine the part of the whole polygon that is shaded.

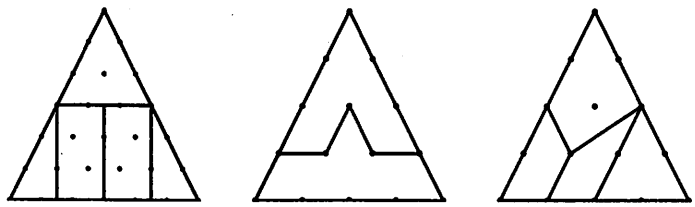


2. What fraction of the whole Geoboard (16 squares) is represented by each of the polygons?



PART 3: Determining parts on a triangular Geoboard

In these problems, the unit of measurement is not a square but an equilateral triangle. Determine what fraction of the whole triangle is represented by each part.

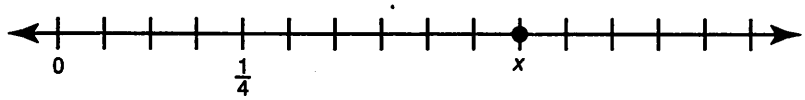


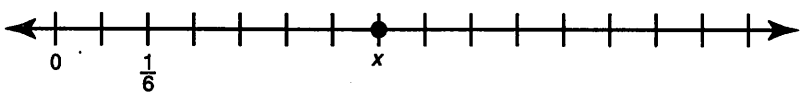
PART 4: Determining fractions of a thermometer

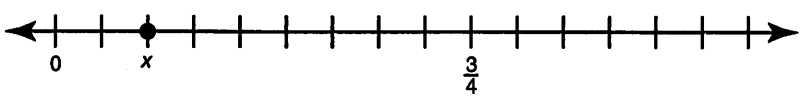
Look at the thermometers on page 85. In each case, determine the fraction of the thermometer that is shaded in two different ways. They are on a separate page so that you can cut them out and fold them if you wish.

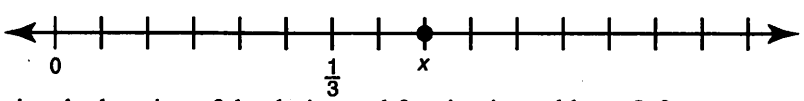
PART 5: Determining fractions on a number line

Determine the value of x in problems 1–4.

1. 

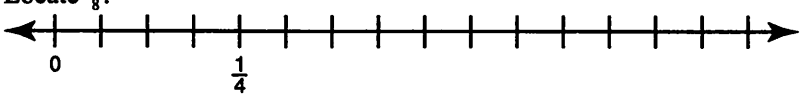
2. 

3. 

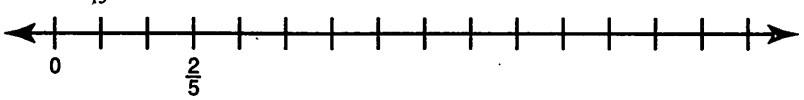
4. 

Determine the location of the designated fraction in problems 5–8.

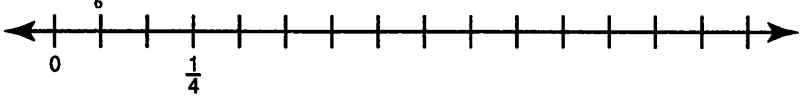
Locate $\frac{5}{8}$.

5. 

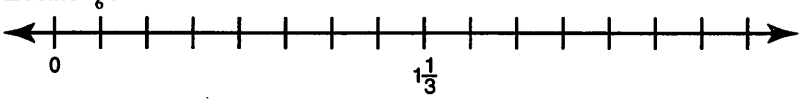
Locate $\frac{7}{15}$.

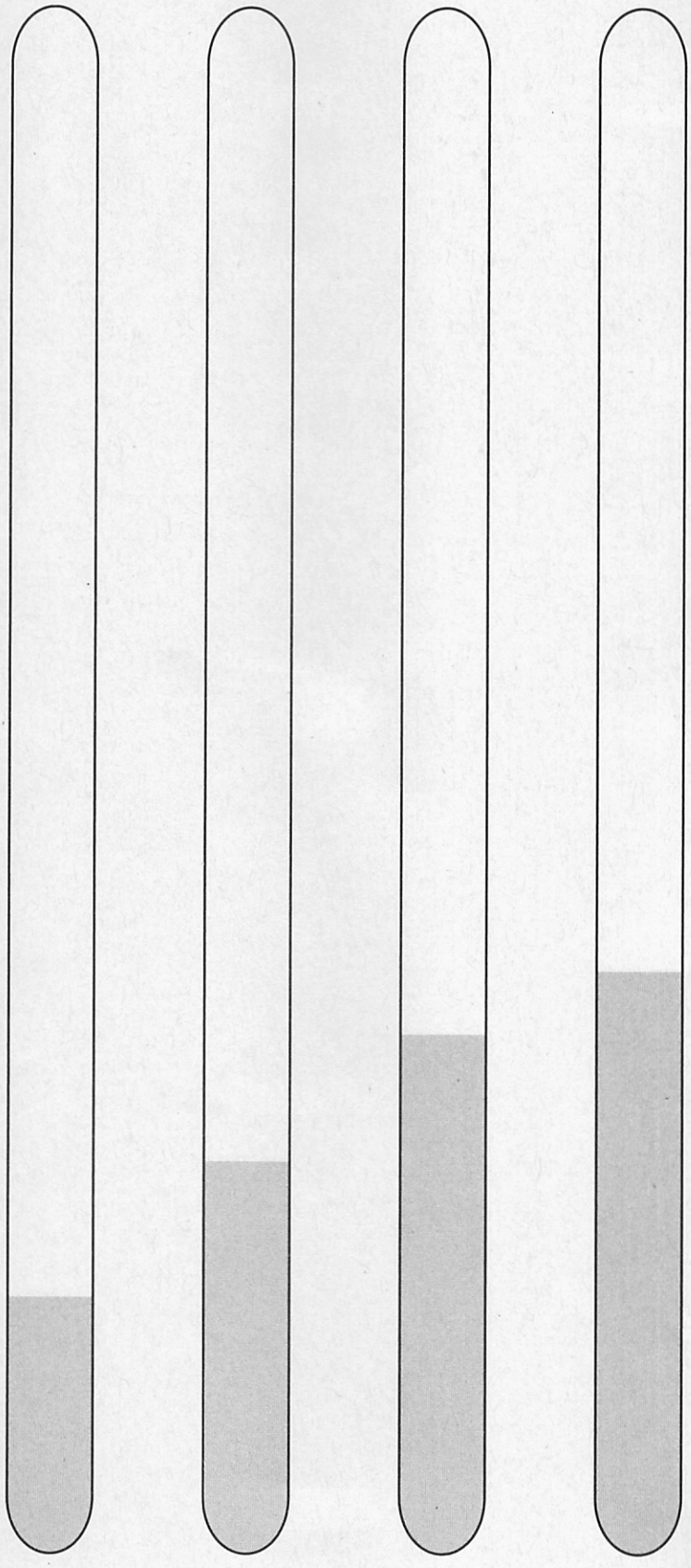
6. 

Locate $\frac{5}{6}$.

7. 

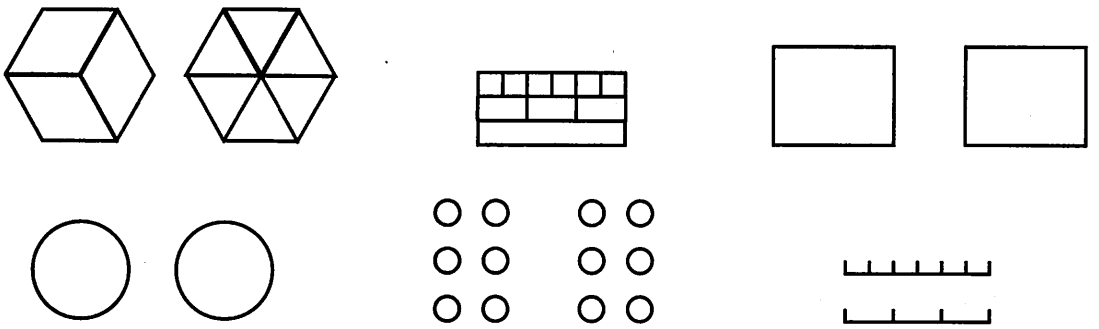
Locate $\frac{5}{6}$.

8. 



EXPLORATION 4.8 Equivalent Fractions

Your instructor will have some of the following manipulatives and concrete materials for you to use: Pattern Blocks, Cuisenaire Rods, Geoboards, circles, discrete models, number line, paper (for folding), etc.



PART 1: Determining equivalence

1. Use one or more of these models to demonstrate the equivalence of $\frac{2}{3}$ and $\frac{4}{6}$.
2. Use one or more of these models to demonstrate the equivalence of $\frac{2}{3}$ and $\frac{8}{12}$.
3. Use one or more of these models to demonstrate the equivalence of $\frac{3}{4}$ and $\frac{6}{8}$.
4. Use one or more of these models to demonstrate the equivalence of $\frac{3}{4}$ and $\frac{9}{12}$.

PART 2: Extending equivalence beyond proper fractions

1. How would you demonstrate that $\frac{6}{6}$ is equivalent to 1?
2. How would you demonstrate that $3\frac{3}{8}$ is equivalent to $\frac{27}{8}$?
4. Are $\frac{3\frac{1}{2}}{7}$ and $\frac{1}{2}$ equivalent?
5. Are $\frac{0}{3}$ and $\frac{0}{8}$ equivalent? Why or why not?

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EXPLORATION 4.9 Developing Fraction Sense

After each part, compare your responses and your justification with those of your partner(s). As before, if you agree, listen to one another's justifications. If you disagree, discuss and debate until you reach agreement.

PART 1: Naming fractions[2]

For each step below, (a) give your answer, (b) explain your thinking (that is, how you came up with your answer), and (c) justify your response (that is, why you believe it is correct).

1. a. Name a fraction between $\frac{1}{6}$ and $\frac{2}{6}$. b. Name another one. c. Is $\frac{1\frac{1}{2}}{6}$ between $\frac{1}{6}$ and $\frac{2}{6}$?
2. a. Name a fraction between $\frac{9}{10}$ and 1. b. Name another one.
3. a. Name a fraction between $\frac{3}{5}$ and $\frac{3}{4}$. b. Is $\frac{3}{4\frac{1}{2}}$ between $\frac{3}{5}$ and $\frac{3}{4}$?
4. Name a fraction that is very close to 1. Now name a fraction that is even closer to 1.
5. Give a value of x that makes the following statement true: $0 < \frac{4}{x} < \frac{1}{10}$.
6. Name a fraction between 0 and $\frac{1}{10}$ that does not have a numerator of 1.

PART 2: Fraction benchmarks[3]

1. "Wall problems" are used by some elementary teachers both as a pre-assessment and to begin a rich conversation about a topic. Do this wall problem, which is useful for beginning our work on fraction sense: Write on the wall (chalkboard or poster paper) everything you know about $\frac{2}{5}$ for example, what it looks like, how large it is, what other fractions or numbers it is close to, etc.
2. a. Place each of the fractions below in one of three groups: closer to 0, closer to $\frac{1}{2}$, or closer to 1. Briefly justify your choice.

$$\frac{3}{8} \quad \frac{2}{7} \quad \frac{1}{3} \quad \frac{21}{30} \quad \frac{4}{5} \quad \frac{7}{11} \quad \frac{31}{181}$$

3. What happens to the value of $\frac{5}{9}$ if
   a. the value of the numerator is increased by 1?
   b. the value of the denominator is increased by 1?
   c. the value of the denominator is decreased by 1?
   d. the value of the numerator and denominator are both increased by 2?
   e. the value of the numerator and denominator are both multiplied by 2?
   f. the value of the numerator is increased by 1 and the value of the denominator is decreased by 1?